# THE WORLD-TIME PARALLEL

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# *Preface*

The emergence of possible-worlds semantics for modal logic in the late 1950s and early 1960s led to a recognition that the structure of modal and temporal logic can be treated in an exactly parallel way, with 'possible worlds' in the one case playing the same role as moments of time in the other. While this has been known now for many years there has been considerable reluctance among philosophers to ask why it should be so, and to embrace its consequences. Most of those who have written on the topic have had the aim of attempting to explain why the formal parallel has little philosophical significance. The present volume is, we believe, the first book-length work to address the phenomenon explicitly and present the case for its power.

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# *Introduction*

*Bird flu is feared, not for what it is, but for what it might become.*
2006 news headline

Today is sunny at Waitarere Beach. The sea is sparkling, the sky is blue, the air is calm and it feels good to be alive.

(1) But it wasn't sunny yesterday. Yesterday was grey, wet and windy, and we were depressed.

(2) But it didn't *have* to be sunny today. It might have been grey, wet and windy, and we would have been depressed.

What have (1) and (2) in common? Begin with (1). Suppose that today is Tuesday, meaning obviously some particular Tuesday. Then (1) is true on Tuesday because there was rain (and it was grey and windy and so on ...) on Monday. That seems like common sense. It also seems common sense that rain is something that can occur on one day — Monday — and fail to occur on another day — Tuesday. What then should be said about (2)? We shall assume in this book that sentences about what is necessary or about what is possible have literal truth values.[1] Our aim is to introduce you to a way of dealing with (2) which is exactly parallel with the way of dealing with (1) that we have just mentioned. This came about with the advent of the possible-worlds semantics for modal logic — the logic of necessity and possibility — in the early 1960s. Corresponding to the *times* Monday and Tuesday and so on, at which things happen, philosophers began to speak of the *possible worlds* in which they happen — so that (2) is true because in some possible but non-actual world there is rain today (at Waitarere), even though *in the actual world* it is sunny. Where is this world? Well, *where* is yesterday's rain? *Now* it is nowhere, but yesterday it was *here* in Waitarere. Similarly the merely possible rain *would have been here*.

Because times and worlds are often in the domain of metaphysicians, metaphysical intuitions are aroused that make philosophers suspicious of the

[1]Chapter 3 of Lycan 1994 is a sustained demonstration of the ubiquity of modal discourse — of sentences about what is or is not possible or necessary. Not all philosophers might agree on the importance of modality. Some feel that although modality might be a part of natural language it should be banished from austere scientific discourse. Smart 1987, p. 182, says "... I want to keep modality (and possible worlds) out of physical theory...". And some linguists have suggested that a sentence like (2) may not be true or false — but may be used merely to express an attitude.

world-time parallel, or unsure of its real import. For a start, many philosophers don't like the assumption of other possible worlds. One of the earliest was Arthur Prior, who, in Prior 1957, had already advocated a temporal interpretation of the operators of modal logic. Prior saw that you could think of modality in the way in which it is common to think of tense. In Prior 1968c he comments on p. 191:

> I wonder whether anybody wants to put forward anything like the following as a piece of serious metaphysics: There really are such objects as possible worlds, and what we loosely describe as propositions of modal logic are in fact predicates of which these objects are the subjects.

He follows this up on p. 192 with

> ... this seems a tall story, and as I have said, I doubt whether anyone seriously believes it. But plenty of people believe an exactly similar story about tenses, i.e. believe that tensed propositions are predicates of `instants', and that there is — really is — an instant at which I unalterably `am' drinking.

Only a very few years after Prior made these observations David Lewis explicitly referred to this passage, and proclaimed that he was one who did accept that other worlds were equally as real as other times.[2] Prior and Lewis represent extremes in this debate: Prior thinking that neither other times nor other worlds are 'real'; and Lewis thinking that *both* are real. What they share is a belief that the cases are parallel, and it is the parallel itself that the present book will be investigating.

But surely it might be said that it is *obvious* that worlds and times are different — that it is *obvious* that modal and temporal talk have a different ontological status. Consider the execution of Charles I in 1649. This actually happened. It is part of reality. Charles I *was* executed. Some years later Oliver Cromwell was offered the crown. He refused. Cromwell *might have* become king, but he *didn't*. King Oliver is *not* part of reality. Doesn't this show that the whole enterprise of examining the world-time parallel is flawed from the beginning?

Our first comment is this. It is no part of the claim that worlds and times are structurally parallel that temporal talk is *synonymous* with modal talk. Worlds

[2]In footnote 6 on p. 185 of Lewis 1970, Lewis says, "A.N. Prior states the indexical theory of actuality in [Prior 1968c] but, sadly, he goes on to say 'this seems a tall story, and ... I doubt whether anyone seriously believes it.'"

and times are different. Tense is about times, and modality is about worlds. But tense and modality have parallel structures — that is, every temporal locution has a corresponding modal locution, and vice versa.

(3) Charles I was executed

does make a claim about what actually happened, while

(4) Oliver Cromwell might have become king

does not. Put in terms of times and worlds (3) is now true in the actual world because Charles I was executed (in the actual world) at some time *t* which precedes the present.[3] (4) is true because there is a possible but non-actual world in which Cromwell accepted the crown. (3) has a structure

(5) It was so that α

while (4) has a structure

(6) It might have been so that α.

The examples we have chosen here draw out the parallel between tense and modality by setting out a past tense sentence and a sentence about an unrealized possibility. The parallel between tense and modality is itself more general so that, instead of the structure of (5) and (6) we can describe the structure of (3) and (4) by exhibiting more general schemata:

(7) it is/was/will be that α

[3]It is customary in (at least philosophical) English to speak of truth 'at' a time, and truth 'in' a world. As far as this book is concerned 'true at' and 'true in' are no more than stylistic variants. Some philosophers have tried to make more of this distinction. For instance Adams 1981, pp. 20-24, restricts truth *in* a world to truths about things which exist in the world in question. This is because he thinks that propositions about things which do not exist in a particular world do not themselves exist in that world, and so cannot be true or false in that world. To the best of our knowledge no philosopher has made a distinction between existing *at* a time and existing *in* a time, but the analogous distinction could certainly be made. Prior for one (Prior 1957, p. 31) claims that there are no facts about things which no longer exist. Prior himself took this to demand a logic in which bivalence has been given up, and he began attempting to study such a logic in Chapter 5 of Prior 1957. We are assuming a classical bivalent attitude to truth, and our use of 'true in a world' is Adams's 'true at a world'.

(8) it is/might have been that $\alpha$.

(7) invokes the *tense* trichotomy of past, present and future, while (8) invokes the *modal* dichotomy of actual and possible. On the face of it this might seem to make the project implausible, since it is lopsided. Explaining how this dichotomy and trichotomy are nevertheless parallel will be one of the tasks of this book. So let us go back to the original claim that (3) is a part of reality but that (4) is not. What might this mean? Well certainly

(9) It is true in reality that Charles I was executed.

But then

(10) It is true in reality that Cromwell might have become king

is equally so, as is

(11) Cromwell might have really become king.

What is not true is

(12) Cromwell became king

but all that that tells us is that something might have been true which never was true. That certainly *is* a truism, but it merely reflects the fact that worlds and times are different, and is of little help in assessing their comparative reality.

Have we been too swift? Charles I's execution is *actual*, Cromwell's becoming king is merely *possible*. Isn't that enough? Perhaps, but we here introduce the other way of marking the difference. In modality we talk of the actual and the possible. Sometimes we speak of the *merely* possible, as what is possible but not actual. The temporal contrast is between the present and the past and future. The first point is that talking of past, present and future cannot provide an analysis of tense. Although the execution of Charles I is not present — it is past — yet it *was* present. In fact anything that is past was once present. There is a famous argument due to J.M.E. McTaggart against the reality of time. One premise of this argument is that everything must be both past and present (and future as well), while another premise is that *nothing* can be both past and present. Whatever this curious argument does or does not show about the unreality of time, the first premise is only plausible in a tensed language, where

it can be said that everything that *is* past *was* present.[4]

What does all this tell us about modality? It is this. The claim is supposed to be that the actual encompasses everything that is so — that there is nothing apart from what is actual. But that turns on what *is* means. Although only the actual perhaps *is* actual, there is much that is not actual but *might have been* actual. Cromwell might actually have become king. Although he didn't, it might have been actual that he did. One might respond that this is to conflate what *is* actual with what might have been actual. And this helps illustrate how the parallel plays out. For consider the claim that Charles I's execution is present. That claim is false. What is true is, presumably:

(13) Charles I's execution *was* present

and (13) seems to mean the same as

(14) Charles I's execution *is* past.

As was said in the discussion of (7) and (8), the modal phrase corresponding to 'was' is 'might have been'. In (13) the 'presentness' of Charles I's execution is qualified by 'was', and, for the same reason, in

(15) Cromwell's becoming king might have been actual

the 'actuality' of Cromwell's becoming king must be qualified by 'might have been'.

But perhaps there is this difference. Perhaps there is no such thing as Cromwell's becoming king, and so (15) is false, or maybe does not even make a claim, since there is nothing which might have been actual except what is actual. Someone who says this will be claiming that while (14) is an alternative way of expressing (3), yet (15) is not an alternative way of expressing (4). The reason is that Cromwell's becoming king only exists if Cromwell does become king, and since he never did there is nothing which might have been actual. If this

[4]Chapter 33 of McTaggart 1927 argues that the alleged contradiction is essential to time itself, and therefore that time is unreal. Certainly no predicate like 'present' can analyse tense. So if tense needs an *analysis* we are in for a vicious regress, just as Mellor 1981 claims on p. 94. As an analysis of tense the regress is vicious because, as Bigelow 1991, p. 5, says, it "presupposes what we are trying to explain." This is in fact McTaggart's own complaint at p. 21. Dyke 2002a argues (p. 141) that even if you can avoid the contradiction at the first level it is only by introducing one at the second level, and so on. She is right that if there *is* a contradiction at any level it won't be removed by going a level up.

is the argument look again at (14). For there *is* no such thing as Charles I's being executed — that is Charles I is not being executed. Of course there *was* such a thing, in that Charles I *was* executed. The defender of (14) can say that the talk of Charles I's execution does not entail that it is now taking place. All that it entails is that it did take place, or is taking place, or will take place. In fact it *did* take place. That is what the truth of (3) amounts to. But that defence is available to the defender of (15). For the claim is not that Cromwell's becoming king *is* taking place, or that it was, or that it will be. It is merely that it *might* have taken place. That is what the truth of (4) amounts to. And nothing has been said so far to show what is wrong with that.

What do we mean by a "structural parallel"? It might help to begin with some explanation of what the parallel is not. Since we began writing this book we have often received comments like the following:

> So you are going to argue in favour of the world-time parallel. That's nice, but it's hardly new. Most metaphysicians already recognise that there is a structural parallel at the level of logic. But we claim that you can admit this parallel without supposing it tells you anything of metaphysical significance. Yes there is a logical parallel between what happens at other times and what happens in other worlds, but there is all the metaphysical difference in the world between the reality of these other worlds and the reality of other times. So we can accept your parallel and yet feel it doesn't help us at all, and that we need take no notice of it.

Some people suppose that when we say there is a structural parallel we mean simply that tense and modality can be formally modelled using the resources of semantical indices – i.e., using possible worlds for modality, and using times for tense. Others may want to know what is "the correct" tense logic or "the correct" modal logic. And they suppose that a claim that there is a logical parallel between time and modality must be wrong when they make the further supposition that a system like S5 might be "right" for modality, but is surely "wrong" for time with both a past and a future operator. (See footnote 11 on page 21 for more about this.) Our book is not about logic in this sense. What we mean is this. If you are faced with an *argument* in the philosophy of modality, there ought to be a corresponding *argument* in the philosophy of time which has the same structure.

Suppose you have two metaphysical arguments, one, **T** say, about time, and another parallel argument, **M** say, about modality. The metaphysicians we have in mind agree that the logical parallel is exact. So imagine a step $T_n$ in

argument **T**. Then there will be a corresponding step $M_n$ in argument **M**, and vice versa. Either $T_n$ — and therefore $M_n$ — is a premise, or it is obtained from some earlier members of **T**. If the latter, then, given that the *logical* parallel is exact, $M_n$ will be obtainable from the earlier members of **M** by the same logical rules according to which $T_n$ was obtained from the earlier members of **T**. Either these rules are good rules, or they are not, but either way the success of the modal argument stands or falls with the success of the temporal argument. So $T_n$ must be a premise, and further, it must be a premise for which no argument is given, since if an argument *is* given you can apply the same procedure to *that* argument. So it must be an unargued premise. Of course it is easy to provide such. If you postulate as unargued premises:

$T_0$: All times are ontologically real

and

$M_0$: Only the actual world is ontologically real

then indeed you will lose the parallel. But unfortunately you then have nothing better to offer the advocate of a different premise than an incredulous stare, and your opponent will simply stare right back at you. To put the point crudely we suggest that if logic is about argument, a metaphysician who gives the $T_0/M_0$ reply is suggesting that logical argument has no place in metaphysics.

We do not in this book claim that there are *never* reasons to treat temporal and modal arguments differently. But we *do* claim that whenever you meet one of these arguments you should very carefully examine the corresponding one. More importantly, as we try to illustrate, it is by no means a trivial matter to be clear about just *what* the corresponding argument is.

Part I of the book sets out what we call an 'indexical' semantics. That is a semantics in which a sentence is held to be true or false, not absolutely, but at a *semantical index*, or more accurately at a sequence of semantical indices. Of these we single out three, a time, a possible world, and a 'person', where a person may be understood abstractly, as for instance in the sense of 'observer' used in relativity theory. It is important to say something here about our use of the word 'indexical'. While we believe that it is the use envisaged by early writers in the semantics of intensional logic[5], it has acquired a number of different uses in recent years. For instance MacFarlane 2009, p. 252 suggests the following definition:

[5]We make some historical remarks about indexicality on p. 34 below.

(1) An expression is *indexical* iff its *content* at a context depends on features of that context.

(2) An expression is *context-sensitive* iff its *extension* depends on features of that context.

We do not, in this book, use 'content' as a technical term, but in Chapter 2 we do offer an account of various kinds of propositions as sets of indices — an untensed proposition is a set of worlds, a tensed proposition is a set of world-time pairs, and so on. An indexical semantics assigns such entities as the semantic content of sentences, and while the truth or falsity of such sentences changes from index to index, the proposition expressed by the sentence does not — since the sentence is true or false at an index iff that index is in the set assigned to the sentence. This means that we are using the adjective 'indexical' to mean what MacFarlane means by 'content-sensitive'.[6] Whether there is, in addition to this kind of indexicality, a need for indexicality in MacFarlane's sense is a question we do not deal with in this book, though we make a few remarks about the content of utterances in Chapter 12.

We end Part I by looking at the possibility that truth at an index can be given an analysis in terms of a primitive notion of actual present truth. This raises the following question: Do we privilege the actual in a way in which we do not privilege the present? Critics of possible-worlds metaphysics may say that it cannot analyse 'plain truth' but only truth in a world. Some philosophers of time – the so-called presentists, such as Craig (2000a and 2000b) – say that others – the so-called eternalists, such as Mellor (1981, 1998) – cannot analyse 'present truth' but only truth at a time. The upshot of our discussion suggests that it may not matter whether you take truth at an index as basic, or simple actual present truth. But more importantly it suggests that whatever you do about this you can do equally for tense and for modality.

To give precision to our discussion of the world-time parallel we give in Part II a formal definition of the syntax and semantics of first-order predicate languages with a selection of tense and modal operators, and look at how they can be used to formalise sentences involving time and modality. The languages

[6]These remarks are in no way intended as any criticism of MacFarlane, who remarks on p. 232 that "we are free to use technical terms in whatever way is most useful", and in fact goes on to make some very insightful observations, many of which we agree with, pointing out confusions in certain current debates. One thing is clear. As a quick glance at the articles in two recent collections on these matters, Preyer and Peter 2005 and 2007, shows, different contributors appear to want to understand the terms of debate in very different ways.

introduced, together with their semantics, are perfectly standard, and have been around since the late 1960s. Their fullest development is in the work of Richard Montague, collected in Montague 1974. Formal modelling of natural language sentences is now common in semantic studies and needs no justification, and in any case our use of it is restricted to the investigation of the world-time parallel, and makes no claim to adequate formalisation of the many and varied phenomena of tense and modality in natural language in any respect other than that parallel. Any treatment in a formal langauge of matters like this is subject to the complaint that it imposes an unwarranted precision and idealisation on an account of our linguistic practices. While such a complaint would undoubtedly be justified if we were making a contribution to the philosophy of language, the formal languages we use, however idealised, are sufficient to establish that the resources of the tense and modal languages needed to express a number of ordinary natural language sentences have to be extended by mechanisms which give the power of quantification over worlds and times. Here too the formal results have been available for a number of years. We give an account of the technique of 'multiple indexing' developed in the 1970s and 1980s. Although the technique is well known it is typically applied either to times alone or to worlds alone. Our treatment highlights the exactly parallel way in which the technique applies to tense and modality.

In Part III of the book we reflect on the implications of what has been shown in Parts I and II. Part III begins by presenting an attitude to ontology which makes it depend on the structure of truths. Whether or not this is a viable attitude, we argue that it applies to the modal case in exactly the same way as it applies to the temporal case. Much in this part of the book examines what various philosophers have said about modality, and looks at what happens when you say the same things about time, or examines what philosophers of time have said, and looks at what happens when you say the same things about modality.

Part IV addresses questions about the nature of the various entities involved in discussions of time and modality. For example, philosophers often ask whether the same individual can exist in more than one possible world. We investigate the analogous question about individuals existing at more than one moment of time. Or you may be concerned about the *nature* of possible worlds, or the nature of what exists in them. Are they abstract or concrete? Is the real world ontologically different from the merely possible worlds? You can ask these questions in just the same way as you can ask about the status of times, as in the dispute between those who think that only the present is real, and those who think that all times are equally real.

We do not ourselves wish to take sides in any metaphysical controversy;

and you will not find in this book a survey of the many views in the philosophical literature on the ontological status of times, or worlds, or anything else.[7] To be sure, there are occasions when we allude to such controversies, and if you feel that we have not taken sufficient account of the literature, do remember that contributing to these controversies is no part of our aim, except where it might bear on the parallel between time and modality. If we have an agenda it is simply this: Given that many philosophers assume that there is a clear difference between worlds and times, it is we believe vital to ensure that arguments for or against the difference are real arguments and not bogus arguments. A large part of this book is intended to establish that many of the considerations which are put forward to show that modal talk and temporal talk are not parallel simply do not hold up. Only when this has been seen and appreciated do we feel that it is appropriate to ask what the *real* differences are. If we appear to be trying to advance the thesis that times and worlds are ontologically equivalent, bear in mind that none of our arguments support any positive metaphysical position. F.H. Bradley once said that metaphysics is the finding of bad reasons for what we believe upon instinct.[8] Maybe the instinct that worlds and times have a different ontological status is right, but we must first clear away the bad reasons.

The book is addressed to anyone with an interest in the semantics or metaphysics (or both) of time and modality. The precise articulation of the world-time parallel does require the resources of formal logic, and we make no apology for that. We have though tried to explain all the technical apparatus we assume, and have tried to make clear at every point just what the formal framework is intended to show. However, to enhance the readability of Part II we have put its more severely technical results into Appendices 1 and 2 on pp. 209-234, which need only be read by those who do not wish to take on trust the claims made in the main text.

[7] A valuable introduction to theories of possible worlds is found in Melia 2003.

[8] *Appearance and Reality*, p. x.

# Part I

# Truth and Indexicality

# 1

# *Semantical indices*

When we say

(1) It is raining in Waitarere

we are speaking in ordinary English. But our concern in this book is not with how to speak English per se. Still less is it concerned with how to teach the meaning of English sentences to anyone who does not understand English. We are simply using English as an example of a language in which sentences are being used which can illustrate the world-time parallel, and we choose English simply because readers of this book will already understand it. We are though concerned with how to analyse the truth or falsity of ordinary English sentences like (1). And we say that (1) is true at a time *t* iff[1] there is rain in Waitarere at *t*. In other words

(2) 'It is raining in Waitarere' is true at a time *t* iff it is raining in Waitarere at *t*.

i.e.,

(3) (1) is true at a time *t* iff it is raining in Waitarere at *t*.

Since what we want is an analysis of (1) we are treating (1) as a sentence of our *object language*. (3) is part of our *metalanguage*. One can present a grammar of Italian in English. Italian is the object language and English is the metalanguage. One can also present a grammar of English in English.[2] The metalanguage in (3)

[1] 'iff' is a standard abbreviation in logic for 'if and only if'.

[2] If the grammar of English is stated in English it would of course be of no use in teaching English to someone who knows no English, but from that it does not follow that it would be incorrect. Readers unfamiliar with the language/metalanguage distinction might think that a sentence like (3) is contentless, or analytically true, but that is not so. If 'is raining' had meant what 'is sunny' means then, without any change in Waitarere's weather it would change the truth value

is a version of English which makes reference to truth at a time. In this chapter we are using as examples of our object language sentences of English, like (1). In later chapters we will introduce predicate tense and modal languages as the formalised object languages in which we will discuss the world-time parallel.

Although (3) has also been written in English there is good reason to think of the metalanguage as 'tenseless' in the sense that, where α is a sentence of the object language, phrases like 'α is true at *t*' should be understood in such a way that it does not even make sense to speak of *when* it is so that α is true at *t*. The metalanguage, as in (3), speaks of a sentence as being true or false *at a time*, where the object language has a sentence, (1), which has no mention of moments of time. For any time, (1) is true at that time iff

(4) It is raining at *t*

is (absolutely) true when *t* is assigned the time in question, and that will be so iff it is raining in Waitarere at the time in question. Even though (4) expresses a temporal fact, since it will be true for some values of *t* and false for others, (4), unlike (1), is understood to be tenseless in that it makes no sense to ask *when* it is true.[3] In that respect it is like

(5) $2 + 2 = 4$

which is simply true, and where there is nothing in it which could be described as tense. (4) is true simpliciter if it is raining at the time assigned to *t* and false simpliciter if it is not. A genuinely tenseless language is one which does not have the resources to speak of *when* its sentences are true. One could read (4) as '*t* is a moment of rain (at Waitarere)', but we have to bear in mind that *any English* explanation of the meaning of (4) is an explanation in a language which is inescapably tensed. What this means is that in saying that (4) is tenseless we are thinking of it as like (5) in that you can't sensibly ask *when* it is true.[4] For the same reason (3) is also tenseless, even though it speaks about a sentence (1) which is of course tensed, since our object language, English, is a tensed

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of (3). (Compare the riddle 'if you call a tail a leg, how many legs does a donkey have?'. The answer is "'Four', calling a tail a leg does not make it one.")

[3]Wahlberg 2010 appears to suggest that this cannot be the end of the matter, since it does not address issues like what it would be for something to be so at a time.

[4]An example closer to our concerns of a genuinely tenseless metalanguage, would be a language of first-order predicate logic (see pp. 59-63 and p. 225).

language.

At this point we should establish some of the logical terminology which we shall have occasion to make use of throughout this book. Begin with ~. Where α is a sentence ~α means 'it is not so that α'. We can specify the meaning of an operator like ~ by the principle:

(6) ~ α is true if α is false and false if α is true.

Other operators can be treated in a similar fashion:

(7) α ∧ β means 'it is so that both α and β' — so that α ∧ β is true if both α and β are true, and false if either one of them is false.

(8) α ∨ β means 'either α or β', in the sense that α ∨ β is true iff at least one of α or β is true.

(9) α ⊃ β means 'if α then β' in the sense that α ⊃ β is true iff α is false or β is true.

(10) α ≡ β means 'if α then β and if β then α' in the sense that α ≡ β is true iff α and β have the same truth value, i.e., iff they are both true or both false. It represents 'iff'.

When tense is involved (6)-(10) need refinement since we cannot speak simply of truth and falsity, but we have to understand it to be truth or falsity at a time. So that (6) has to be stated as

(11) For any time *t*, ~ α is true at *t* if α is false at *t*, and false at *t*, if α is true at *t*.

Tense logicians use the letters *F* to mean 'it will one day be the case that', and *P* to mean 'it was once the case that'. With this notation we can express the sentence

(12) It once rained in Waitarere

as

(13) *P*(It is raining in Waitarere)

In saying this we make no claims about the syntax of English or any other natural language. Indeed that had better be so, since this is supposed to be a *logical* investigation of the world-time parallel, not a grammatical discussion about natural language. What our logical language does is provide a framework in which a sentence like (13) can be understood as built up from a simpler sentence in just the way that a formula of logic or mathematics can be built up out of simpler components. This reflects the view that we understand sentences of the languages we speak not by learning them one at a time, but by building them up from the words in them together with their grammatical structure. The languages of formal logic are too simple to provide a realistic model of natural language, but they are sufficient for studying the world-time parallel. Notice already a divergence between (13) and how we would express the past truth of (1) in English. If in English we were to say

(14) It was the case that it is raining at Waitarere

the present tense of 'is' is in tension with the prefix 'it was the case'. It would be more natural to say

(15) It was the case that it *was* raining in Waitarere

but our aim is to explain how the truth of (13) depends on the truth of (1) at this or that time.[5] For that purpose we write $t < t'$ to mean that time $t$ is earlier than time $t'$. We can then say that (13) is true at a time $t$ iff there is some $t'$ such that $t' < t$ and (1) is true at $t'$. This is clearly independent of the meaning of the particular sentence in question, and we may state this as a schematic principle, where $\alpha$ is any sentence:

(16) $P\alpha$ is true at $t$ iff there is some $t'$ such that $t' < t$ and $\alpha$ is true at $t'$

[5]This reflects A.N.Prior's view that the present is semantically basic. (See for instance Prior 1957, p. 10, and Prior 1968a. (P. 171 in Prior 2003.) In a language like English it is more likely that $P$ operates on a pre-tense form — something like ***shout Bugsy*** where ***shout*** is the infinitival form. Despite the simplicity of classical tense logic, natural language, as to be expected, needs refinements. Here is a more dramatic case of the phenomenon illustrated by (15). Assume that ***yesterday*** $\alpha$ is true at a time $t$ iff, where $t'$ is a moment in the day preceding the day in which $t$ is a moment, $\alpha$ itself is true at $t'$. If 'Bugsy shouted yesterday' has the form ***yesterday P shout Bugsy*** it would be true at $t$ iff Bugsy shouts at some $t''$ which precedes some $t'$ where $t'$ is on the day preceding the day in which $t$ occurs — obviously the wrong result.

In this logic *P* is a sentential operator. We could put this by saying that in this logic *tense* is a feature of whole sentences. It is important to stress this since in later chapters we shall be discussing work which concerns whether verbs like 'is' are tensed or tenseless, and what is crucial to learn at this stage is that from the point of view of logic the tensed/tenseless distinction is one which can only be drawn at the level of whole sentences. This comes out more clearly in a sentence like

(17) A child prodigy studied at this school.

In (17) the obvious implication is that the person was a prodigy *at the time of studying*. So that whatever the tense is doing it must be captured in a manner which takes you back to a time when a person who was then a prodigy was then studying. (17) would be formalised as

(18) *P*(A child prodigy studies at this school).

What (18) makes clear is that the 'pastness' is captured only by *P*, and that not only is the *studying* something which happened in the past, which is what the surface English grammar might suggest, but also the being a prodigy. And indeed (18) gets the truth conditions right, since it is true at a time $t$ iff there is some $t' < t$, such that

(19) A child prodigy studies at this school

is true at $t'$. There are of course refinements to do with what is called 'aspect', by which a distinction is made between 'studies' and 'is studying', and the semantics for the English tense system is far more complex than our little logical language allows. But within its limits it will suffice to illustrate the world-time parallel. We shall take up the appropriate representation of (19) in Chapter 5 when we introduce tense and modal predicate logics.

To express

(20) It will one day rain in Waitarere

we use the operator *F* to mean 'it will one day be the case that', and express (20) as (21)

(21) *F*(It is raining in Waitarere)

We can say that for any sentence α

(22) $F\alpha$ is true at $t$ iff there is some $t'$ such that $t < t'$ and α is true at $t'$.

Then (21) is true at $t$ iff there is some $t'$ such that $t < t'$ and (1) is true at $t'$. In (20) the verb 'rain' is in the infinitival form, whereas in (21) the embedded (1) is in the present tense. 'will' in English is what is called a 'modal' verb.[6] Modal verbs like ***will***, ***would***, ***can***, ***could***, ***may***, ***might*** and the like are marked by three features. (i) They do not have a separately marked third person singular. (ii) They are negated by a ***not*** which follows the verb rather than by 'does not'. (iii) They take a 'naked infinitive'. Thus we say 'Bugsy might sneeze', not 'Bugsy mights to sneeze', and we say 'Bugsy might not sneeze', rather than 'Bugsy doesn't might to sneeze'.[7] This already indicates the strength of the parallel, but at present all we need note are the similarities between past and future. A sentence like

(23) One day a child prodigy will study at this school

can be formalised by analogy with (17) as

(24) $F$(A child prodigy studies at this school).

Using $P$ and $F$ it is easy to define an operator $H$ in such a way that $H\alpha$ means that α always *has* been so, and an operator $G$ so that $G\alpha$ means that α is always *going* to be true. $G$ and $H$ are straightforwardly related to $F$ and $P$:

$G\alpha \equiv \sim F\sim\alpha$
$H\alpha \equiv \sim P\sim\alpha$[8]

To say that α is always going to be true is to say that it is not so that it will one

[6]Perhaps a reason for this difference between past and future is the view that *in the case of the future* actuality and possibility may be hard to tell apart. In any case examples with ***will*** in them will correspond with examples with other modal words.

[7]An interesting case here is the verb ***need***, which can be either a modal or an 'ordinary' verb. Thus we can say either 'Tallulah need not be here tomorrow', or 'Tallulah doesn't need to be here tomorrow.'

[8]Alternatively we can take $G$ and $H$ as primitive and define $F$ and $P$, which is how we shall proceed in Chapter 5.

day be false, and to say that α has always been true is to say that it is not so that it was once false. One can also introduce operators ***always*** and ***sometimes***, where ***always*** α means that α is and always has been and always will be so, and ***sometimes*** α means that α was either once so, or is now so, or will one day be so. ***always*** α and ***sometimes*** α do not change their truth values with the passage of time. In place of ***sometimes*** we shall use ⊗, and in place of ***always*** we shall use ⊕. If time is linear we may define these operators in terms of *F*, *P*, *G* and *H*:

$$\otimes\alpha =_{df} (F\alpha \vee \alpha \vee P\alpha)$$
$$\oplus\alpha =_{df} (G\alpha \wedge \alpha \wedge H\alpha)$$

What we have so far may be called an *indexical* treatment of tense, in the sense that the semantics of tensed sentences is given by specifying whether they are true or false at a time. The time is what can be called a *semantical index*, or an *index of truth*. This is a neutral term because the question of what indices are needed is a somewhat open-ended one. By an index of truth we merely mean something with respect to which a sentence is determined for truth or falsity. Calling something an index of truth says nothing about its intrinsic nature. In fact, keeping questions of the nature of the indices separate from their role as indices of truth enables us to study the logical parallel. (Other authors use the term 'indexical' differently from us, and we shall note these divergent uses from time to time.)

The indexical treatment can easily be extended to modality. If we use the symbol *M* for 'might' we can formulate

(25) A child prodigy might study at this school

as

(26) *M*(A child prodigy studies at this school).

Modal notions involve possibility. Correlative with possibility is *necessity*, and we write *L*α to mean that it is necessary that α. Among truths, some are necessarily true, they couldn't have been otherwise. Others, while no less actually true, might easily have been false. And among falsehoods some, though false, might have been true — they are possible — while others are not possible, they could not have been true. The relations between necessity and possibility can be characterised by a *square of opposition*. The impossible is what *has to be false*, i.e. 'not possibly' means the same as 'necessarily not'. Similarly 'possibly not'

means the same as 'not necessarily. There are also similar squares in the temporal case:

| | | | | | |
|---|---|---|---|---|---|
| $L\alpha$ | $L{\sim}\alpha$ | $G\alpha$ | $G{\sim}\alpha$ | $H\alpha$ | $H{\sim}\alpha$ |
| $M\alpha$ | $M{\sim}\alpha$ | $F\alpha$ | $F{\sim}\alpha$ | $P\alpha$ | $P{\sim}\alpha$ |

The way to interpret these squares is this. In each case the top two formulae cannot both be true, while the bottom two formulae cannot both be false. Finally, each formula is equivalent to the negation of the diagonally opposite formula. As the definitions of *G* and *H* on p. 17 make clear, if α is always going to be true than it is not so that ~α will ever be true, and anlogously with *H* — if α has always been so, then~α was never so, and so on.

Necessary truths include the facts of mathematics and logic. Among those actually true but not true by necessity are such facts as that someone is talking to you, or that the Number 2 bus runs to Miramar. To account for modality we speak of sentences as true or false in different possible worlds. Thus

(27) The number two bus goes to Miramar

is true in world *w* iff the number two bus does indeed go to Miramar in *w*. Since it doesn't *have* to do so there will be worlds in which it does not, and since it *could* do so there will be worlds in which it does. In the case of modality an extra complexity appears which does not seem to arise in the temporal case. That is the phenomenon of different senses of possibility. If we ask

(28) Can people fly to the moon?

one answer is yes, because it has been done. Another answer is no, because the equipment is not ready, even though it could have been ready. Take a sentence like

(29) Auckland might be the capital of New Zealand.

If someone asserts (29) it probably indicates ignorance. If they know their New Zealand geography they would know that Auckland isn't the capital. In order to express possibilities which are known not to be realised, English combines modal words with the past tense. So that the possibility that the decision to transfer to Wellington in 1865 might have gone the other way could be expressed as

(30) Auckland might have been the capital of New Zealand.

In ordinary English these sentences can be expressed by various combinations of modal words and tense auxiliaries like ***may***, ***might***, ***might have***, and so on. Yet what they all seem to have in common is that possibilities are *contextually constrained*. In (29) it is our knowledge which rules out certain possibilities; in (28) perhaps it is technological facts; in other cases it might be economic constraints. More basic will be the constraints of the laws of nature in the physical universe, and so on. The truths of mathematics and logic are a limiting case of senses of possibility. For there is *no* sense of 'possible' in which 2 + 3 = 4, though of course the *symbol* '4' might have stood for the number 5. While we may not be able to jump more than 3 metres high, or travel faster than light, or disappear at will, these facts are because of the way the world is. The 'worlds' in which these things are so, although possible worlds, are not possible for us because of various constraints, and we will speak of *logical* possibility or *logical* necessity when we mean something which could or must be true without any constraints whatsoever.[9] A sentence $\alpha$ is said to be *necessary* in a world $w$ provided $\alpha$ is true not only in $w$ but in all the other possible worlds as well, and $\alpha$ is *possible* in $w$ if $\alpha$ is true in *some* possible world, whether or not $\alpha$ is true in $w$ itself. Unadorned, the picture gives us only logical necessity and possibility. Logical necessity, for which we shall use the symbol $\Box$, and logical possibility, $\Diamond$, refer to what happens in all worlds, or to what happens in at least one world. For any formula $\alpha$, $\Box\alpha$ is true at any world iff $\alpha$ is true at *every* world; and $\Diamond\alpha$ is true at any world iff $\alpha$ is true in *at least one* world. Since there is nothing quite comparable to the linear ordering of times, and since $L$ and $M$ can represent many different kinds of necessity, $\Box$ and $\Diamond$ cannot be defined in terms of $L$ and $M$, and must be taken as primitive. Still, just as $\oplus\alpha$ and $\otimes\alpha$ cannot change their truth value from time to time — if they are ever true they are always true — so $\Box\alpha$ and $\Diamond\alpha$ do not change their truth values from world to world. While statements of the form $L\alpha$ and $M\alpha$ can be contingent, statements of the form $\Box\alpha$ and $\Diamond\alpha$ are never contingent.

One of the features which we believe has confused discussions of the world-time parallel is that the only modalities which are often considered are the logical modalities. The reason why this is unfortunate is that the semantics for $\Box$ and $\Diamond$ makes clear that any sentence of the form $\Box\alpha$ or $\Diamond\alpha$ has a truth value which is absolute in the sense that it is constant from world to world. This is

[9]Some philosophers contrast logical with metaphysical necessity. We make no such distinction, and discuss this on p. 63.

important since in the temporal case $P\alpha$ and $F\alpha$ can change their value with the passage of time, a fact which has some importance in the argument by McTaggart referred to in the introduction on p. 4, since in that argument it is crucial that whatever is now future will one day be past, and so on.[10] In order to see why the difference between $P$ and $F$ on the one hand, and $\Box$ and $\Diamond$ on the other doesn't change the question we shall begin by explaining how necessity is analysed in the semantics of modal logic. Modal logic is concerned not only with what is so, but with what must be or might be so. To say that something is possible in some sense other than that of logical possibility will still be to say that it is true in some world; but not just *any* possible world will do. It must be a world which is possible in the relevant sense. In modal logic we say that for any given world $w$ certain worlds are *accessible* from $w$, or possible relative to $w$, and $\alpha$ is then held to be necessary in $w$ if $\alpha$ is true in every $w'$ accessible from $w$, and possible in $w$ if $\alpha$ is true in at least one world accessible from $w$. Corresponding to $t < t'$ in the temporal case we may write $w\mathrm{R}w'$ to indicate that $w'$ is accessible from $w$. We can then specify the meaning of a possibility operator $M$ based on an accessibility relation R by saying that $M\alpha$ is true in $w$ iff there is some $w'$ such that $w\mathrm{R}w'$ and $\alpha$ is true in $w'$. An associated necessity operator $L$ can be defined as $\sim M\sim$. The stage we have reached is this. In a tensed language sentences are evaluated for truth or falsity with respect to a moment of time, and the meaning of the tense operators $P$ and $F$ is specified using a temporal ordering $<$. In a modal language sentences are evaluated for truth or falsity with respect to a possible world, and the meaning of the modal operators $M$ and $L$ is specified using an 'accessibility' relation, usually written R. The temporal ordering is usually thought to have a fairly precise mathematical structure, typically the real line, whereas the modal accessibility relation need not have such a structure. This may indeed be an important difference between time and modality, but the fact that it *is* a difference does not of itself tell us much until we can evaluate just what kind of difference it is, and just what the implications of the difference are. In the temporal case it seems that all we need is $<$. But in the modal case there are a whole family of $L$s and $M$s, each with its own accessibility relation.[11]

Although statements about necessity and possibility, modal statements,

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[10]This difference may be what is driving the criticism on p. 202f of Dyke 1998 of the modal version McTaggart's argument presented in Cresswell 1990a.

[11]For that reason one cannot ask what the *logic* of time is, or what the *logic* of modality is, but only what the logic is of a particular modal or tense operator. Thus S5 is the logic of both $\oplus$ and $\Box$, but not of $G$ or $H$, and not of most of the $L$ operators. (For S5 and other modal and tense systems see any introduction to modal and tense logic. S5 is discussed on pp. 58-62 of Hughes and Cresswell 1996.)

abound in ordinary discourse there are, as we have observed, many different kinds of possibility and necessity, and it is doubtful whether there are more than a handful which are statements about *logical* necessity or possibility.[12] By varying the accessibility relation one can account for the many different kinds of necessity and possibility, and rely on context to make clear just what kind it is. This book is not the place to develop a theory of context,[13] so we simply assume that our logical language contains a family of $L$ and $M$ operators, each with its own accessibility relation. Since we shall usually only be concerned with one such operator at a time we shall not mark the context-dependence explicitly.

We have stressed the distinction between the tensed and modal object language and the tenseless and modal-free metalanguage with its quantification over times and possible worlds. What must be equally stressed is that in the latter kind of language words like 'past', 'present' and 'future' have no place. Nor have modal words like 'necessary' or 'possible'. In speaking of events as past, present or future this has to be in a tensed language, and in speaking of them as necessary or possible this has to be in a modal language. Philosophers of time frequently talk about a tenseless 'is'.[14] Tense, and perhaps even modality, is of course a grammatical phenomenon. A linguistic study of tense is a study in syntax. Thus it may be said that while English has a past tense, in the form of an inflected verb, or indeed a present tense, it has no future tense; since what we call a future tense is a modal auxiliary applied to the infinitive. And it may be said that the infinitive is a tenseless form of a verb, and that even the present tense is formed by an operator applied to the infinitive. This may all be true, and important, but for the present it is beside the point. Although one can talk about the tense of a verb, the key classification is, as we pointed out on p. 16, the tense of a whole *sentence*. In a natural language like English there are no grammatically tenseless sentences, and while there might be languages which, unlike English, are grammatically 'tenseless', yet we would be very surprised to find a natural language whose sentences could never be true at one time and false at another.

Prior 1967, p. 102f, gives a pretty example of the kind of problem you get into when *in the same language* some expressions are tensed while others are

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[12]See chapter 8 of Lycan 1994, especially pp. 171-178. For an interesting attempt to see all modal sentences in terms of restrictions on the logical modalities see van Fraassen 1977.

[13]A context-dependent account of modality is given in Kratzer 1977.

[14]For instance Smith 1993, p. 7, defines a 'B-sentence' as one which "does not contain a tensed copula or verb", and places "parentheses around the relevant copulae and verbs" to indicate that they are tenseless. Tooley 1997, p. 18 refers to what he calls the 'tenseless sentence', "there are (tenselessly) dinosaurs".

untensed. Prior points out that if *a* is a 'non-temporal' proposition, and *p* is what he calls simply a 'temporal', then if *a* is (timelessly) true, $a \wedge p$ is equivalent to *p*, and is therefore a temporal proposition, while if *a* is (timelessly) false then $a \wedge p$ would appear to be timelessly false. He is considering the possibility that the *syntax* of a formal language might dictate whether or not a proposition is tensed or tenseless, and he points out that any formation rule which depends on the truth value of *a* would be 'a very awkward formation rule indeed' (*op cit*, p. 103.) Prior of course believes that truth itself is tensed (1967, p. 198f) and would no doubt complain that trying to explain the semantics of a tensed language in terms of an untensed metalanguage is going the wrong way round. In any case, his point about formation rules makes clear why a genuinely tenseless sentence could not be conjoined with a tensed sentence, and if it cannot then a genuinely tenseless 'is' could not occur in a language with tense.

But perhaps there is a kind of 'tenselessness' in which a tenseless sentence *can* occur in a tensed language. Look at

(31) Charles I was executed in 1649.

In tense logic 'in 1649' can be treated as a tense operator, and (31) can be represented as

(32) [**1649**](Charles I is executed)[15]

by analogy with (13) on p. 14, where the tense of the embedded sentence in the scope of the past operator *P* remains in the present tense. Where α is a sentence we can give a semantics for [**1649**] as follows:

(33) [**1649**]α is true at *t* iff α is true at some time during 1649.

and, as a consequence,

[15]We are treating '1649' in sentences like (31) as the proper name of a particular time, There is of course a use of the phrase 'in 1649', which means something like 'at a time 1649 revolutions of the sun around the earth after the time conventionally regarded in western Christianity as the birth of Jesus Christ.' In this latter sense, although 1649 might happen to refer to a particular time, if things had turned out differently that expression might perhaps have referred to some different time. On this see the dispute between Smith 1993 and Dyke 2002b. The discussion in the present chapter is about times themselves, not about dates, which are conventional ways of referring to times. There are of course scholars like Prior, pretty certainly, and Smith (*op cit*) and Sider 2001, p. 110f, possibly, who are sceptical of the whole idea that times are in any sense 'things'.

(34) [**1649**]α

will be true or false independently of what time it is.[16] There is a sense in which both (33) and (34) are tenseless. (33) is a sentence in a genuinely tenseless metalanguage. But (34) is also tenseless, though in a slightly different sense. Although (34) is a sentence in a tensed language, and therefore a sentence which it makes sense to assess for truth or falsity at a time, it is nevertheless a sentence whose truth value does not change over time, since its truth or falsity at *any* time depends on the value of the embedded α at 1649.

In order to address the modal parallel we need to be clear about just what the modal correlate of tense is. Firstly one can make the kind of distinction we have been stressing between a tensed *language* and an untensed language: If the language is untensed then, while its sentences may state that something happens at a certain time, that in no way calls into question the temporal status of a metalanguage sentence itself. The question of that status simply does not arise. In a tensed language the situation is different. From the point of view of the formal languages of tense logic a tenseless sentence like (34) is, as we have said, one which it is either false or else makes no sense to speak of as having once been true but no longer true, or as being true now but as going to be false.[17] So the modal correlate of a tenseless sentence would be one which it is false or makes no sense to speak of as something which might have been true but happens not to be, or as something which is true but which could have been false. In other words the modal correlate of a tenseless sentence is a non-contingent sentence. For that reason, in discussing the parallel it is sufficient to replace 'tensed' by 'contingent' and 'tenseless' by 'non-contingent'. If we held tense to be a feature of the *verb* we would have to say that corresponding to the tenseless 'is' we should consider the non-contingent 'is'. Philosophers have tended not to speak of the non-contingent 'is', and perhaps for this reason have failed to appreciate the force of the world-time parallel. For instance it is often said that there is only one world — the actual world.[18] If we take the parallel seriously we

[16](31) is of course a little more complicated than (32) in that it is not clear what to say about its truth value before 30 January 1649. But whatever should be said about (31), (32) behaves as stated in the text.

[17]One could perhaps make a distinction here between a sentence which is contingently tenseless, i.e., which happens not to change its truth value from time to time, and one like (33) which is necessarily tenseless since, although (33) can change its truth value from world to world, within each world it cannot change its truth value from time to time.

[18]Russell 1919, p. 169

must look at the claim that there is only one time — the present. If the 'is' is tensed then there *is* only one time, although there were and will be others. If the 'is' means 'is, was or will be' then since there were many past times, and will be many future times, it is *not* true that there is only one time. Analogously in the modal case. If the 'is' is the contingent 'is' then there may indeed only *be* one world, but there might have been many others. So consider a sentence like

(35) Cromwell becomes king in world *w*.

(35) is non-contingent, since the addition of 'in world *w*' means that if (35) could be so then it is so. Why do we say the latter? Well, the analysis we are trying to motivate is that if Cromwell could have become king then there is a possible world in which he did become king. It is part of the *nature* of such a world that it is a world in which Cromwell becomes king. It would not be possible for him to fail to become king *in that world*, for a world in which he fails to become king *is not that world*. Does this show a difference between times and worlds? Charles I was executed in 1649, but he didn't have to be. It is not part of the nature of 1649 that Charles I was executed then. Forbes 1985, pp. 77-79, argues that times and worlds are ontologically different because times can, but worlds cannot, be distinguished from what 'occupies' them. He seems to have in mind that, while it can be contingent whether or not something happens at a given time, it is never contingent whether or not it happens in a given world. Such a reaction is to misconceive the role of times and worlds. Certainly (31) can have different truth values in different worlds, while (35) cannot. But (35) can have different truth values at different times, while (31) cannot. Times explain tense not modality, and so, when we describe the nature of a world, in this context 'nature' is a modal term. What *is* the case is that, given that Charles's execution took place in 1649 there is no other time at which that execution in fact took place. Of course it *might have* taken place at another time, or not at all, but all that that means is that times alone are not sufficient for modality.

In this chapter we have presented an indexical semantics for temporal operators, and a parallel indexical semantics for modal operators. The parallel of course is subject to the proviso that in the case of times we may assume a unique linear ordering with a precise metric, and there seems nothing comparable in the case of worlds.[19] Whether this proviso is something with ontological significance

[19]Despite what some philosophers seem to have assumed (see for instance Prior 1957, p. 52), the claim of a structural parallel between worlds and times is not of course so strict as to entail that because there is a natural linear ordering of moments of time there must be an analogous natural ordering of possible worlds. For a start, there are many different accessibility relations between

is not something we shall address directly in this book, though one might make an argument of the following kind. Given that there are many senses of 'necessary' and 'possible, and given that each sense seems to have *some* structure, it might perhaps emerge that the temporal linear ordering is just a special case of modal accessibility.[20]

Here is a way in which modality might give a plausible account of our experience of the passage of time. At any given moment many possibilities are open to us. As the moment passes, one of these becomes actual. If we look at things in this light we may think of modal facts in a given world at a given time as facts which depend on the temporal facts in that world *up to that time*. The idea is like this — take a time $t$ in a world $w$. Particular regions of (logical or physical) space have been occupied up to $t$. If the passage of time is, as we have suggested, connected with the openness of the future, a being situated at $t$ may not be in a position to pick out any one particular world of the many which are just like $w$ up to $t$, but diverge thereafter. This might be an ontological matter, or it might be an epistemic matter. But for practical purposes there may be things we want to be committed to at $t$, *without* being committed to the actual particular future of $w$. If we want to know whether to take an umbrella, it is good to know whether it *might* rain, even if we cannot know whether it *will* rain. As situated beings we might be held to have access to only a fraction of the truth, and we may often want to make claims which are not hostage to whatever else might be true, so we need a way of making statements which are not vulnerable. If you say that it might rain tomorrow or that it probably will or won't then that statement is not open to the same kind of falsification as the claim that it *will* rain tomorrow. If so possibility may well have its genesis in our locatedness in a restricted portion of the world.

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worlds, to say nothing of relations like the world-similarity relation described on p 66 below, used in the evaluation of counterfactual sentences. The most that can be said might be that whereas modality includes many relations among worlds, temporal discourse ultimately depends upon just one — the earlier/later relation between times. But even if this should be so an argument would still be needed to explain why it should have any ontological significance.

[20]Lucas 1989, on p. 199f, suggests that with enough non-logical modalities, one can determine a linear temporal ordering. If he is right the connection between time and modality might be even closer than the structural parallel this book is investigating.

# 2

# *Philosophical entities*

So far we have spoken of truth at times or truth at worlds. But of course many interesting sentences in natural language are true or false with respect to *both* a time and a world. Thus, in place of (3) you would have

(1) (1) is true at a time *t* in a world *w* iff it is raining in Waitarere at *t* in *w*.

When both times and worlds are involved it is convenient to use the terminology of set theory, whereby, if *w* is a world and *t* is a time then $\langle w,t\rangle$ denotes the pair consisting of *w* and *t* in that order.[1] So we may speak of a sentence $\alpha$ as true or false at a $\langle w,t\rangle$ pair, meaning simply that it is true (or false) in world *w* at time *t*.

Time and modality are intimately connected. A person situated in a world at a time may be deliberating what to do. An attractive way of modelling this deliberation is to imagine the ways in which things could develop from that time. One picture of time sees it as a single string of moments in a linear earlier/later relation. Then the two ways things might develop could be represented by two such strings. Corresponding to sentence (2) in the introduction (p. 1), we could have

(2) It might have been raining today in Waitarere.

And now we see a role for the 'have' in (2). Given that today is sunny, a world with rain today does not 'branch out' from today. But if we go back to yesterday (Monday) there are at least two ways things could have developed. For *then* both

[1] Where an index comprises more than one component, as $\langle w,t\rangle$ does, it is sometimes convenient to describe each component as a 'co-ordinate' of the index. But we also speak of *w* and *t* themselves as being indices, so that we can either talk of the world co-ordinate or the world index in $\langle w,t\rangle$. The context should make clear that there is no confusion.

(3) It might rain tomorrow

and

(4) It might not rain tomorrow

could be true. And (2) could be understood as claiming that it was once the case (on Monday) that there might be rain the next day (since (3) was true yesterday) or there might not (since (4) was also true then). This book is not the place for a discussion of the interplay between tense and modality in natural language, except to observe how natural the 'might have' locution in (2) is. (2) is true at $t$ in $w$ iff at some $t'$ on the day preceding $t$ there is a world $w'$ which coincides with $w$ up to a time at or before $t'$ and branches — in some appropriately restricted way — from $w$ thereafter; and (1) is true in $w'$ at $t$.[2] Because the 'might-have-been' rain (the rain in $w'$) has to take place at $t$, accessibility would have to involve sets of times as well as worlds. Many kinds of possibility could then be based on restricting the allowable branching in different ways. Although this speaks of 'world histories' it is perfectly straightforward to express it using a family of time-dependent accessibility relations. For any $t$, $wR_tw'$ iff $w'$ is one of the worlds 'left open' at $t$. This allows what is possible to vary with time, and if time is understood to be open in the future but fixed in the past, then one ensures that if $t < t'$ then the worlds accessible at $t'$ are never more than those accessible at $t$.[3] Using a family of context-dependent accessibility relations, we have a very natural explanation of where necessity and possibility, and all the other modal notions that natural language is peppered with, comes from.

Times and worlds may be fine for tense expressions and modal expressions, but consider

(5) I am hungry

To know whether (5) is true we need more than just a time and world. We need to know who ***I*** is. For that it is customary to add a reference to a person. (5) is

[2]The branching futures model has been articulated in works like McCall 1994. McCall stresses the fact that at each point in a tree made up in this way there are many futures but only one past. As time passes the branches are 'pruned' and the 'trunk' grows. This way of looking at a branching futures model is sometimes described as a 'growing block' universe, and features in works such as Tooley 1997. The present book takes no position on the vexing question of how to treat the perceived asymmetry between past and future.

[3]This kind of necessity can be called 'historical necessity' and is studied in Chapter 7 of Prior 1967, where on p. 125, he gives postulates for it which satisfy the modal system S5. For more on this see Cresswell 2007, Thomason 1984, pp. 136-139, von Kutschera 1997 and Wölfl 1999.

true at $\langle w,t,p\rangle$ iff $p$ is hungry at $t$ in $w$. Or take a sentence like

(6) You are hungry

(6) will be true at $\langle w,t,p\rangle$ iff, where $p'$ is the person addressed by $p$ at $t$ in $w$, (5) is true at $\langle w,t,p'\rangle$. Notice that this framework permits a $\langle w,t,p\rangle$ triple where $p$ does not even exist at $t$ in $w$. This fact is needed to explain the truth of a sentence like

(7) There was a time at which I didn't exist.[4]

The use of a $\langle w,t,p\rangle$ triple may help in analysing other words besides the personal pronouns. Consider a sentence like

(8) It is raining in Palmerston

(8) would be true at $\langle w,t,p\rangle$ iff

(9) It is raining

is true at certain triples of the form $\langle w',t',p'\rangle$. How does Palmerston come into it? One possibility is this. We have been taking it that $p$ is a person, but from a formal point of view its nature doesn't really matter. One could say that (9) is true at $\langle w,t,p\rangle$ iff it is raining at $t$ in $w$ at a location which contains $p$, and that (8) is true at $\langle w,t,p\rangle$ iff (9) is true at $\langle w,t,p'\rangle$ for some $p'$ located in Palmerston at $t$ in $w$. That at any rate is how you might show that these triples are enough for many more expressions than you might suppose.

Each item of one of these kinds, worlds and times, people, or whatever, may be described is an index of truth (or a semantical index) in the sense described on p. 18. There is a view of meaning, which dates at least from Wittgenstein 1921, that each sentence in a language has a meaning by being associated with a set of truth conditions.[5] If something like this is right — if you

[4]Meyer 2006, pp. 32-34, even envisages the possibility that what *times* there are may vary from world to world. If it makes sense to say things like 'If 2005 had not existed the war in Iraq would not have lasted so long', we might need to allow a sentence to be true at $\langle w,t,p\rangle$ even if $t$ doesn't exist in $w$. (We have said that $t$ may not exist in $w$, but, given a time $t$, one could equivalently say that $w$ doesn't exist at $t$.)

[5]Carnap 1947 endorses the view on p. 10 with the following words: "to know the meaning of a sentence is to know in which of the possible cases it would be true and in which not, as Wittgenstein has pointed out."

can specify the meaning of a sentence by specifying at which indices it is true — then the way is open for identifying the meaning of a sentence with a set of indices. For then a sentence will be true at an index if that index is in the set which is the meaning of the sentence, and false if it is not.

In chapter 5 of Montague 1974 Richard Montague provides an account of philosophical entities such as propositions, properties, (intensional) relations, events and the like, all in terms of set-theoretical constructions out of indices: times, worlds and individuals, where 'individuals' may be thought of as a catch-all phrase for whatever else you may need. Montague perhaps was claiming to do metaphysics, but the mechanism of constructing philosophical entities out of indices — what we call the indexical approach — is in fact compatible with a wide range of metaphysical positions. The most extreme is to say with Montague that the philosophical entities just *are* the set-theoretical constructions. Less extreme is to say that in so far as *anything* is a proposition, property or what have you, these will do. This is a kind of quasi-realism which says that the postulation of a philosophical entity of this kind can do no more than claim that reality is apt for being understood in this way. Less extreme still is an attitude like that proclaimed on p. 9 of Barwise and Perry 1983 to the ontological status of their 'situations'. That is that although there really are (or may be) such things as propositions, properties, and so on, the job of a theory is to *model* these things with 'abstract' entities.[6] Finally one could, like Plantinga (see Chapters 4 and 16), adopt a metaphysics of such entities as somehow really there, and then produce structures, using your favourite metaphysical principles, which turn out to behave sufficiently like the set-theoretical ones to make the latter sufficient to the purposes in hand. The fact that this range of responses is available is all to the good, since it means that you can get on with the job of describing the semantic facts using indices and sets, without commitment to any of these metaphysical positions. It is important to stress this since many metaphysicians use 'indexical' in a more loaded way than we do. (See the remarks on p. 142.)

In order to make clear how precisely we proceed, and to avoid any confusion, we shall now give a 'glossary' of the terms we use to denote the set-theoretical entities which are (or do duty for) the corresponding 'philosophical' entities. The indices out of which we construct these entities include worlds, times, and persons, where a 'person' index can be understood as an abstract observer in that there can be an index composed of a world, a time, and a person,

[6]What they say about 'abstract' situations is that: "They play no role in the causal order. People don't grasp them, see them, move them, or even know or believe them. But as semanticists we can use them to classify real situations." (*loc cit.*)

even if there is no person in that world at that time. In addition there has to be an open-ended domain of 'things' or 'individuals' which include whatever you may quantify over. Some of these may be set-theoretical constructions out of others, and we have to assume that precautions are taken to avoid set-theoretical or other paradox. Here is the glossary, where the usual 'philosophical' name is in bold, and our set-theoretical rendering is on the right:

**untensed (eternal) proposition** set of possible worlds

A proposition *a* may be said to be *true* in a world *w* iff $w \in a$. Two propositions will be logically incompatible iff they contain no world in common. One proposition will entail another iff it is a subset of the other. A proposition is *impossible* if it is true in no worlds — that is to say the impossible proposition is the empty set $\varnothing$. From this we see that there can be no impossible worlds. For, on the assumption that an impossible world *w* is a world at which an impossible proposition is true, we have that if *w* is an impossible world then $w \in \varnothing$. But since $\varnothing$ is empty there can be no such *w*.

| | |
|---|---|
| **tensed proposition** | set of world-time pairs |
| **situated proposition** | set of world-time-person triples |
| **property** | function from individuals to propositions[7] |
| **n-place relation** | function from n-tuples of individuals to propositions |

One could just as easily speak of an n-place property, and one could describe a property as a 1-place relation. A 0-place property would be a proposition.[8] Because there are three different types of proposition there are three different types of relation. Properties and relations are understood *intensionally* because they map individuals onto propositions not just onto truth-values.[9] We have defined an untensed proposition as just a set of worlds. In a tensed language every sentence has to denote a tensed proposition. Any set of worlds induces a

[7]What we have called a property Russell sometimes called a propositional function. (See for instance Russell 1905, p. 489n) Properties here are understood as what Lewis 1986a p. 59 calls 'abundant' — being any way at all of dividing the world. But there are also 'sparse' properties, which represent natural ways of dividing the world, and may be what some philosophers have in mind when they ask whether of not there is thus and such a property.

[8]Not all philosophers seem to think so. Thus Field 1978, p. 39, eschews propositions but assumes a "truth-theoretic semantics that assigns properties rather than sets to predicates."

[9]On the arbitrariness of 'intensionality' see Lewis 1974.

set of world-time pairs in a trivial way: If $a$ is a set of worlds, the corresponding set $a^*$ of world-time pairs is simply the set of pairs $\langle w,t\rangle$ such that $w \in a$. That is, for any time $t$ whatsoever, $\langle w,t\rangle \in a^*$ iff $w \in a$. Similarly with triples. Any subset A of a given set B is of course equivalent to a function from B to the two truth values 1 and 0, since the 'characteristic function' $C_A$ of A may be defined so that for any $a \in$ B, $C_A(a) = 1$ if $a \in$ A and $C_A(a) = 0$ if $a \notin$ A; and conversely, given such a function C the set A associated with it can be defined by saying that $a \in$ A if $C_A(a) = 1$ and $a \notin$ A if $C_A(a) = 0$. In modelling philosophical entities using set theory one must be alive to the fact that apparent alternatives are often equivalent.

An alternative treatment of an untensed property would be as a set of individual-world pairs. Suppose ω is a function from individuals to untensed propositions. The claim that $a$ has property ω in world $w$ is just the claim that $w \in \omega(a)$. The associated set of pairs ω* will consist of those $\langle w,a\rangle$ pairs such that $w \in \omega(a)$, i.e., we have the equivalence

$$w \in \omega(a) \text{ iff } \langle w,a\rangle \in \omega^*$$

Equally, a tensed proposition can be equivalently described as a function from times to untensed propositions. So that where ω* is a tensed proposition the associated function ω satisfies

$$\langle w,t\rangle \in \omega^* \text{ iff } w \in \omega(t).$$[10]

In the terminology of this book untensed (or 'eternal') propositions are sets of worlds, while tensed propositions are sets of world-time pairs, and the semantic questions are questions about whether sentences or utterances express tensed or untensed propositions.

An untensed property can also be described as a function which associates each world $w$ with the set of things which have that property in $w$. This set can be called the *extension* of the property in $w$. If ω is a function from individuals to sets of worlds then the extension of ω in $w$ is the set $\{a: w \in \omega(a)\}$. The extension of a tensed property in $w$ at $t$ will be $\{a: \langle w,t\rangle \in \omega(a)\}$, and so on. The extension of an n-place relation in a world at a time will be a set of n-tuples. (Just as an ordered pair consists of two things, possibly the same one repeated,

[10]There have been accusations that some theorists of time have conflated propositions and propositional functions. Thus Tooley 1997, on p. 123-127 quotes Ducasse 1941 protesting about this alleged confusion. While there may be disagreement here there is no conflation or confusion.

in some definite order, so an n-tuple consists of n things, possibly with repeats, in some order.)

The treatment of propositions as sets of worlds or world-time pairs is not uncontroversial, and perhaps is not something most philosophers accept. The most common criticism is that sets of possible worlds do not discriminate between distinct propositions (such as for instance distinct true propositions of mathematics or logic) which are necessarily equivalent. For two propositions *a* and *b* will be equivalent iff they contain the same set of worlds, or world-time pairs, or $\langle w,t,p \rangle$ triples. In particular there is only one necessary proposition — the class of all indices — and only one impossible proposition — the empty set. Such distinctness is usually held to be crucial to the semantics of such sentences as those reporting propositional attitudes. The semantics of propositional attitude sentences and other questions which bear on the possibility of distinct but logically equivalent sentences raise issues of extraordinary complexity. But these questions have been tackled in various ways by those working in the possible-worlds tradition in semantics. (See for instance Cresswell 1985 and Richard 1990. The essential idea here is that just as say 6+6, 5+7 and 9+3 all evaluate to the same number, 12; so different semantic structures can evaluate to the same proposition, so that a proposition is *not*, on this account, a structured entity which reflects the structure of a sentence used to express it, but is the *result* of evaluating that structure.) All that we will say here is this. A proposition is something which is true or false in a possible world at a time, and perhaps with respect to a person and to whatever other indices may be required. So that, whatever the metaphysics of such entities may be, *at least* they must have the properties that fall out of our set-theoretical construction. The set-theoretical approach makes clear just what the primitives are and what the principles of construction are. The syntax and semantics of the regimented tense and modal languages that we shall use to illustrate our claims are presented in Chapter 5.

How many indices might we need for natural language sentences? Worlds, times and persons need not be the only semantical indices. It is for instance possible to think of a predicate as a sentence true at an index. Thus *New Zealander* could be true at *x* if *x* is a New Zealander, and false at *x* if *x* is not a New Zealander. (See Cresswell 1996.) For

(10) The pen is on the table

we might require a value for a particular pen and particular table, as well as a

time and a possible world.[11] Another family of possible indices which have been discussed recently concern whether statements about knowledge, gradable adjectives or predicates of 'taste' require an index to set the relevant standard. (See for instance the essays in Preyer and Peter 2005.) In this book we are principally concerned with worlds and times, and occasionally with persons. It's not obvious that semantic indices are the best way to deal with phrases like 'the pen' or 'the table' or 'knows', but whether or not that is so, we have little to say about how such phenomena should be handled.

One might consider that there is an ontological difference between 'at *t*' and 'in *w*'. In a dated sentence like (31) on p. 23 a time, 1649, is referred to by name. In (35) on p. 25, by contrast, the world is only referred to by the letter *w*, and there seems no way in which we can refer to any particular possible world — except perhaps to the actual world. But look more carefully. We have treated '1649' as if it were the name of a single moment. But 1649 is *not* a single moment. It is a family of moments. And this can make a difference. Consider the sentences

(11) In 1649 it was frosty in England in January

(12) In 1649 it was not frosty in England in July

Assume, as seems reasonable, that both of these are true. (11) says that there is frost in the first month of 1649. (12) says that there is none in the seventh month. January and July can themselves be further divided, and in fact this process can be carried on infinitely. What we call a moment of time is the limit of this infinite process of division. In this light it becomes easier to see how the same method which allows us to refer to and track different times, can also be used to refer to and track different worlds. One way we mark out a range of worlds is by counterfactual conditionals. We say things like

(13) If Cromwell had become king in 1657, the 1689 revolution would not have occurred.

[11] Early discussions of contextual indices are found in Lewis 1972 (see p. 175) and Montague 1974 (see, p. 98). Both of these articles are based on work done in the late 1960s. The word 'index' is used as a more neutral term than 'world' on p. 148 of Scott 1970. Temporal indices (though not under that name) are found even earlier — see Prior 1957. As we noted in the introduction, and will see in chapter 12 (p. 142), the word 'indexical' has been used in a number of different ways, some of which might rule out phenomena which we would call 'indexical'.

While this may not mark off a collection of worlds with the same precision that 1649 marks out a collection of temporal moments, yet sentences like (13) do have a place in ordinary discourse, and do seem to pick out, however imprecisely, a range of possibilities.

One mark of a semantical index is the way it treats negation. If *i* is an index then ~α is true at *i* iff α is not true at *i*. Look at the semi-formal sentence

(14) ***it rains on Monday***

which means that it rains on (some particular) Monday. In that respect ***On Monday*** is like [**1649**] in (32) on p. 23. Suppose (14) is true. Then perhaps it rains in the morning. If so (14) seems to mean that it rains *at some time* on Monday. And if that is so then it would seem that

(15) ***On Monday ~it rains***

could also be true. But of course (15) is not what is usually meant when we say

(16) It didn't rain on Monday.

*That* usually means

(17) ~ ***On Monday it rains***.

and we do not have

(18) (15) ≡ (17).

For these reasons ***On Monday*** does not pick out a unique semantical index. The failure of (18) is established by finding two sub-times $t_1$ and $t_2$ of Monday, where it rains at one but not at the other. A temporal index is a sort of minimal sub-time which cannot be further divided in this way, and the classical conception of truth involves a completion assumption that there are such minimal indices, and that truths such as (14) depend on them. This conception is quite general and does not depend on what indices really are. Take for instance

(19) Someone is hungry.

If we formalise this as

(20) ***someone*** *x* ***hungry*** *x*

then you can certainly have both (20) and

(21) ***someone*** *x* ~***hungry*** *x*,

and you certainly don't have the equivalence

(22) ~ ***someone*** *x* ***hungry*** *x* ≡ ***someone*** *x* ~***hungry*** *x*.

For (19) to be true you have to find someone, say Bugsy, who is hungry. For ***someone*** the appropriate indices are people, and it is a person who is needed to 'complete' (19) to make it true. If no such completion is available then it is false, just as (14) is true if completed by a (minimal) time, and false if there is no such time. For a 'complete' index like Bugsy we have no possibility of ambiguity in

(23) ~ ***hungry Bugsy***.

Notice that Bugsy is a complete entity only in a 'logical' sense. To ask whether Bugsy himself is complete is to be guilty of a category mistake. All that one means is that the truth of a sentence like (19) is to be analysed in terms of the satisfaction of the predicate ***hungry*** by an appropriate kind of entity. Similarly in the case of a time one might ask whether times are complete. To the best of our knowledge no one has raised this question, and it is hard to see what sense could be given to it. However, although the question of the completeness of *other* indices seems not to have arisen, it *has* arisen in the case of possible worlds. The idea goes something like this. (See for instance Barwise and Perry 1983, p. 52.) The users of human languages, human beings, are finite creatures. So the entities involved in the semantics of the languages they use must be finite, and cannot be made up of things, like worlds, which determine the truth values of all propositions. If this is the argument it is unclear why it should apply only to worlds. Apart from worlds the indices that principally concern us are times and people, and it is not at all clear that it even *makes sense* to ask whether or not they are complete — or if it does make sense it is, as we have remarked, only in a rather peculiar 'logical' sense. The reason is not hard to understand. The fact that (23) above does not allow the kind of scope ambiguity which makes (22) fail is not a principle about the nature of any particular kind of index, but is a principle about the difference between a term like ***someone*** and a term like ***Bugsy***. It is a principle about the nature of truth. Our attitude to truth is classical.

While we offer no analysis of truth, we assume that every sentence which has a truth value has just one — it is either true or false. So that if truth or falsity applies at an index then every sentence is either true or false at that index, but not both. This rules out incomplete indices — indices where a sentence may be neither true nor false.

Similar arguments apply concerning what are called 'inconsistent' indices — indices where a sentence is *both* true *and* false. There have been critics of possible worlds who claim that if you accept *possible* worlds then you should also accept *impossible* worlds — worlds where a sentence can be both true and false. On p. 39 of Lycan 1994 we read "semantics needs impossible worlds." But the reasons he immediately lists are that semantics needs impossible *propositions*.[12] An impossible proposition is something true at *no* index, and so an impossible world would be a world at which something true at no index is true. Obviously the fact that there are no such indices does not mean that there are no impossible propositions. Our claim is that it is not coherent to ask whether an *index* is possible or impossible. The situation can be illustrated by sentences of the form $\alpha \wedge \sim\alpha$. Provided $\sim$ and $\wedge$ are *classical* negation and conjunction — that is, provided they obey their standard truth tables — there can be no index at which $\alpha \wedge \sim\alpha$ is true. We might of course choose a non-standard interpretation of $\sim$ and $\wedge$, but, as Perszyk 1993, p. 206f notes, you don't need to be in an impossible world to do *that*.

An impossible world is a world at which something impossible is true — i.e., a world at which something true at no world is true — and there are no such worlds. The temporal analogue would be a time at which something which is never true is true — and there are no such times. The analogue of an 'impossible' person would be what we might call a 'negative' person — a person such that something true of no one is true of that person — and there are no such people. Suppose that (5) is true for Bugsy, and suppose that all the other indices are held constant, so that we can assume that in $\langle w,t,\text{Bugsy}\rangle$ $w$ is the actual world and $t$ the present time. What kind of negative person would Bugsy have to be to make both (5) and

(24) I am not hungry

[12]In fairness to Lycan it must be pointed out how easy it is to suppose that the need for impossible propositions can lead to the postulation of impossible worlds. For instance Cresswell 1973, pp. 39-44 flirted with them under the name 'heavens', but recants in Cresswell 1985a — mainly because it became obvious that heavens were unable to perform any useful semantical work. The passage discussed from Lycan 1994 is based on parts of Lycan 1979. Impossible worlds are also defended in Yagisawa 1988. Yagisawa's article is criticised in Perszyk 1993. A defence of the incoherence of impossible worlds is found in chapter 3 of Stalnaker 2003, pp. 55-67.

true of him? Once you raise the question in this form you can see that this too is a category mistake. The issue is not the nature of the index — in this chapter *nothing* has been said about what indices really *are* — it is the classical nature of truth itself that stops (5) from being both true and false at a given index. There is nothing about the *index* that stops it. While one can *say* that indices are therefore complete and consistent we think that this is a misguided way of raising the question. If that is right then we can rule out incomplete or inconsistent times or worlds in advance of any metaphysical consideration of what times or worlds are. And indeed so far nothing we have said has committed us to any view about their metaphysical nature. We have argued that natural language semantics demands truth at indices, and that a classical view of truth demands that sentences which have truth values at all have exactly one at each index — and this remains so, whether the indices are worlds, times, people, or anything else.

# 3

# *Situated truth*

Imagine two business executives who each suddenly realise the truth of

(1) The meeting begins in five minutes.[1]

Executive A knows that it is now five minutes to ten, but thought the meeting was at eleven, and has just recognised the truth. Executive B knows that the meeting is at ten, but thought the time was only half past nine. B's ignorance may be expressed by saying that B has just realised the truth of

(2) It is five minutes to ten.

(2) is an interesting sentence in terms of its empirical content. Contrast it with (1). For things to be as executive A thinks they are the world would have to be different. For things to be as B thinks they are the world is the same but the time is different.[2] In the case of (2) there is no comparable distinction. If it is five minutes to ten there is *no* world in which it is not then five to ten. For such a world would be one in which at five to ten it is *not* five to ten. What this means is that a correct account of the objects of knowledge has to treat them as tensed

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[1]This example is inspired by Perry 1979. A similar point is made in Lewis 1979a, and will be discussed below. King 2003, p. 196, says "it is hard to make sense of the idea that the things I believe may change truth value across time and location." The case of the executives shows that very often these *do* seem to be the kind of things we can come to believe. Much of the argument of King paper depends on the assumption that they are not, so it is perhaps necessary to defend the view of the objects of attitudes as situated propositions.

[2]Mellor 1981, p. 29 asks: "If it were now the eighteenth century, how different would the world be?" Mellor treats this as a substantive question — he regards believers in tense as thinking that these two worlds could be different, and that, moreover, they could differ even in their *tenseless* facts. Bigelow 1991, for instance, advocates 'worlds' in which the things which occur in them have 'intrinsic' properties like pastness, presentness or futurity — so that there could be a world just like ours except that it is now tomorrow. From the perspective of this chapter it matters little whether you want to say that a 'world' is a 'world-at-a-time', or whether it is not, so long as you recognise the difference between the executives A and B in their attitude to (1).

propositions. (1) is true at $\langle w,t \rangle$ iff

(3) The meeting begins

is true at $\langle w,t' \rangle$ where $t$ precedes $t'$ by five minutes. (2) is true iff $t$ is five minutes to ten. In the case of (1) we notice two kinds of ignorance. Executive A knows what time it is, but not what world it is. So A may be said to learn that the actual world is one in which the meeting is at ten for (unlike executive B) A knows that it is ten o'clock in five minutes. B knows that the actual world is a world in which the meeting is at ten, but doesn't know what time it is.

Yet another source of ignorance has been discussed in Perry 1977 and 1979, and in Lewis 1979a. Perry 1977 considers the case of the madman Heimson who thinks he is Hume. There is no world in which Heimson is Hume. Yet Heimson does not suffer from a *logical* delusion. Lewis argues that Heimson's attitude is an attitude *de se* — an attitude concerning himself. He ascribes to himself the property of being Hume. Semantically the sentence

(4) I am Hume

is a sentence true at any time in any world of Hume, but false of everyone else.[3] So it seems that we require the three indices, worlds, times and persons postulated in chapter 2. We can call this phenomenon *situatedness*. Situatedness refers to the fact that in speaking of a sentence as expressing what a person knows, we imagine a particular person in a particular world at a particular time. So the basic notion is truth at a world at a time for a person. This is apt to provoke the criticism that truth at an index does not account for truth simpliciter. In Chapter 4 we shall look at a framework which incorporates a primitive unanalysed notion of absolute truth. In the present chapter we shall examine an indexical analysis of absolute truth. Indexicality has a number of traps. One is the following. For simplicity consider for a start only 'eternal' propositions — propositions like that expressed by

(5) Wellington is, was, or will be, the capital of New Zealand in 2005.

[3]There is an additional complexity about the word ***I***. The sentence 'Heimson believes that I am Hume' does not mean the same as 'Heimson believes that he is Hume', since ***I*** protects its referent in embedded contexts. In that respect it is like the words ***now*** and ***actual*** to be discussed in Chapter 7 below. (Problems caused by combining epistemic and modal notions are discussed in Egan, Hawthorne and Weatherson, 2005, but lie beyond the scope of the present book.)

(5), if true at all in a world, is true at every time in that world, and so the proposition it expresses may be treated as if it were simply a set of worlds. (See p. 31.) We must carefully distinguish between the proposition *a* and the proposition that *a* is true in a world *w*. For any (eternal) proposition *a*, if *a* is a set of worlds and *w* is a world then if $w \in a$ that is a set-theoretic and, therefore, a necessary fact. The proposition that Wellington is the capital of New Zealand in 2005 in *w* is in fact necessary — if Wellington is the capital of New Zealand in 2005 in *w*; or impossible — if it is not the capital in *w*. So it can hardly be the proposition that *a* is true in *w* — i.e. that Wellington is, was, or will be, the capital of New Zealand in 2005 in *w* — that (5) expresses. If you ask whether something is true without qualification then the answer to your question is a proposition. Your question and the answer both occur in a world and the proposition is either true or false in that world. If Tallulah tells you that a proposition is true and it is, then Tallulah has spoken truly. If Tallulah tells you that it is true and it is not, then Tallulah has spoken falsely. Tallulah has not told you that it is true or false in this or that possible world, even though her telling occurs in a possible world; and even though, in each possible world in which the telling occurs, the proposition is true or false according as that world is in the set of worlds which is the proposition. But suppose you can ask whether *a* in more than one world, and that the very same asking — is *a* true? — can occur in both $w_1$ and $w_2$; and that the telling of the answer — the very same telling — can occur in both $w_1$ and $w_2$. Here's how it goes schematically, for a proposition *a*, say, that Wellington is the capital of New Zealand, such that $w_1 \in a$ but $w_2 \notin a$:

(6) In $w_1$ Bugsy asks whether *a*

(7) In $w_2$ Bugsy asks whether *a*

Suppose that the very same event of Bugsy's asking occurs in two worlds $w_1$ and $w_2$. (It is an asking whether *a* in both worlds.)

(8) In $w_1$ Tallulah tells Bugsy that *a* is true

(9) In $w_2$ Tallulah tells Bugsy that *a* is true

Suppose that Tallulah's telling (the very same telling) also occurs in both $w_1$ and $w_2$.

(10) $w_1 \in a$ so *a is* true in $w_1$

(11) In $w_1$ Bugsy comes to know that $a$

(12) $w_2 \notin a$ so $a$ is *not* true in $w_2$

(13) In $w_2$ Bugsy does not come to know that $a$.

This little exchange is not intended to be committal on whether or not the same event *can* occur in more than one world, or even on what kind of thing an event is. But it *is* intended to show that an indexical account of absolute truth is compatible with allowing the very same event to occur in more than one world. A seemingly easier way of doing this might be available to those who follow Lewis 1986a in supposing that an individual can exist in no more than one possible world and at no more than one instant of time — in the sense that truth simpliciter for such a language user just means truth in that language user's world at that language user's time. This response is not available if you think that an individual can exist in more than one world or at more than one time. Luckily our diagnosis does not presuppose that an individual could not exist in more than one world or at more than one time. Everything we have said is compatible with the very same asking and answering occurring in more than one world.

A person who is ignorant and has a false belief thinks that a world which is *not* actual *is* actual. Suppose that Bugsy mistakenly thinks that Auckland is the capital of New Zealand. It may seem therefore that Bugsy thinks that the actual world is a world in which Auckland is the capital of New Zealand. In a sense he does, but in another sense he does not. Consider the following dialogue between us and Bugsy in a world $w^*$, the actual world, in which Wellington is the capital of New Zealand:

US Do you believe that Wellington is the capital of New Zealand?

BUGSY Of course not. Auckland is the capital.

US Now imagine a world $w^*$ in which Wellington is the capital of New Zealand.

BUGSY No problem.

US Do you believe that Auckland is the capital of New Zealand in $w^*$?

BUGSY Of course not. Wellington is the capital in $w^*$.

US Gotcha. $w^*$ is the actual world. In believing that Auckland is the capital of New Zealand you believe that Auckland is the capital of New Zealand in the actual world. You are mistaken of course in that though you believe that $w^*$ is $w^*$, and in fact $w^*$ is actual, you do not believe that $w^*$ is actual. So that even if $w^*$ is actual, believing that a proposition is true in the actual world is not the same as believing that it is true in $w^*$.

So why does Bugsy believe that Auckland *is* the capital of New Zealand? Because he does not believe that $w^*$ — the actual world — *is actual*. One thing this shows is that the content of Bugsy's belief that '$w^*$ is not actual' is *not* to be analysed as '$w^*$ is not $w^*$'.

But how can this be? What is it to ask, in a world $w$, whether $a$, or be told whether $a$, or come to know whether $a$, where $a$ is a set of worlds? Knowing what kind of world is actual can be explained using an epistemic version of the modal accessibility relation discussed on p. 21. This use of accessibility dates at least from Hintikka 1962. Hintikka's account treats the epistemic operator 'knows that' as a modal operator. For each person $p$ there is a relation of 'epistemic accessibility', $R_p$, (strictly $p$-accessibility to indicate the dependence on $p$) between worlds. $wR_pw'$ holds iff, as far as what $p$ knows in $w$ is concerned, the world could be $w'$. We now say that, in a world $w$, $p$ knows whether $a$ iff $a$ is true in every world $w'$ such that $wR_pw'$. The set of all worlds $p$-accessible from $w$ can be described as the *knowledge set* associated with $p$ in $w$. Suppose a proposition $a$ were *not* true in some $w'$ in $p$'s knowledge set in $w$, i.e., that $w' \notin a$. If $w'$ is in $p$'s knowledge set in $w$, that means that, for all $p$ knows in $w$, $w'$ could be the way the world is. So $p$'s knowledge set in $w$, i.e. what $p$ knows in $w$, *allows* the world to be $w'$, and $a$ is false in $w'$. So $p$'s knowledge allows $a$ to be false, which is to say that $p$ does not know that $a$. Contrariwise, suppose that $p$ does not know that $a$. Then, for all $p$ knows in $w$, there is a world $w'$ such that $w' \notin a$. So at least one world in which $a$ is false, i.e., at least one $w' \notin a$, must be in $p$'s knowledge set. Whatever may be said about this account of knowledge, it does make clear how and why knowing in a world $w$ the proposition that $a$, is different from knowing in $w$ the proposition that $a$ is true in $w$. Notice what would happen if the proposition that Bugsy comes to know in $w_1$ were the proposition that $w_1 \in a$. In the first place, as we said on p. 41, that proposition is either necessary or impossible. But even apart from that difficulty, if the proposition that Bugsy is asking about in $w_1$ is the proposition that $w_1 \in a$ then the proposition that Bugsy

is asking about in $w_2$ is the proposition that $w_2 \in a$, i.e. a different proposition.

An analogous problem arises in the case of tense. Craig 2000a, p. 95, writes:

> We want to know, not what makes *Jim races tomorrow* true on June 1, but what makes it true that *Jim is racing tomorrow* or that *Jim is racing*. If sentence types need truth makers, then we need tensed facts as the truth makers of such tensed sentence types. For if there are no tensed truth-makers, then it is inexplicable why *P* is true—not true at *t* mind you, but simply true.

The solution is this. Suppose that Tallulah asks Bugsy whether Jim is racing. Tallulah's asking occurs at a time, say 4.00pm on 1 June. When Bugsy tells Tallulah that Jim is racing he is *not* telling her that Jim is racing at 4pm, since Tallulah may already know that, but may not know that it is 4pm. The 'tensed' proposition (see p. 31) that Jim is racing is the set of $\langle w,t \rangle$ pairs such that Jim is racing in world $w$ at time $t$. This is different from the proposition that Jim is racing at 4pm, which is just the set of worlds in which Jim is racing at 4pm — or equivalently the set of all pairs $\langle w,t \rangle$ at which Jim is racing in $w$ at 4pm, and $t$ is any moment at all, whether or not Jim is racing at $t$. Just as Bugsy's coming to know in world $w$ that $a$ is true is *not* his coming to know that $a$ is true in world $w$, so Tallulah's coming to know that Jim is racing is not coming to know that Jim is racing at 4pm. It is just coming to know that Jim is racing. Of course if Tallulah knows that it is 4pm then she can conclude that Jim is racing at 4pm. But knowing that it is 4pm demands a proposition which is essentially tensed — that is to say a proposition which cannot simply be a set of worlds, but has to be a set of world-time pairs. The semantic moral is that coming to know something at an index is coming to locate the index in a set of indices. Cases like (4) require the addition of the 'person' index, because truth is now truth at a $\langle w,t,p \rangle$ triple, as introduced on p. 29. If your index is $\langle w,t,p \rangle$, i.e., if you are $p$ and the time is $t$ in a world $w$, then you know who you are iff the 'knowledge set' associated with $\langle w,t,p \rangle$ is included in $\{\langle w',t',p' \rangle : p' = p\}$. The term 'knowledge set' for triples works just as it does for worlds, as described on p. 43 except that the relativity to $p$ is now incorporated into the $\langle w,t,p \rangle$ triple: For any triple $\langle w,t,p \rangle$, i.e., for a person $p$ at $t$ in $w$, any other, or the same, triple $\langle w',t',p' \rangle$ is in $\langle w,t,p \rangle$'s 'knowledge set' iff nothing in what $p$ knows at $t$ in $w$ would prevent $p'$ from being $p$, or would prevent $t$ being $t'$, or $w$ being $w'$. For $p$ to know something in $w$ at $t$ requires that it couldn't fail compatibly with what $p$ then knows. For this to happen it would have to be true in every $\langle w',t',p' \rangle$ in $\langle w,t,p \rangle$'s knowledge set.

You, $p$, know, at $t$ in $w$, what time it is iff the knowledge set associated with $\langle w,t,p\rangle$ is included in $\{\langle w',t',p'\rangle : t' = t\}$. You would be propositionally omniscient iff the knowledge set associated with $\langle w,t,p\rangle$ is included in $\{\langle w',t',p'\rangle : w' = w\}$. If what has been said is correct, there can be knowledge relating to other indices beside the world index. One can know, or fail to know, what time it is, and one can know, or fail to know, who one is. This does of course assume that we can have the same time or individual in different worlds, but our discussion of possibility in Chapter 1 suggested many senses of 'possible' can be modelled by a structure for time in which worlds are 'time lines' and possible futures are lines which all coincide up to a certain time (the present) but branch thereafter. Such structures appear to assume individuals and times which can be part of different world-histories.

The account we have given also shows how to address another example often discussed in the literature, Prior's

(14) Thank goodness that's over

(Prior 1959). To avoid problems we shall take a version of (14) which makes it straightforwardly an object of a propositional attitude:

(15) I know that the 1960 exams are over

Suppose (15) is uttered on 1 September 1960. Then a first shot at formalising (15) might seem

(16) I know that the 1960 exams are over by 1 September 1960.

(16) won't do, since the proposition

(17) The 1960 exams are over by 1 September 1960

is true at any $\langle w,t,p\rangle$ iff the exams are over by 1 September in $w$, whereas

(18) The 1960 exams are over

is true at $\langle w,t,p\rangle$ iff the exams are over by $t$ in $w$. If we analyse this as above we have that (15) is true at $\langle w,t,p\rangle$ iff every $\langle w',t',p'\rangle$ in the 'knowledge set' associated with $\langle w,t,p\rangle$ is a $\langle w',t',p'\rangle$ at which (18) is true — i.e., is such that the exams are over by $t$ in $w$. By contrast (16) is true at $\langle w,t,p\rangle$ iff every $\langle w',t',p'\rangle$ in

the 'knowledge set' associated with $\langle w,t,p\rangle$ is a $\langle w',t',p'\rangle$ at which (17) is true — i.e., is such that the exams are over by 1 September 1960 in $w$. But suppose $p$ knows that the exams will be over by 1 September, but doesn't know it is yet 1 September. Then even when $t$ *is* 1 September, $p$ may not know that the exams are over, because even though every $\langle w',t',p'\rangle$ at which the exams are over at 1960 in $w'$ may be in the knowledge set of $\langle w,t,p\rangle$, yet some $\langle w',t',p'\rangle$ at which the exams are over $t'$ in $w'$ may not be.[4]

Perry 1979, p. 14, claims these 'new-fangled propositions' do not solve the problem. He correctly observes that his believing 'that I am making a mess'[5] is 'true for me', cannot express what he believes when he believes that he is making a mess, since anyone can believe that that proposition is true for Perry. Quite so, but recall what was said about worlds in the case of (5). Where $w^*$ is the actual world we have

(19) In $w^*$ Wellington is the capital of New Zealand.

Now Perry is a good logician, and Perry understands the proposition that Wellington is the capital of New Zealand, so

(20) Perry knows in $w^*$ that in $w^*$ Wellington is the capital of New Zealand.

This does not guarantee that Perry knows that Wellington is the capital of New Zealand, because *Perry may not know that* $w^*$ is actual. In fact if he does not know that Wellington is the capital of New Zealand then he will *not* know that $w^*$ is actual, for all that he knows, with respect to the actual world, that the proposition 'Wellington is the capital of New Zealand' is true in it. Knowing that a situated proposition is true for Perry at 10am is no more knowing that situated proposition than knowing that a certain (untensed) proposition is true in a certain world is knowing that proposition. We are labouring this point because articles like Perry's bring out so powerfully the phenomenon of situated knowledge that it is important to see that the 'new-fangled' propositions really are up to the task of solving the problem. An author who sees this clearly Lewis 1979a, and it is worth showing that Lewis's account of belief in terms of the self ascription of properties is exactly equivalent to our account in terms of situated propositions.

[4]This point is also made in Sider 2001, p. 20.

[5]Perry's paper begins with a story in which he followed a trail of sugar in a supermarket in order to tell the shopper that the shopper was making a mess — until he realised that *he* was the shopper.

For Lewis a *property* is simply a set (See Lewis 1979a, p. 515). But that is because for Lewis an individual exists only at a particular time in a particular world. Since we do not wish to commit ourselves to the view that a person cannot exist at more than one time or in more than one world we can construe a Lewisian 'person' as a $\langle w,t,p \rangle$ triple, and then, what for us is a situated proposition is for Lewis a property. In our terms a located individual $\langle w,t,p \rangle$ would self-ascribe a property $\omega$ iff the knowledge set associated with $\langle w,t,p \rangle$ is included in $\omega$.

Knowing which world you are in may be less common than knowing who you are or what time it is, and one possible difference between worlds and times might be sought by contrasting the case of executives A and B. Are there serious *epistemic* differences between executive A's ignorance and executive B's ignorance? It is a temporal matter what the time is. Maybe we never know *exactly* what the time is, but it seems that we can get close about that in a way in which we may not be able to about the world. In the case of time it seems all we need is a good clock. Is there any equivalent in the case of a world? It is a contingent matter how the world is. We never know completely what the world is like. We do not even come close. What is the modal equivalent of looking at a clock? Well, it's like the experience of becoming propositionally omniscient, except that you have to know what time it is to make use of it — much as you have to know what world you are in to use the clock. Suppose someone says to you:

> Over the next week I'm going to flash before your eyes a number of different possible worlds — scenes which display *everything* that could be going on. Of the possible scenarios which will flash by the one that occurs at 4.45pm on Thursday 26 September 2008 is the one which shows the actual world. I'm arranging the presentations in such a way that you will be put to sleep between each presentation for a varying length of time, so that there will be no way of predicting from the presentations themselves just what time it is.

The set of episodes forms a kind of modal clock,[6] which enables someone who knows what time it is to work out what world they are in. The example is of course no more realistic than the example of Lewis's omniscient gods. No doubt there is a factual difference. But it would seem to need more argument to show that the difference between the two kinds of ignorance leads to an ontological or

[6]Except that there is no commitment to displaying *every* possible world in this series of presentations. Leibniz (as discussed in Adams 1974) considers this further picture.

metaphysical difference between time and modality. We are, presumably, in a world in which it is comparatively easy to tell what time it is, but difficult to tell what world we are in.

# 4

# *The privileged position*

There is a view of time which says that all times are equally real, and that the appearance of anything special about today is illusory. This view is sometimes called the 'eternalist' view of time, and is contrasted with what is called 'presentism' — the view that only the present is real.[1] There is a view of modality called 'possibilism', which says that all possible worlds are equally real — that other worlds are, at least metaphysically, as real as the actual world in the way that other times are often thought to be metaphysically just as real as the present. This contrasts with 'actualism' — the view that only the actual is real.

In this chapter we address one alleged difference between presentism and actualism. One might be tempted to say the following. Among the many possible worlds there is one which is metaphysically privileged in a way none of the others are. It is actual. This is a powerful picture.[2] One might suppose that the temporal analogy fails at this point. No presentist thinks that (say) 8.15 am NZDT on 31 December 2008 is more real than any other time; yet actualism holds that one world is privileged. Surely that is a difference? But is it? What is it to regard a world or time as privileged? A tempting option for the actualist is to designate

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[1]There is sometimes an additional constraint put on presentism, that the domain of quantification is restricted to the present. (See for instance the articles in Part I of Zimmerman 2004, though note the slightly different view in Hinchliff 1996.) The version of presentism described in this chapter is what Cresswell 2006 calls 'alethic presentism', because it is the claim that although all truth is present truth there can be present truths about past and future individuals. We do take presentism to claim that the only kind of *existence* is present existence, because the only kind of truth is present truth. But we take the question of existence to be separate from the question of the appropriate domain of quantification. (See p. 73 below.)

[2]It was assumed in Lewis 1968, and in the presentation in Cresswell 1972 of the 'Tractarian' model introduced here on p. 203 below. Bricker 2001 p. 29 f suggests that realism about the actual should "hold that the actual and the possible must differ in kind" and that it is "an absolute fact as to which among all the worlds has been actualized." The temporal counterpart would be that it is an absolute fact which moment is present. Armstrong 1989, p. 3, defines 'naturalism' to be the doctrine that 'nothing at all exists except the single world of space and time'. The temporal counterpart would be that nothing at all exists except the present spatial world.

one of the possible worlds as the actual world, @, and define actual truth as truth in @.[3] In the early days of the possible worlds semantics for modal logic, things went somewhat as follows: You postulated what is now called a *frame* — a set of worlds and an accessibility relation; and then you added a distinguished 'real world', which we shall continue to call @.[4] When the language is interpreted in such a structure you can call a sentence true *simpliciter* iff it is true at @. From the point of view of pure logic the 'worlds', including @, are simply indices at which wff are true or false. The analogous temporal move would be to designate one moment as a privileged present. So what we need to do is see whether there are objections to such privileging in the case of tensed languages; and if there are, see whether they apply in the same way in the case of modal languages.

The indexical semantics appears to offer a reductive account of contingent truth. For on the indexical theory the meaning of a sentence is given, at least in part, by its truth conditions. For an eternal sentence α (see p. 40) one can treat the meaning $a$ of α as a set of possible worlds, and

(1) α is true in $w$ iff $w \in a$.

What then do we say about a person who is asking whether α is true simpliciter? One answer is to note, as on p. 41, that an *asking* whether α is true is an asking which occurs in a world — and the answer 'α is true' to an asking in a world $w$ is a correct answer in $w$ iff $w \in a$. The effect of this is to reduce the metaphysical notion of contingent truth to a notion of logically necessary truth. If propositions are thought of as sets of possible worlds then if $a$ is such a set, $a$ is true in $w$ iff $w \in a$. In a sense the *very essence* of $a$ is to specify which worlds it is true in, and it is the nature of $w$ and the nature of $a$ themselves which make it so that $a$ is true in $w$. Nothing *more* is needed. Truth in a world, it may be said, is internal to the nature of the proposition. And the case is exactly similar for temporal propositions. A tensed proposition $a$ may be thought of as a set of world-time pairs, so that $a$ is true at $t$ in $w$ iff $\langle w,t\rangle \in a$. It is still no doubt important to have an analysis of metaphysically necessary truth, but the indexical theory of actuality in terms of truth at a world allows a reductive analysis of contingency in exactly the same way as the indexical theory of the present in terms of truth at

[3]It has become customary to refer to the actual world as @, which was the symbol used in Lewis 1968.

[4]Kripke 1959 p. 3 and 1963 p. 68f used the phrase 'model structure' to denote the set of worlds, the (accessibility) relation, and the distinguished world, which he called G. A model then assigns values from this structure to the atomic sentences in a way which makes every wff true or false at any given world, and a wff is *true simpliciter* in the model iff it is true at G.

a time may be thought to give a reductive analysis of tense. What then are the logical objections to privileging some particular time, say 2011? Look at a sentence like

(2) 2008 was once present

assuming that it is 2011. If we have designated 2011 as the present time, then (2) would seem to mean the same as

(3) 2008 was once 2011.

Of course we *could* use (2) to mean the same as (3), but then it would be trivially false, and we would lose any explanation of what makes 2011 *present*. For 2008 was never 2011, and any connection between tense and the present has been lost. Clearly this is not a viable way of proceeding. Doesn't a presentist rather say something like the following? Either the present is a single entity which was once realised by moments in 2008 and is now realised by a moment in 2011 and will be realised by moments in 2012, *or* being present is a *property* of moments in 2008, 2009 and so on — a property which things have but once lacked or will lack, or which lack but once had or will have. Now consider the parallel. Could (should) an actualist say that actuality is a single entity which might happen to be realised by $w_1$, but which could have been realised by $w_2$? Or might one say that being actual is a property that $w_1$ has but $w_2$ might have had? Either way it would seem that being actual cannot mean the same as being $w_1$, even if $w_1$ is actual. We say

(4) @ might not have been actual.

If we have designated the actual world as @ then, by analogy with (3), (4) would mean the same as

(5) @ might not have been @.

As with the temporal case we *could* use (4) to mean the same as (5), but then we would lose any explanation of what makes @ *actual*.[5]

We have already (p. 44) referred to Craig's complaint that the indexical semantics for tense only defines truth at a time, and not truth simpliciter.

[5]See van Inwagen 1980, p. 424, and Meyer 2006, p. 30f

Similarly, an actualist might complain that a possibilist only defines truth at a world, and not truth simpliciter. In order to approach this question we shall present two versions of actualism, those developed by Robert Adams and Alvin Plantinga. Adams's version is presented on pp. 224-228 of Adams 1974. Adams has just considered a number of theories of actuality, including the indexical theory, and finds them all wanting.[6] He then writes:

> I prefer, therefore, a different approach, which I call an *actualist* theory of actuality, as opposed to the theories discussed in Sections II-IV above, which I call *possibilist.* They begin with the whole system of possible worlds and see the actual world first of all as a possible world, a member of that system.
>
> I propose to begin, instead, with the actual world, to treat talk about the system of possible worlds as a way of talking about a proper part of the actual world, and thus to gain, so to speak, a standpoint outside the system of possible worlds from which judgments of actuality which are not world-relative may be made. *Actualism*, with respect to possible worlds, is the view that if there are any true statements in which there are said to be nonactual possible worlds, they must be reducible to statements in which the only things there are said to be are things which there are in the actual world and which are not identical with nonactual possibles. The actualist will not agree that there are nonactual possible worlds, if the notion of possible worlds is to be regarded as primitive. *Possibilism*, with respect to possible worlds, is the view that there are nonactual possible worlds and that the notion of a possible world is not to be analyzed in terms of actual things. The difference between actualism and possibilism may be seen in some cases as a difference in order of analysis, but it is not a trivial difference. As we shall see, it may involve the difference between an absolute and a world-relative concept of truth. (Adams 1974, p. 225)

The crucial sentence in this quotation is the last, which distinguishes between 'an absolute and a world-relative concept of truth', and we shall explore this difference. Adams follows up on p. 227 with

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[6]Adams's discussion of the indexical theory is on pp. 214-220, though the only version of the indexical theory that Adams appears to consider is David Lewis's, whereby no ordinary individual can exist in more than one world or at more than one time. Most of Adams's objections depend on this (inessential) feature.

> We must distinguish between the notion of truth and the world-relative notion of truth *in* a possible world. In the true-story theory of actuality, the notion of truth is presupposed, if not as primitive at least as prior to the notion of actuality, since the latter is analyzed in terms of the former.

In addition to an absolute notion of truth Adams also has primitive propositions and primitive modality. To be specific:[7]

> Let us say that a world-story is a maximal consistent set of propositions. That is, it is a set which has as its members one member of every pair of mutually contradictory propositions, and which is such that it is possible that all of its members be true together. The notion of a possible world can be given a contextual analysis in terms of world-stories. Of the following statement forms, for example, (1), (3), and (5) are to be analyzed as equivalent to (2), (4), and (6), respectively.
>
> (1) There is a possible world in which *p*.
> (2) The proposition that (*p*) is a member of some world-story.
> (3) In every possible world, *q*.
> (4) The proposition that (*q*) is a member of every world-story.
> (5) Let *w* be a possible world in which *r*. In *w*, *t*.
> (6) Let *s* be a world-story of which the proposition that (*r*) is a member. The proposition that (*t*) is a member of *s*.
>
> A similar contextual analysis can now be given to the notion of actuality. 'In the actual world, *p*' is to be analyzed as 'The proposition that (*p*) is true.'

Adams does not apply his construction to the temporal case, but it is not difficult to do so. Take the sentence

> (6) Mt Ruapehu is erupting

[7]The numbering here is Adams's. A word should be said about Adams's use of 'maximal consistency'. Maximality can be understood so that for every proposition *p* the set contains either *p* or not-*p*; but in logic 'consistency' is a proof-theoretical notion, defined syntactically in a formal system of logic. Adams has to have in mind an intuitive notion like 'jointly possible'. The difference is not trivial. Since most formal systems are 'finitary' then any set which is not consistent has a finite inconsistent subset, and it is not at all clear that this is a reasonable condition to impose on the intuitive notion of joint possibility. (See Menzel 1989 and Cresswell 2006b.)

(We have resumed our own numbering from now on.) Should we put (6) into a maximally possible set of propositions? Well that depends. (6) was briefly true in July 1996 and has been mostly false since then. How then can we interpret (6) in tune with Adams's actualist assumption of primitive truth? The answer is to follow Prior and take simple truth to be actual present truth, and take propositions to be things which can change their truth value as time passes. One says that a set of such propositions is simultaneously possible iff it is possible that it was once so that they there were all true, or that it is so that they are all true, or that it will be so that they are all true. A world is then a maximal simultaneously possible world-story In this construction of course a 'world' $w$ — i.e., one of Adams's 'world-stories' — is a world at a time, and what are normally called worlds will be chains of these, where the structure of time is determined by the truth or falsity of the tensed truths which are taken as primitive.

There is however what seems a problem about this construal of Adams's construction. For there seems nothing in it to distinguish between cases in which a maximally possible set of propositions represents a prior or subsequent time in the actual world, or whether it represents a time in another possible world. The solution is this. If one is to construe Adams as assuming that primitive truth is present truth, then, like Prior, one must assume not only primitive modality, but also primitive tense. This indeed was a large part of Prior's motivation for developing tense logic. So suppose that $w$, in this version of Adams's theory is the set of all propositions which are now actually true. Among these will be propositions of the form '*p was* true' or '*q will be* true'. Then suppose that $w'$ is a maximally possible set of propositions with the property that, according to the actual world-story, either it was once so that *all* the propositions of $w'$ were (together) true, or that it will one day be so that all the propositions of $w'$ will (together) be true. In such a case $w'$ is an earlier or later time in the actual world. More generally, where $w$ is *any* maximally possible set of world-stories, then, for any other $w'$, we can say whether or not $w'$ is an earlier or later part of the same world as $w$, by using truth in $w$ in place of actual truth. What this means is that everything in Adams's actualist metaphysics can be understood in terms of Prior's conception of primitive truth as tensed.[8]

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[8]Remarks on p. 188f of Prior 1967 suggest that Prior was aware of the points made in the text. A presentist construction of times as abstract entities based on taking truth simpliciter to be present truth is found in chapter 3 of Bourne 2006. On p. 56 Bourne acknowledges a debt to Adams. Cappelen and Hawthorne 2009, advocate a view like Adams's. On p. 3 they call the claim that truths simpliciter might have been otherwise 'contingency', and on p. 4, they call the claim that there are propositions which are true simpliciter but will be false or were once false 'temporality'. They stress that both these are compatible with what they call truth simpliciter. Not all philosophers are happy about taking simple truth as metaphysically primitive. See Meyer 2006, p. 31f.

In order to see how to use this idea to provide an explicit construction of times and worlds we shall turn to the view presented in Plantinga 1976. Plantinga bases his metaphysics on *states of affairs* (p. 257). Although he is inclined to distinguish states of affairs from propositions he hopes that little of what he says will be affected if you think that states of affairs *are* propositions, provided of course that you don't analyse propositions as sets of possible worlds. As an example of a state of affairs Plantinga instances Quine's being a distinguished philosopher. He contrasts this with Quine's being a distinguished politician. While both states of affairs *exist*, only the former is actual or *obtains*. A *world* is a maximal state of affairs, i.e., *w* is a possible world iff *w* is a state of affairs such that it is possible for *w* to obtain, and, for any state of affairs *s*, if it is possible for both *w* and *s* to obtain then it is not possible for *w* to obtain and *s* not to obtain. All of this presupposes that we have a sufficiently robust notion of a possible state of affairs, but Plantinga takes the notion of states of affairs as 'obvious'. We also need to take as primitive what it is for a state of affairs to *obtain*, so that we can define the actual world @ as the world which obtains.[9] What then of the status of @? Plantinga on p. 258 is explicit that @ is an abstract entity just like all the other worlds. Its actuality consists in its being a state of affairs which (actually) obtains. Its maximality guarantees that it is the *only* world which actually obtains.

Plantinga (1976 p. 263) carefully distinguishes Quine's being the author of 'Word and Object' (which is a contingent state of affairs) with Quine's being the author of 'Word and Object' in @, which is a necessary fact, whose necessity Plantinga uses to establish Quine's essence. Plantinga notes that this assumes that "it is *truth* that is the basic notion." In particular he comments that even when @ is the actual world "truth-in-@ is to be defined in terms of truth plus modal notions." And of course even if it is so that *p* is in fact true iff it is true in @, that cannot be the *analysis* of *p*'s truth, since *p* would still be true in @ *even if* @ were not actual and *p* were false. On p. 259 Plantinga defines 'true in *w*' so that

(7) *p* is true in *w* iff it is impossible for *w* to obtain and *p* to be false.

In examining the temporal parallel we need to be a little more precise. Take Plantinga's example, and let us suppose that Quine's philosophical distinction dates from the publication of 'The problem of interpreting modal

[9]We continue to use Lewis's symbol @. Plantinga 1976, p. 259 calls the actual world α. We use @, even in quoting Plantinga, because we use α as a metavariable for sentences of wff of a language.

logic' in the 1947 *Journal of Symbolic Logic*, and ended at his death in 2000. Plantinga was writing say in 1976, and the state of affairs of Quine's being a distinguished philosopher then obtained, though presumably now does not. What then of a 'world' in Plantinga's sense? Should we include David Lewis's being more famous than Quine? Whatever we may say about the relative fame of these two we would put it into Plantinga's 'actual' world depending on whether Lewis was *then* more famous than Quine. Notice that Plantinga's construction of worlds depends on taking 'possible' as given. That is to say we begin with a language in which there are modal words. But in speaking of Quine as a distinguished philosopher we also have *tense*. Plantinga in his 1976 article does not talk about time but he has done so on p. 90f of Plantinga 1985, and it is not at all hard to see that everything he says about worlds can easily be made to apply to times. We can classify states of affairs in the following way. Quine's being a distinguished philosopher is a state of affairs which obtains (or does not obtain) not only in a possible world but also at a moment of time. Following Plantinga 1985 call a state of affairs *s temporally invariant* if it is not possible for *s ever* to obtain (i.e., if it is not possible that *s* ever did, or does or ever will obtain) unless *s always* obtains. An example would be Quine's being a distinguished philosopher in 1976, where this is understood to be true, even today, because in 1976 he *was* distinguished. We can then define a possible world as a temporally invariant maximally possible state of affairs — i.e., (i) *w* is a temporally invariant state of affairs, and (ii) it is logically possible that *w* obtain, and (iii) for any temporally invariant *s* if it is possible for both *w* to obtain and for *s* to obtain then it is not possible for *w* to obtain and for *s* not to obtain.

Corresponding to temporally invariant states of affairs there are modally invariant states of affairs, where a state of affairs *s* is *modally invariant* if it is never so that *s could* obtain unless *s must* obtain. An example would be its being 1976. On the assumption that '1976' is a proper name (relative say to NZDT) then in 1976 it is necessary that it is 1976, and when it is not 1976 it is necessary that it is not 1976. We might then define a time *t* as a maximal modally invariant state of affairs. We can now say that *s* obtains in *w* at *t* iff it is possible both that *w* obtain and that it is, was or will be that the time is *t* and *s* obtains. (On p. 235 we will give a precise formal account of how to obtain these models from Plantinga's metaphysics.[10]) The state of affairs of Quine's being a distinguished philosopher now corresponds in an obvious way to the set of pairs $\langle w,t\rangle$ such that it is now *t* and is actually *w*, and Quine is a distinguished philosopher at *t* in *w*,

[10]Discussion of a similar construction in terms of world propositions is found in Prior and Fine 1977. Prior certainly took propositions as tensed, and thought of a world as a world at a time.

where *w* is a temporally invariant world, and *t* is a modally invariant time. The stage we have reached is this. It does indeed seem that we can think of possible worlds as constructed with the aid of primitive actual truth and primitive modality. But we have also seen that this construction can equally be applied to the construction of times with the aid of primitive present truth and primitive tense. In this respect presentism is just like actualism. Being present is a temporally varying fact in the sense that although 2008 was once present it is no longer present, so that privileging the present is quite different from privileging a particular moment, for the moment which is now privileged once was not and soon will not be. Similarly, privileging a possible world is a contingent matter in that the world which is in fact privileged might not have been.[11]

The reductive theory of tense claims that metaphysical theorising should be done in an untensed language. One can talk in a tensed language or in an untensed language. In a tensed language the present *is* privileged, the past *was* privileged, and the future *will be* privileged. If you eschew a tensed language then indeed no moment is privileged. As we noted on p. 22 the modal counterpart of an untensed language is a non-contingent language. One can talk in a contingent language or a non-contingent language. In a contingent language the actual *is* privileged while the merely possible might have been privileged. By contrast then, just as a tenseless language cannot privilege the present, a non-contingent language cannot privilege the actual. So there seems no distinction between the way the present is privileged and the way the actual is privileged.

[11]For some reflections on the theological implications of such questions see Rini 2007.

# Part II

# Predicate Logic: Tense and Modal

# 5

# *A formal language*

The principal theme of Part II of this book is that an adequate appreciation of the world-time parallel demands the resources of formal logic, and the purpose of this chapter is to set out a description of a language of modal and tense logic, and provide an indexical semantics for it. The chapter presents fairly standard material, and can be omitted by those who are familiar with intensional predicate logic. It does however establish our notation, and will provide a reference chapter for later passages in which we present material in ways which might otherwise appear sloppy. In discussing these formal languages we shall not be concerned with axiomatising their logic, but solely with a presentation of an indexical semantics. We shall first set out a description of the languages of (untensed and non-modal) first-order predicate logic. We shall then show how they can be extended to become formalised tense and modal predicate languages. The notation and terminology we shall use is from Hughes and Cresswell 1996.

In specifying a formal language we first provide a list of its primitive symbols, and then a mechanism for specifying which finite sequences of these symbols count as well-formed formulae (wff). A language $\mathfrak{L}$ of the (first-order or *lower*) predicate calculus (LPC) contains as primitive symbols

(a) For each natural number n ($\geq 1$) a set of n-place *predicates* $\varphi$, $\psi$, $\chi$, ... etc., where this set is possibly finite, or even empty for some n, but at most denumerably infinite.
(b) A denumerably infinite set of *individual variables*, $x, y, z$, ... etc.
(c) The five symbols $\perp$, $\supset$, $\forall$, (, and).

The well-formed formulae (wff) of $\mathfrak{L}$ are those and only those finite sequences of symbols which satisfy the following formation rules:

**FR1** Any sequence of symbols consisting of an n-place predicate followed by

n (not necessarily distinct) individual variables is a wff.

**FR2** $\perp$ is a wff. (Wff formed in accordance with FR1 or FR2 are called *atomic wff*.)

**FR3** If $\alpha$ and $\beta$ are wff so is $(\alpha \supset \beta)$.

**FR5** If $\alpha$ is a wff and $x$ is an individual variable then $\forall x\alpha$ is a wff.

Other symbols may be introduced by definitional abbreviation

[Def $\sim$] $\sim\alpha =_{df} (\alpha \supset \perp)$
[Def $\vee$] $\alpha \vee \beta =_{df} (\alpha \supset \beta) \supset \beta$
[Def $\wedge$] $\alpha \wedge \beta =_{df} \sim(\alpha \supset \sim\beta)$
[Def $\equiv$] $\alpha \equiv \beta =_{df} (\alpha \supset \beta) \wedge (\beta \supset \alpha)$
[Def $\exists$] $\exists x\alpha =_{df} \sim\forall x\sim\alpha$

$\supset$ and $\perp$, together with the defined symbols $\sim$, $\vee$, $\wedge$ and $\equiv$, are discussed on p. 14. The symbol $\forall$ is used to say that everything satisfies a certain condition, and is called the *universal quantifier*, while the symbol $\exists$ is used to say that something satisfies a certain condition, and is called the *existential quantifier*. Suppose we wanted to express the fact that all cats are animals. If we interpret the one-place predicates $\varphi$ and $\psi$ as, respectively, 'is a cat' and 'is an animal' then we can express this fact by the wff

(1) $\forall x(\varphi x \supset \psi x)$.

(1) says that for every $x$, if $x$ is a cat then $x$ is an animal. Alternatively, if $\psi$ is a two-place predicate and is interpreted as 'chases' then we can express

(2) every cat chases a cat

as

(3) $\forall x(\varphi x \supset \exists y(\varphi y \wedge \psi xy))$.

In a wff of the form $\forall x\alpha$, $\alpha$ is said to be the *scope* of the quantifier $\forall x$. An occurrence of a variable $x$ in a wff $\alpha$ (not as part of a quantifier) is said to be *free* or *bound* in $\alpha$. If $x$ does not lie within the scope of any quantifier which contains $x$ it is said to be *free* in $\alpha$. If $x$ is free in $\alpha$ it is said to be *bound by* $\forall x$ in $\forall x\alpha$. Thus in the wff

(4) $\forall x(\varphi x \vee \psi xy)$

the $x$ occurring after $\varphi$ and $\psi$ is bound (by the quantifier $\forall x$ at the beginning) but $y$ is free. Note however that even when we are speaking of (4) we say that $x$ is free in $\varphi x \vee \psi xy$, since no quantifier containing $x$ occurs in *that* expression, though of course $x$ is not free in (4) itself. Note also that it is *occurrences* of variables that are bound or free, and that the same variable may occur both bound and free in the same formula, as, e.g., $x$ does in $\forall x \varphi x \supset \varphi x$. Note thirdly that in a wff like $\forall x(\varphi x \vee \forall x \psi x)$ the first occurrence of $x$ is bound by the outermost (initial) quantifier, and the second by the innermost quantifier.

In order to interpret wff of LPC we use what is often called a 'universe of discourse', or in technical terms a *domain*. The quantifiers are then said to *range over* the domain. This means that they refer to everything or to something from the domain in question. So to interpret a wff of LPC we must specify a domain D, and must interpret the predicates in that domain. Consider (1). If D is a domain which includes animals then some subset of D will be those animals which are cats. If A is the set of those members of the domain which are cats, then A will be the interpretation of the predicate $\varphi$ when $\varphi$ means 'is a cat'. And if B is the set of animals in D then B will be the interpretation of the predicate $\psi$ when $\psi$ means 'is an animal'. For a two-place predicate it is a little more complicated. If $\psi$ means 'chases' then we need to consider pairs from the domain. If C is the set of pairs $\langle u,v \rangle$ where $u$ and $v$ are both in D and $u$ chases $v$, then C will be the interpretation of $\psi$ when it means 'chases'. To get an interpretation, or *model*, for a language $\mathfrak{L}$ of LPC we form the pair $\langle \mathrm{D,V} \rangle$ where D is any class of objects we please, and V is a function such that where $\varphi$ is an n-place predicate in $\mathfrak{L}$ then $\mathrm{V}(\varphi)$ is a class of n-tuples from D. The idea is that $\langle u_1, \ldots, u_n \rangle \in \mathrm{V}(\varphi)$ iff $u_1, \ldots, u_n$ (in that order) stand in the n-place relation which is the meaning of $\varphi$.

We must now give rules for evaluating wff of $\mathfrak{L}$. In defining $\langle \mathrm{D,V} \rangle$ we have made no mention of the individual variables. The reason is this. The kind of LPC wff that we are ultimately interested in are those like (1) and (3) in which there are no free variables. Such wff are called *closed* wff, or sometimes *sentences*. The simplest kind of closed wff is a wff like

(5) $\forall x \varphi x$.

In any interpretation $\langle \mathrm{D,V} \rangle$ (5) will be true if $\mathrm{V}(\varphi) = \mathrm{D}$, and false otherwise, and this fact does not depend on the value of $x$. By contrast consider a wff with a free variable, say

(6) $\varphi x$.

Is (6) true or false? Well, it depends on what $x$ is. We could of course require V to give values from D to the individual variables as well as to the predicates. But in obtaining the value of (5) from (6) we need to refer to all the possible values $x$ might have. For this reason it is convenient to separate the value-assignment to the individual variables from the model itself. So we say that $\mu$ is a *value-assignment* based on a model $\langle D,V\rangle$ provided that, for every variable $x$ in $\mathfrak{L}$, $\mu(x)$ is a member of D. We shall use 1 and 0 for the 'truth values' true and false respectively, and then write

$$V_\mu(\alpha) = 1$$

to mean that $\alpha$ is true in the model $\langle D,V\rangle$ when the individual variables are given the values assigned them by $\mu$. Thus for atomic wff we have

[V$\varphi$] $V_\mu(\varphi x_1 \ldots x_n) = 1$ if $\langle \mu(x_1), \ldots, \mu(x_n)\rangle \in V(\varphi)$ and 0 otherwise.[1]

What [V$\varphi$] means is that $\varphi x_1 \ldots x_n$ is true, with respect to $\mu$, iff the n-tuple made up from the individuals $\mu$ assigns to $x_1, \ldots, x_n$, is in the set of n-tuples that V assigns to $\varphi$. For $\perp$ and $\supset$ the procedure is obvious.

[V$\perp$] $V_\mu(\perp) = 0$

[V$\supset$] $V_\mu(\alpha \supset \beta) = 1$ if either $V_\mu(\alpha) = 0$ or $V_\mu(\beta) = 1$ and 0 otherwise.

The complexity comes with the quantifiers. Take $V_\mu(\forall x\alpha)$. Obviously we want $V_\mu(\forall x\alpha)$ to be true when $V_\mu(\alpha) = 1$. But this only tells us that $\alpha$ is true when $x$ is given the value that $\mu$ assigns it. For $\forall x\alpha$ to be true we want $\alpha$ to be true whatever value from D is assigned to $x$. So we have to think, not just of $\mu$, but of any assignment $\rho$ which is unconstrained in the value it gives to $x$. That is, $V_\rho(\alpha) = 1$ for any such $\rho$. But we must be careful. For consider the wff $\forall x\varphi xy$. In evaluating this wff with respect to $\mu$ we permit $\rho$ to give any value whatsoever to $x$, but we need to keep the same value for $y$ as $\mu$ gives, since $y$ remains free in $\forall x\varphi xy$. So we say that $\rho$ is an *x-alternative* of $\mu$ iff for every variable $y$ except (possibly) $x$, $\rho(y) = \mu(y)$. We then say

[1] Following Tarski 1936 some logicians prefer to speak of those sequences of members of the domain which *satisfy* the predicate, and thus by extension which satisfy a wff with free variables. (See for instance Mendelson 1964, p. 50.) Thus $\langle u_1,\ldots,u_n\rangle$ satisfies $\varphi x_1 \ldots x_n$ iff $\varphi$ is true, in the interpretation, of that n-tuple, i.e. iff $\langle u_1,\ldots,u_n\rangle \in V(\varphi)$. Such logicians would only speak of truth and falsity, strictly understood, in the case of closed wff.

[V∀] $V_\mu(\forall x\alpha) = 1$ if $V_\rho(\alpha) = 1$ for every $x$-alternative $\rho$ of $\mu$, and 0 otherwise.

By Def ∃ this means that we have

[V∃] $V_\mu(\exists x\alpha) = 1$ if there is an $x$-alternative $\rho$ of $\mu$ such that $V_\rho(\alpha) = 1$, and 0 otherwise.

What we notice about such a language is that, except for the relativity to an assignment to the variables, the truth value of any wff is not relative to any index, such as a time or a world. And if the wff is a closed wff, without any free variables, one can prove that even the relativity to the assignment to the variables does not affect the truth value. It is in this sense that $\mu$ and $\rho$ are only mechanisms for establishing the truth values of whole sentences. And so there is an absolute truth value for the wff which represent the kind of natural language sentences we are interested in. There is of course the relativity to V, because the very same predicate which is interpreted in one way by some V could equally be interpreted in another way by some V′ different from V. This relativity only matters when we are comparing the meaning of an expression in one language with its meaning in another language — or rather, when we are contrasting what a predicate means according to one interpretation with what it means according to another.

It is important to contrast relativity to an interpretation with the kind of relativity to an index which we have been looking at in earlier chapters, and which we are about to introduce when we extend predicate languages with the addition of the temporal and modal operators. Early objections to modal logic by W.V.Quine suggest that he believed that the only acceptable notion of necessity was in terms of logical *validity*, in the sense that a wff of LPC is *logically valid* iff it is true for every assignment to its variables in *every* interpretation. The failure to appreciate the difference between logical necessity — truth in every possible world — and logical validity — truth according to every interpretation — is we believe responsible for an unfortunate confusion in recent philosophy. Some philosophers have begun to make a distinction between logical possibility and what is sometimes called *metaphysical* possibility (with correlative senses of necessity).[2] Another way of marking this supposed distinction is by talking of

[2]Points similar to those made in the text are also made in Stalnaker 2003, p. 203. Jackson 1994, pp. 67-86 contains a defence of the view that work by Kripke 1972 and Putnam 1975 and elsewhere give no support to the claim that logical (or as Jackson calls it 'conceptual') necessity is a different kind of necessity from metaphysical necessity. See also Cresswell 1994, pp. 72-93, and Chalmers 1996, pp. 136-138.)

*narrowly* logical possibility and *broadly* logical possibility. Our use of the term logical possibility is always in the sense of metaphysical possibility — if by this is meant that a proposition is logically possible if it is true in at least one possible world without restriction, and logically necessary if it is true in every possible world without restriction — but we should say a few words about why we think that the recent use of the distinction between logical and metaphysical possibility is unfortunate. In the early days of modal logic Quine accused its proponents of confusing use and mention.[3] Logical truth, he claimed, is an appropriate classification of *sentences* — or perhaps even of schematic sentences. Thus we can say that $\alpha \supset \alpha$ is logically true, since its truth is independent of the meaning of $\alpha$ and is a consequence of the meaning of the logical symbol $\supset$. Quine was certainly right that the logical truth of the *schema* can be understood in that way; but all that that establishes is that every instance of the schema is *true*, and tells us nothing about necessity. The schema only expresses a proposition when $\alpha$ is given a particular meaning. The meaning of $\supset$ does indeed guarantee that when $\alpha$ has as its semantic value a particular proposition then $\alpha \supset \alpha$ will express a proposition which is not only true, but necessary; but that is because the truth table for $\supset$ is constant from world to world, and ensures that that particular instance of $\alpha \supset \alpha$ is true in every possible world, whether or not the proposition expressed by $\alpha$ is true in that world or false in that world. The sense in which the proposition expressed by $\alpha \supset \alpha$ for a particular $\alpha$ is *necessary* is just metaphysical necessity — truth in every world. It is not modal logicians who are guilty of use-mention confusion but those who treat logical and metaphysical possibility and necessity as different kinds of possibility or necessity applying to the same objects.

In this book, when we talk about the relativity of a sentence to a time and a world we do not have in mind that the expressions in the sentence change their meanings from index to index. Although a sentence in a tensed and modal language may change its *truth value* from time to time or world to world, it is the same proposition which is the meaning of the sentence which has a different truth value at a different index. For that reason, although the formal truth definitions we shall provide in this chapter all make explicit reference to a particular interpretation, we shall in later chapters frequently proceed as we did in earlier chapters by speaking simply of a sentence as true or false at an index, having

[3]See Quine 1960, p. 195f. Barcan 1961 (p. 5 of Barcan 1993) says that Quine seems to believe of modal logic "that it was conceived in sin: the sin of confusing use and mention"; and in Quine 1961, p. 323, Quine says that she "struck the right note". From another perspective it may be suggested that it was Quine who was guilty of a sin; the sin of confusing the form/content distinction with the necessary/contingent distinction.

some particular but only partially specified interpretation in mind.

The languages of tense and modal predicate logic extend LPC by including operators such as the *F, P*, *L*, *M*, □ and ◊ that we have already met. A tense-modal predicate language of the kind we shall assume in this book will add to LPC these operators together with two new operators, a one-place operator *K* to represent 'knows that', in the sense given on p. 44, where *K* refers to some unspecified 'knower' held constant for each occurrence of *K*, and a two-place modal operator □→ to represent the 'counterfactual conditional', where $\alpha \,\Box\!\!\rightarrow\, \beta$ is to mean that if α were true then β would be too. For convenience we shall take *G*, *H*, *L*, *K*, □ and □→ as primitive. The formation rules then extend FR1-FR5 by

**FR6** If α is a wff so are $G\alpha$, $H\alpha$, $L\alpha$, $\Box\alpha$ and $K\alpha$
**FR7** If α and β are wff so is $(\alpha \,\Box\!\!\rightarrow\, \beta)$

We may define *F*, *P*, ◊, ⊕ and ⊗ in the usual way:

[Def *F*] $F\alpha =_{df} \sim G\sim\alpha$
[Def *P*] $P\alpha =_{df} \sim H\sim\alpha$
[Def ◊] $\Diamond\alpha =_{df} \sim\Box\sim\alpha$
[Def ⊕] $\oplus\alpha =_{df} (H\alpha \wedge \alpha \wedge G\alpha)$
[Def ⊗] $\otimes\alpha =_{df} (P\alpha \vee \alpha \vee F\alpha)$

We can also define a 'possibility' sense of *K*, whereby $K_{\Diamond}\alpha$ means that 'for all the agent knows α could be true':

[Def $K_{\Diamond}$] $K_{\Diamond}\alpha =_{df} \sim K\sim\alpha$

To interpret ordinary LPC all we need to provide is a domain D and a value assignment V to the predicates. For tense and modal languages we need some extra resources to reflect the dependence on times and worlds that we have seen in previous chapters. We shall, in this chapter, ignore the person co-ordinate, though the remarks made on p. 29 ought to indicate how it would go. We need the following:

(i) A set W of 'possible worlds'.
(ii) An 'accessibility' relation R between worlds, where $w_1 R w_2$ means that $w_2$ is a possible world from the point of view of how things are in $w_1$. R interprets *L*. As we explained in Chapter 1 there are many senses of 'necessary' and 'possible', and we are

using R as the relation for one of these without specifying which. In this book it will not be necessary to consider sentences in which there are a number of different *L*s with their corresponding *M*s.

(iii) A set T of moments of time.

(iv) A (transitive and irreflexive) linear ordering $<$ on T so that $t_1 < t_2$ means that $t_1$ precedes $t_2$. I.e., no time precedes itself, and if $t_1 \neq t_2$ then either $t_1 < t_2$ or $t_2 < t_1$. $<$ is used to interpret *G* and *H*

(v) For any $t \in$ T, K is a four-place relation between worlds and times, used to interpret *K*. *K*α is intended to mean that some particular agent (unspecified but assumed the same throughout the formula) knows, at $t$ in $w$, that α is true. I.e., if K($w$,$t$,$w'$,$t'$) holds then $\langle w',t' \rangle$ is in the agent's 'knowledge set' at $t$ in $w$.[4] The motivation for such an account is described in Chapter 3 on p. 44, though in that chapter the agent *p* was explicitly introduced as a third index. As well as interpreting *K*, the relation K is also an example of an accessibility relation which depends on both worlds and times.

(vii) Finally, to interpret □→ there is a four place 'counterfactual' relation C, such that for any $t \in$ T and $w_1$, $w_2$ and $w_3 \in$ W, C($t$,$w_1$,$w_2$,$w_3$), written $w_2 <_{\langle w_1,t \rangle} w_3$, means that, at $t$, $w_2$ is 'closer' to $w_1$ than $w_3$ is. Closeness is used to convey the idea that if you want to know whether β would be true if α were, and α is not true, you look for a world as much like ours as it can be but in which α is true, and see whether β is true there. Sentences like this are called 'counterfactuals' because their typical use is when α is false and known to be false. That is why they involve possible worlds. The possible worlds interpretation of α □→ β is that β is true in worlds which are as like ours as possible but where α is true.[5] This means that both α and β are in the scope of what is in effect a two place modal operator. It is the determination of what the standards are for a world's being 'like' ours which make counterfactuals context-dependent, and

[4]We adopt the convention that an italicised *K* is the sentential operator in the language being interpreted, while the unitalicised K represents the epistemic 'accessibility' relation, which interprets *K*.

[5]This semantics was developed around the same time by Stalnaker 1968, Lewis 1973a and Åqvist 1973. Another one-place operator which could be introduced would be an *L* to represent what is sometimes called 'historical necessity'. (See the discussion on p. 159 below.)

correspond with the phenomenon mentioned on p. 19 of the different senses of ***possibly***.[6]

We must finally say something about how values are given to the atomic formulae. Suppose that φ is an n-place predicate. In ordinary LPC it is sufficient to specify, of any n-tuple of individuals $a_1$, ..., $a_n$ whether or not they satisfy φ, so that V(φ) is simply a set of n-tuples from D. For a modal and tense language the interpretation V(φ) of an n-place predicate φ will be an n-place relation in the sense of p. 31 — i.e. a function from n-tuples of individuals to sets of world-time pairs. As remarked there, this is equivalent to letting V(φ) give a set of n-tuples for each world-time pair, which is called the *extension* of the predicate in that world at that time. Even simpler we can let an n-place predicate be assigned a set of n+2-tuples of the form $\langle a_1,...,a_n,w,t\rangle$ where $a_1$,..., $a_n$ are all in D, $w$ is in W and $t$ is in T, and that is what we shall do. Among the predicates we may, if we wish, distinguish one, $\varphi_=$, as the *identity* predicate, where $V(\varphi_=)$ is the set of quadruples of the form $\langle a,a,w,t\rangle$ for $a \in$ D, $w \in$ W and $t \in$ T. We frequently write $\varphi_= xy$ as $x = y$, provided we are careful to distinguish between talking of identity in the metalanguage, as when we said above things like $V_\mu(\alpha) = 1$, and its use to refer to a two-place predicate in the object language. $\sim\varphi_= xy$ is frequently written $x \neq y$.

We can sum this up by calling $\langle$W,R,T,<,K,C,D,V$\rangle$ a model, and the next task is to show how any model leads to an assignment of truth values with respect to any formula at any time in any world. We write $V_\mu(\alpha,w,t) = 1$ to mean that α is true in $w$ at $t$, according to this model, with respect to an assignment μ to its variables, and $V_\mu(\alpha,w,t) = 0$, to mean that it is false in $w$ at $t$. The full definition goes like this:

[Vφ] $V_\mu(\varphi x_1...x_n,w,t) = 1$ if $\langle\mu(x_1), ... ,\mu(x_n),w,t\rangle \in V(\varphi)$ and 0 otherwise.
[V⊥] $V_\mu(\perp,w,t) = 0$
[V⊃] For any wff α and β, and for any $t \in$ T, $V_\mu(\alpha \supset \beta,w,t) = 1$ if either

[6]The purpose of introducing these operators is to give a sense of what these languages are capable of expressing. We do not claim to be giving a definitive account of the semantics of knowing or of counterfactuals, except as illustrating how they can be treated. In a general semantics for an intensional logic the meaning of an n-place operator $O$ is given by an n+1-place 'neighbourhood' relation $R_O$, where $R_O$ relates any $\langle w,t\rangle$ pair to n-tuples of sets of $\langle w,t\rangle$ pairs. A model for a language using this more general semantics is a quadruple $\langle$W,R,D,V$\rangle$, where W is a set of 'worlds', R a family of neighbourhood relations with $R_O$ for each operator $O$, D a domain of individuals and V an assignment to the predicates as before. Truth with respect to an assignment μ is defined in the usual way with the addition of a rule for evaluating operators:
[V$O$] Where $|\alpha_i|_\mu$, for $1 \leq i \leq n$, denotes the set $\{\langle w',t'\rangle : V_\mu(\alpha_i,w',t') = 1\}$ then $V_\mu(O(\alpha_1,...,\alpha_n),w,t) = 1$ iff $\langle w,t\rangle R_O\langle|\alpha|_\mu,...,|\alpha|_\mu\rangle$.

$V_\mu(\alpha,w,t) = 0$ or $V_\mu(\beta,w,t) = 1$; otherwise $V_\mu(\alpha \supset \beta,w,t) = 0$.

[V∀] $V_\mu(\forall x\alpha,w,t) = 1$ if $V_\rho(\alpha,w,t) = 1$ for every $x$-alternative $\rho$ of $\mu$, and 0 otherwise.[7]

[V*G*] For any wff $\alpha$ and for any $t \in T$, $V_\mu(G\alpha,w,t) = 1$ if for every $t' \in T$ such that $t < t'$, $V_\mu(\alpha,w,t') = 1$; otherwise $V_\mu(G\alpha,w,t) = 0$.

[V*H*] For any wff $\alpha$ and for any $t \in T$, $V_\mu(H\alpha,w,t) = 1$ if for every $t' \in T$ such that $t' < t$, $V_\mu(\alpha,w,t') = 1$; otherwise $V_\mu(H\alpha,w,t) = 0$.

[V*L*] $V_\mu(L\alpha,w,t) = 1$ if $V_\mu(\alpha,w',t) = 1$ for every $w'$ such that $w\mathrm{R}w'$; otherwise $V_\mu(L\alpha,w,t) = 0$.

[V*K*] $V_\mu(K\alpha,w,t) = 1$ if $V_\mu(\alpha,w',t') = 1$ for every $w' \in W$ and $t' \in T$ such that $\mathrm{K}(w,t,w',t')$; otherwise $V_\mu(K\alpha,w,t) = 0$.

[V□] $V_\mu(\Box\alpha,w,t) = 1$ if $V_\mu(\alpha,w',t) = 1$ for every $w' \in W$; otherwise $V_\mu(\Box\alpha,w,t) = 0$.

[V□→] $V_\mu(\alpha \mathrel{\Box\!\!\rightarrow} \beta,w,t) = 1$ if there is some $w'$ such that $V_\mu(\alpha,w',t) = 1$ and $V_\mu(\beta,w',t) = 1$, and if there is any $w''$ such that $V_\mu(\alpha,w'',t) = 1$ and $V_\mu(\beta,w'',t) = 0$, then $w' <_{\langle w,t\rangle} w''$. Otherwise $V_\mu(\alpha \mathrel{\Box\!\!\rightarrow} \beta,w,t) = 0$.

Using the appropriate definitions we have the following derived rules:

[V*F*] For any wff $\alpha$ and for any $t \in T$, $V_\mu(F\alpha,w,t) = 1$ if for some $t' \in T$ such that $t < t'$, $V_\mu(\alpha,w,t') = 1$; otherwise $V_\mu(F\alpha,w,t) = 0$.

[V*P*] For any wff $\alpha$ and for any $t \in T$, $V_\mu(P\alpha,w,t) = 1$ if for some $t' \in T$ such that $t' < t$, $V_\mu(\alpha,w,t') = 1$; otherwise $V_\mu(P\alpha,w,t) = 0$.

[V*M*] $V_\mu(M\alpha,w,t) = 1$ if $V_\mu(\alpha,w',t) = 1$ for some $w'$ such that $w\mathrm{R}w'$; otherwise $V_\mu(M\alpha,w,t) = 0$.

[V$K_\Diamond$] $V_\mu(K_\Diamond\alpha,w,t) = 1$ if $V_\mu(\alpha,w',t') = 1$ for some $w' \in W$ and $t' \in T$ such that $\mathrm{K}(w,t,w',t')$; otherwise $V_\mu(K_\Diamond\alpha,w,t) = 0$

[V◇] $V_\mu(\Diamond\alpha,w,t) = 1$ if $V_\mu(\alpha,w',t) = 1$ for some $w' \in W$; otherwise $V_\mu(\Diamond\alpha,w,t) = 0$.

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[7]This means that we take the quantifier to be interpreted with respect to the whole domain. Some authors interpret the quantifier with respect to a restricted domain so that when $\forall x\alpha$ is evaluated at $w$ and $t$ it ranges only over those entities which exist in $w$ at $t$. We discuss this matter in Chapter 6. In the second place, assignments to the individual variables are given absolutely and not relative to an index. This has the consequence that all wff of the form $x = y$ are true or false absolutely and do not vary in truth value from index to index. As a related feature the language contains no function symbols or complex terms. Our preference is to paraphrase these away in the manner of Russell's theory of descriptions (Russell 1905) as applied to intensional languages in Smullyan 1948, or to treat 'terms' as quantifier-like expressions, as in Montague 1974, p. 249, or Cresswell 1973, pp. 130-133. As for names we incline to some such view as that argued for in Bach 1981, where a word like 'Aristotle' means 'the person called Aristotle', and what it is to be called a name is analysed in the manner advocated in Kripke 1972. (These latter issues are independent of the world-time parallel.)

[V⊕] $V_\mu(\oplus\alpha,w,t) = 1$ if $V_\mu(\alpha,w,t') = 1$ for every $t' \in T$; otherwise $V_\mu(\oplus\alpha,w,t) = 0$.

[V⊗] $V_\mu(\otimes\alpha,w,t) = 1$ if $V_\mu(\alpha,w,t') = 1$ for some $t' \in T$; otherwise $V_\mu(\otimes\alpha,w,t) = 0$.

We mentioned in Chapter 1 that the metalanguage in which the truth conditions are expressed is completely tenseless and modal-free. It simply does not make sense to suppose, say, that $V_\mu(\alpha,w,t) = 1$ today, but that $V_\mu(\alpha,w,t) = 0$ yesterday, or that although it happens to be that $V_\mu(\alpha,w,t) = 1$ yet it might have been so that $V_\mu(\alpha,w,t) = 0$. Of course there are things you can say which might be mistaken for such claims. For if $t$ is today you might have $V_\mu(\alpha,w,t) = 1$, but if $t$ is yesterday you might have $V_\mu(\alpha,w,t) = 0$. But all that shows is that if $t \neq t'$ you can have $V_\mu(\alpha,w,t) = 1$ but $V_\mu(\alpha,w,t') = 0$. Similarly you can have $w \neq w'$ and $V_\mu(\alpha,w,t) = 1$ but $V_\mu(\alpha,w',t) = 0$. Or again you might say that, although $V_\mu(\alpha,w,t) = 1$, if $\alpha$ had meant something different then $V_\mu(\alpha,w',t) = 0$. But this is simply to say that you can have two different value assignments V and V′, such that $V_\mu(\alpha,w,t) = 1$ but $V'_\mu(\alpha,w,t) = 0$.

The semantics presented here makes use of set theory. We have explained our attitude to the set-theoretical construction of the entities we refer to in Chapter 2. Set theory can be presented as a first-order theory in which the only predicate is $\in$, where $x \in y$ means that $x$ is a member of $y$. Although set theory can be combined with tense and modality it is normally held not to make much sense to ask when $x \in y$, or to suppose that even though $x \in y$, yet this might not have been so. That at any rate is how we are taking it when we use set theory in our metalanguage. This fact does have metaphysical consequences. For in a debate about what there is it is important to consider the status of the metalanguage. If the metalanguage is the kind we have been assuming in this chapter then modal and temporal qualifiers do not apply to it. Given that these languages make explicit reference to times and worlds it may be thought that this already begs the question against the possibility of meaningful metaphysical debate. That is a question which will occupy us in Chapter 11.

In the next few chapters we shall apply the tense and modal language to the formalisation of natural language sentences. When we do so we shall represent predicates by boldface italic English words or phrases to indicate that that is how the predicate is to be interpreted. Thus in formalising (17) on p. 16 we shall use the phrases ***child prodigy*** and ***study at this school*** as one-place predicates.

# 6

# *The non-existent*

In interpreting the formulae of any predicate language we have to specify a domain D of individuals, and it is by no means a trivial question what D should contain. In the special case of modal and tense languages there is the question of whether D should contain things which do not presently exist but which did or will, or things which never actually existed but which might have. We shall try to show that there are at least prima facie reasons for permitting (as on p. 68) the quantifiers to range over the whole domain, and not just over the things which exist in the world in question at the time in question. At any rate we will give reasons for supposing that whatever you say on this question about times you should also say about worlds. We will begin by considering tense, and will try to isolate what we regard as some important semantical features of tensed language. We will then go on to show that modal sentences share these same features. Take the simple tensed sentence (17) on p. 16, relabelled here as (1):

(1) A child prodigy studied at this school.

From the point of view of English grammar the tense (at least at the surface level) attaches to the verb 'studied'. Yet it is not hard to see that the semantics of (1) is more complex. Suppose someone utters (1) in 2004, but the prodigy studied at the school in question in 1984. Obviously the person who studied is not *now* a child-prodigy. The person was a prodigy in 1984. That was the motivation for introducing the temporal operators $P$ and $F$ on p. 14. $P\alpha$ may be read as 'it was once the case that $\alpha$' and $F\alpha$ as 'it will be the case that $\alpha$', though we need to remember that these rather unnatural English renderings are merely our way of indicating how to read the operators. We may formalise (1) as

(2) $P\exists x(\textbf{\textit{child prodigy}}\ x \wedge \textbf{\textit{study at this school}}\ x)$

where the complex predicate ***study at this school*** has been left in the present tense to indicate that the past tense of the sentence in this logical language is

introduced by *P*, and not, as in English, by the verb. (2) represents the sense in which (1) is probably intended, and in which it is assumed to be true. The false sense would be represented by:

(3) $\exists x(\textbf{\textit{child prodigy}}\ x \wedge P\ \textbf{\textit{study at this school}}\ x)$.

The difference between (2) and (3) lies in the scope of the operator *P*. In (2) *P* is said to have *wide scope*, because it is outside the scope of ∃, while in (3) *P* has narrow scope. Sometimes a narrow-scope reading is preferred as in:

(4) Every old man was once a baby

which, if it is to be true, has to be represented as

(5) $\forall x(\textbf{\textit{old man}}\ x \supset P\ \textbf{\textit{baby}}\ x)$

rather than by

(6) $P\forall x(\textbf{\textit{old man}}\ x \supset \textbf{\textit{baby}}\ x)$.

In tense logic the future is represented with the operator *F* replacing *P*. Thus

(7) A child prodigy will study at this school.

is formalised as

(8) $F\exists x(\textbf{\textit{child prodigy}}\ x \wedge \textbf{\textit{study at this school}}\ x)$

or

(9) $\exists x(\textbf{\textit{child prodigy}}\ x \wedge F\ \textbf{\textit{study at this school}}\ x)$.

(8) entails that at some time in the future someone who is then a prodigy will study at the school, while (9) claims that someone who is now a prodigy will study at the school.

Now look at a sentence only minimally different from (3)

(10) $\exists xP(\textbf{\textit{child prodigy}}\ x \wedge \textbf{\textit{study at this school}}\ x)$

(10) is different from (3) because it does not claim that anyone who is *now a child prodigy* once studied at this school. Still (10) may be held to entail that

something which was once a prodigy did, with the implication that that something must *still exist*, even if it is no longer a child prodigy. (2), by contrast, is held to make no such claim, since an ∃ *inside the scope of* a *P* can be read as saying that there *was* a prodigy who studied at the school and that when the prodigy was a prodigy and studied at the school the prodigy *then* existed. In the case of (10) it might be thought that, because the quantifier ∃ is outside the scope of *P*, it must refer to things which still exist. But whether or not this is so there are certainly natural language sentences which appear to involve present truths about things which no longer exist. An example is

(11) A deceased philosopher once studied at this school.

On the assumption that dead people no longer exist the problem is with 'deceased'. (11) cannot be formalised as

(12) $P\exists x(\textbf{\textit{deceased}}\ x \wedge \textbf{\textit{philosopher}}\ x \wedge \textbf{\textit{study at this school}}\ x)$

since (12) entails that the philosopher was deceased while studying at the school — and even if some students might appear moribund they are not literally deceased. There seems no option but to say something like

(13) $\exists x(\textbf{\textit{deceased}}\ x \wedge P\ \textbf{\textit{philosopher}}\ x \wedge P\ \textbf{\textit{study at this school}}\ x)$

but only if ∃ in (13) is permitted to range over things which no longer exist. Does that mean that there can be non-existent philosophers? Well that depends. In (13) the predicate ***philosopher*** is inside the scope of *P*, and (13) does not claim that the now deceased being is still a philosopher. So the fact that ∃ is permitted to range over erstwhilia (as we'll call no longer existing individuals) does *not* entail that there are non-existent philosophers. Perhaps ***philosopher*** is a predicate which is only true of an individual at a time at which that individual exists. There is in fact a difference between natural language quantification and logical quantification. Words like 'a' or 'every' in English require a noun to make sentences which contain them grammatical. You can't say

(14) A studied at this school

or

(15) Every was once a baby

unless of course 'a' and 'every' are names or dummy names. One might claim that natural language amalgamates two separate functions. There are the 'pure' quantifiers, $\forall$ and $\exists$, and there are the restrictors 'child prodigy', 'old man' or 'deceased philosopher'.[1] You would then say that all but a very few of these restrictors are existence-entailing. You can't be a child prodigy or an old man when you don't exist, though you can be a deceased philosopher. Presumably also you can only study at a school when you exist. In fact there is very little you can do when you don't exist, and that is why (3) has to be about someone who now exists, and why

(16) $\exists x(\textbf{\textit{old man}}\ x \wedge P\ \textbf{\textit{study at this school}}\ x)$

also entails that the entity in question now exists. But (13) need not have that implication. For we can allow $\exists$ and $\forall$ to range over things which exist at *any* time, not just at the present, and if they are interpreted this way then (2) and (10) are equivalent.[2] The reason the domain of quantification might be thought to be tensed might be the idea that the quantifier $\exists$ in $\exists x\alpha$ is to be read as 'there *is* an $x$ such that $\alpha$'. But long ago Ruth Marcus ([Barcan] Marcus 1962) protested that you can read $\exists x\alpha$ as 'for some $x$, $\alpha$', and the tense, if there is one, is in $\alpha$. All we need to note is that $\exists x\alpha$ is (now) true iff for some value of $x$, $\alpha$ is now true. Put another way, the quantifier is not a verb, and is neither tensed nor tenseless.

Modal sentences have a structure just like tensed sentences. If we use $M$ as a modal operator meaning 'it might be that',

(17) A child prodigy might study at this school.

can be represented in a manner exactly analogous to (8) and (9) with $M$ replacing $F$:

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[1] In the languages we are presenting the only 'quantifiers' are the one-place $\forall$ and $\exists$. Quantifiers like ***most*** are essentially two place, but even here we can distinguish between the quantifier and the restrictors. In a sentence like 'most deceased philosophers are not worth studying' the 'most' can range over non-existent individuals.

[2] This view accepts the truth of the tensed version of what is known as the *Barcan Formula*: $\mathrm{BF}P_{\equiv}\ \exists xP\alpha \equiv P\exists x\alpha$ and $\mathrm{BF}F_{\equiv}\ \exists xF\alpha \equiv F\exists x\alpha$. The name comes from a formula first introduced (for modal operators) in Barcan 1946, p. 2, Axiom 11. The modal equivalent of BF is $\Diamond\exists x\alpha \supset \exists x\Diamond\alpha$ where the main operator is $\supset$ rather than $\equiv$. (Actually Barcan uses strict implication as the main operator.) She proves the converse as theorem 37 on p. 7. BF appears to have been first called the 'Barcan Formula' in Prior 1957, p. 26. In Prior 1956 it is shown derivable in S5. In tense logic the 'bridging laws' connecting past and future ($p \supset \sim F\sim Pp$ and $p \supset \sim P\sim Fp$) ensure the derivability of the equivalential laws stated here in standard predicate tense logic. This is noted on p. 147 of Prior 1967, who goes on to discuss non-standard tense logics which block the derivation.

(18) $M\exists x$(***child prodigy*** $x \wedge$ ***study at this school*** $x$)

or

(19) $\exists x$(***child prodigy*** $x \wedge M$ ***study at this school*** $x$).

Now it is a fact that tigers exist, but unicorns do not. That is

(20) Tigers exist

is true, while

(21) Unicorns exist

is false. On the assumption that (20) means that there is at least one tiger (20) would be formalised as

(22) $\exists x$ ***tiger*** $x$

while (21) would be

(23) $\sim\exists x$ ***unicorn*** $x$.

In (22) and (23) there is no predicate of existence, and perhaps the 'received' view is that existence is part of the meaning of the 'existential' quantifier. Whether or not this *is* the received view it is something we do not want to be committed to. If existence in (22) and (23) is not captured by $\exists$ how is it captured? The claim is simple. It is captured by the predicates ***tiger*** and ***unicorn***. We live in a tigered world, but not in a unicorned world. If you read $\exists x$ as 'for some $x$', what you see is that questions of existence arise *at the point of predication*, and *not* at the point of evaluating the quantifier. It might be thought that ***unicorn*** is not existence-entailing — but that would be hasty. Could anything be a unicorn if it didn't exist? (23) is certainly compatible with an answer 'no'. For if you have to exist to be a unicorn, and no existent thing is a unicorn, then nothing at all, existent or not, really is a unicorn. This is not because $\exists$ is restricted to what actually exists, but because to *actually be a unicorn* you must exist.

Nevertheless, unicorns are presumably *possible*, and it might be thought that although nothing in the realm of the actually existent is a unicorn, yet something in the realm of the possible is a unicorn. This observation must be handled with care. If nothing existent is actually a unicorn then nothing which is

merely possible is *actually* a unicorn either. It is not the quantifier which is ontologically important — it is the *predicate* ***unicorn***.

(24) $M\exists x$ ***unicorn*** $x$.

and

(25) $\exists xM$ ***unicorn*** $x$

could both be true because in both of them ***unicorn*** is in the scope of $M$.

W.V. Quine was bothered about possible individuals, beloved of Wyman — the profligate metaphysician whose 'ontological slum' contains possibilia. In Quine 1953a p. 3 he writes

> Pegasus, Wyman maintains, has his being as an unactualized possible. When we say of Pegasus that there is no such thing, we are saying more precisely, that Pegasus does not have the special attribute of existence. Saying that Pegasus is not actual is on a par, logically, with saying that the Parthenon is not red; in either case we are saying something about an entity whose being is unquestioned.

and on p. 4

> Wyman's slum of possibles is a breeding ground for disorderly elements. Take, for instance, the possible fat man in that doorway; and, again, the possible bald man in that doorway. Are they the same possible man, or two possible men? How do we decide? How many possible men are there in that doorway? Are there more possible thin ones than fat ones? How many of them are alike?

Look at what would happen if you took what is written on the relevant page of Quine 1953a and 'translated' it into the analogous point that might be made by a philosopher, whom we shall call Quine*, who thinks of things which no longer exist in the way Quine thinks of merely possible things. 'Quman' in what follows is the temporal equivalent of Quine's 'Wyman'. Quman's slum contains erstwhilia, which Quine* disavows. So Quine* 1953 p. 4 would read

> Quman's slum of erstwhiles is a breeding ground for disorderly elements. Take for instance the erstwhile fat man in that doorway; and again the erstwhile bald man in that doorway. Are they the same erstwhile man or two erstwhile men? How do we decide? How many erstwhile men are

there in that doorway? Are there more erstwhile thin ones than fat ones? How many of them are alike?

An erstwhile man is a man who used to exist but exists no longer. Could such a man be fat? Yes if that means once fat, but no if it means now fat. Then we ask how many erstwhile men are in the doorway — and we must distinguish between asking how many men who no longer exist *are now* in the doorway, and asking how many men (who no longer exist) *ever were* in the doorway. Begin with the statement:

(26) There is an erstwhile man in the doorway.

An erstwhile man no longer exists; but Quman holds that quantifiers range over erstwhilia just as much as over presently existing things. If we take Quine* seriously ***erstwhile*** would have to be a predicate and (26) would be represented as:

(27) $\exists x(\textbf{\textit{erstwhile}}\ x \wedge \textbf{\textit{man}}\ x \wedge \textbf{\textit{in doorway}}\ x)$

where ***erstwhile*** is to mean 'no longer exists' and ***in doorway*** is to mean 'is in the (relevant) doorway'. But there is an immediate problem. If ***erstwhile*** $x$ really does mean that $x$ no longer exists, then (27) has to be talking about the present time. If ***erstwhile*** $x$ is written as $\sim$***exist*** $x \wedge P$ ***exist*** $x$ we get

(28) $\exists x(\sim\textbf{\textit{exist}}\ x \wedge P\ \textbf{\textit{exist}}\ x \wedge \textbf{\textit{man}}\ x \wedge \textbf{\textit{in doorway}}\ x)$.

How do we construe ***in doorway***? Well it must be (now) true of $x$ iff $x$ is *now* in the doorway. But $x$ no longer exists, and so there can be *no* erstwhile man now in the doorway, and (28) is false. Notice that this is *not* because the quantifier $\exists$ is at all restricted. There is nothing in this analysis of (26) which goes any way to suggest that $\exists$ cannot apply to Socrates, and the sentence

(29) $\exists x(\sim\textbf{\textit{exist}}\ x \wedge P\ \textbf{\textit{exist}}\ x \wedge P\ \textbf{\textit{man}}\ x \wedge P\ \textbf{\textit{in doorway}}\ x)$

could well be true. The point is that if the language contains a predicate like ***erstwhile*** then sentences containing it must be relativised to time. Although Socrates may *now* be an erstwhile being, in 404BC he was very much alive. Similarly with ***possible***. As with ***erstwhile*** the problem is that 'possible' is an adjective, and in 'possible man' it is tempting to think that Quine supposed

'possible' to be a predicate which is true of some things but false of others. Why is this misguided? It is misguided because *M* is a sentential operator and can have scope. And the issue of scope is crucial in the present case, because it is of the utmost importance to decide whether *M* has scope only over ***man*** or over ***in doorway*** as well. If we formalise

(30) There is a possible man in the doorway

analogously with (28), as

(31) $\exists x(\sim$***exist*** $x \wedge M$ ***exist*** $x \wedge$ ***man*** $x \wedge$ ***in doorway*** $x)$

then (31) is false for the same reason as (28) is false, since a man who does not exist cannot *actually* be in a doorway. This is not, as we have stressed, because $\exists$ is restricted to the actual — it is not so restricted in our formal languages, any more than it is restricted to the present. Perhaps Quine's worry about (30) is that there is supposed to be a sense in which it is true. (31) must be distinguished from

(32) $\exists x(\sim$***exist*** $x \wedge M$ ***exist*** $x \wedge M$ ***man*** $x \wedge M$ ***in doorway*** $x)$

which might well be given an interpretation analogous to that offered for (29) under which it is true.[3]

Still, one might wonder whether the modal cases really are just like the temporal cases. Predicates which are not temporally existence-entailing are predicates like ***deceased*** or ***famous***, where Socrates' present fame does not rest on his continued existence. The most natural predicates of this kind tend to be applicable to an individual in terms of that individual's causal properties. What Socrates did when alive has caused certain effects today, effects which constitute his now being famous; effects such as that we are using him to make a philosophical point. For that reason modal examples are more difficult to come by if you think that an individual can only have causal effects in worlds in which it exists. Things which are merely possible, it will be said, leave no causal traces, and so no predicate analogous to ***famous*** can apply to them. Nevertheless, by a roundabout argument we can suggest that there are predicates that are true of

[3]Another sentence which is obviously true is of course $M\exists x($***man*** $x \wedge$ ***in doorway*** $x)$. The general lesson is that the situation that Quine parodies is susceptible of different interpretations, but they can only be expressed precisely in a formal language of the kind he eschewed. Points similar to those made in this chapter are also made on pp. 194-198 of Williamson 1999.

individuals in worlds in which those individuals do not exist. Consider the sentence

(33) If Tallulah had not existed she would not have been famous.

(33) seems the kind of statement that could well be true. But if it is true it would seem that ***exist*** has to be a predicate. Further it seems that ***~exist*** and ***~famous*** are both true in such a case. Since most obvious properties are existence-entailing it is natural, as here, to use their negations to get ones which are not existence-entailing.[4] To formalise (33) we need the counterfactual operator $\Box\!\!\rightarrow$ discussed on p. 66. If (33) is true of Tallulah there is *someone* of whom it is true, and so we have:

(34) $\exists x(\sim\!\boldsymbol{exist}\ x \Box\!\!\rightarrow \sim\!\boldsymbol{famous}\ x)$.

It is tempting to think that fictional characters might provide easier examples of things which have properties when they don't exist. And it has even been suggested that ordinary relations like loving are such, since Othello loves Desdemona even though neither of them exist. (Bigelow 1996, p. 37.) But that seems a mistake. For Othello's loving Desdemona does not take place in the actual world. It takes place in Shakespeare's play, and in Shakespeare's play both Othello and Desdemona exist. Put in terms of possible worlds, in any world in which Othello loves Desdemona, *in that world* they both exist. This is just like the possible fat man who is *actually* in Quine's doorway. Whatever is actually in a doorway must exist. One reason we have not chosen fictional characters as the bearers of properties which do not entail existence is because of the doubt about whether in naming them we really are referring. The impossibility of referring to particular merely possible individuals has often been thought a block to admitting them into one's ontology.[5] The example discussed above has been

[4]Salmon 1987, p.56, instances not existing as a non-existence-entailing property, and later in that same article, pp.93-98, he discusses other non-existence-entailing properties. Fine 2005, p. 328 even wonders whether a predicate like 'man' might not be existence-entailing, though he is more inclined to deny that it makes sense to think of such natural kind predicates as applying at a time at all.

[5]See for instance Jubien 1975, p.3, and Marcus 1993, p. 204. Marcus writes: "It is not, I will propose, the general absence of 'identification *conditions*' that make possibilia problematic. It is that possibilia cannot be objects of reference at all." Then follows (pp. 208-212) a discussion of the causal theory of reference. Lewis 1978 (p. 404) secures fictional reference by putting a narrator *into* the story. The storyteller pretends to be referring. The referring thus takes place in the story worlds and in those worlds there may well be the right sort of causal connection between a name

especially chosen not to presuppose any interworld reference. In a world in which Tallulah does not exist, no one in that world will be able to refer uniquely to her by name — at least not if the causal theory of reference (popularised in Kripke 1972) is correct. So we had to start off in a world in which she does exist, and only then consider what would be true about her in worlds in which she does not exist.

The same issue arises in the temporal case, since, although we can refer to things which used to exist, it is not clear that we can refer to things which do not yet exist. Mellor 1981 points out that, although Elizabeth II can refer to George III, *he* could not have referred to her. Mellor comments:

> This suggests to many that George III's world contained no future Elizabeth for him to refer to, and also lacked means whereby he could refer to her as even a hypothetical future individual. (1981, p. 32.)

Mellor is concerned to argue that from the fact that George III could not refer to Elizabeth II it does not follow that she is "absent from the future of his world". For similar reasons, from the fact, if it is a fact, that we cannot *refer* to fictional objects, it does not follow that our quantifiers cannot range over them.

If we construe 'exists' in such a way that things can exist at some times in some worlds but not in others, then we see that it is no more than a predicate which applies to things of a certain kind. It applies to those things which, in some rough and ready sense, have causal powers at certain times in certain worlds. It is the locus of these powers which gives us a notion of 'exists' to apply to the things in question. We have suggested that existence is a predicate, but perhaps it is better to say that there are many notions of existence. Does Socrates exist? Well, no if you mean that he is still alive, but yes if you mean that he was once alive. In a sense Socrates is part of actual reality in a way in which Sherlock Holmes is not. Is Socrates alive today? — No. Is Socrates a philosopher today? Well, perhaps yes and no. The moral may be that each predicate should be taken on its merits. When we are right about predication there is nothing more to any question of existence.

It has sometimes been suggested that if we countenance possibilia — things which could have existed but do not — then we should also countenance impossibilia — things which neither do nor could have existed — things like 'the

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and the thing named.

round square' which are supposed to exist at 'impossible worlds'.[6] This would be like saying that if we countenance actual beings who do not presently exist but who once did, we should also countenance actual beings who never did and never will exist. Lycan 1994, p. 39, says "I can think of no direct argument for 'non-existents' that does not support impossibilia by parity of reasoning." What might such reasoning be? Consider the sentence:

(35) There are no unicorns but there could have been

(35) can be formalised as

(36) $\sim\exists x$ ***unicorn*** $x \wedge M\exists x$ ***unicorn*** $x$.

On the indexical semantics for $M$ we have, where $\alpha$ is a sentence,

(37) If $M\alpha$ is true in a world $w$ then there is a world $w'$ in which $\alpha$ is true.

So, if (36) is true, then, from its second conjunct, there is a world $w'$ in which

(38) $\exists x$ ***unicorn*** $x$

is true. But if (38) is true in $w'$ there must be something, $u$, which is a unicorn there. If ***unicorn*** is a natural kind term then presumably nothing actual *could* be a unicorn, and so $u$ is a non-actual possibile. What then is the parallel argument for impossibilia? It cannot depend on a sentence like

(39) There are no round squares but there could have been

since the round square is supposed to be *not* possible. I.e.

(40) There are no round squares and there could not have been.

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[6]There is one sense in which 'impossibilia' may be quite respectable. If you think that ***exists*** means existence in space and time then perhaps many things that we may want to talk about — say numbers or pure sets — cannot exist. But we can certainly quantify over them, and presumably they are not what authors like Lycan have in mind when they speak about impossibilia. The 'round square' example is attributed to Meinong on p. 480 of Russell 1905. For a defence of a Meinongian view of non-existence see Parsons 1980.

To ensure that there is no doubt about the impossibility replace 'square' by 'not round'. (40) can then be represented as

(41) $\sim\exists x(\boldsymbol{round}\, x \wedge \sim\boldsymbol{round}\, x) \wedge \sim M\exists x(\boldsymbol{round}\, x \wedge \sim\boldsymbol{round}\, x)$.

Assume then that (41) is true at a world $w$. What is the parity of reasoning which leads to an impossible object? Presumably we require an impossible world $w'$ at which $\exists x(\boldsymbol{round}\, x \wedge \sim\boldsymbol{round}\, x)$ is true. Then we need a principle parallel to (37), and the only option seems to be

(42) If $M\alpha$ is false at a world $w$ then there is an impossible world $w'$ at which $\alpha$ is true.

But (42) is a principle unknown to modal logic, and there is no way in which it could be supposed to follow from (37) by 'parity of reasoning'. In fact it looks like arguing as follows. If

(43) Someone is on the road

is true then there must be a (positive) person $p$ who is on the road. Therefore, by parity of reasoning, if

(44) No one is on the road

is true then there must be a negative person on the road. We find it difficult to see what else could be meant here by 'parity of reasoning', and if this is what Lycan has in mind then there would seem to be some confusion. One of the things that Russell 1905 succeeded in establishing was that there are many sentences whose surface grammatical form might suggest quantification over the non-existent, but whose logical structure carries no such entailment.[7] Take Russell's example

(45) The present king of France does not exist

There is a sense in which (45) is true. Leaving aside the uniqueness of the denoting phrase and taking ***king of France*** as a simple predicate, the sense in

[7]It is perhaps this which has encouraged the myth, strongly implied on p. 169f of Russell 1919, and on pp. 5-8 of Quine 1953a, that Russell's theory allows the avoidance of *all* such quantification.

which (45) is true can be represented as

(46) $\sim\exists x$ ***king of France*** $x$.

(46) is as true today as it was in 1905, but it gives no more reason to suppose that there is a present king of France who no longer exists than (44) gives to suppose a negative person on the road. The other analysis of (45) permitted by Russell's account is where 'the present king of France' has what he called 'primary occurrence'. This sense of (45) can be represented as

(47) $\exists x$(***king of France*** $x \wedge \sim$***exist*** $x$)

On the assumption that you can't be a king of France without existing[8] (47) would indeed, *if true*, establish the existence of a non-existent king of France. (46) is true, but (47) is not merely false, but is necessarily false, *even if* $\exists$ *should range over non-existents*; and any argument for possibilia which treats (47) as an analysis of (45), in the sense in which (45) is true, is fallacious. Consider the analogous argument for impossibilia. Although (46) is true it is not necessarily true, since there could have been a present king of France, and many erstwhile beings *were* in their day, kings of France. But there could not have been a round square. I.e. there is a sense in which

(48) The round square cannot possibly exist

is true. By analogy with (46), (48) can be formalised as the second conjunct of (41):

(49) $\sim M\exists x$(***round*** $x \wedge \sim$***round*** $x$)

(49) is true, but suppose one were to represent (48), not as (49), but as

(50) $\exists x$(***round*** $x \wedge \sim$***round*** $x \wedge \sim M$ ***exist*** $x$)

then, by parity of reasoning, the argument for impossibilia based on (50) would be just as fallacious as an argument for possibilia based on (47). Parity of reasoning does indeed show that these two bad arguments fail in the same way.

[8]It seems obvious to us that you must exist to be a king, and we shall stipulate that we are using the predicate in such a way.

What it does *not* show is that a *good* argument for possibilia such as the one based on (36) and (37) can justify a bad argument for impossibilia such as one based on (41) and (42).

# 7

# *Multiple indexing*

In Chapter 5 we showed how to provide a semantics in terms of times and worlds for a tense and modal predicate language. The metalanguage involves quantification over both these kinds of entities. But there have been philosophers, A.N.Prior for one, who seem to have thought that the use of tense operators obviates the need for quantification over times, and there are philosophers, such as Graeme Forbes, who seem to think that taking modality as primitive doesn't commit you to possible worlds. While it is true that the expressive capacity of the tense and modal languages described in Chapter 5 is less than full quantification over worlds, it is equally so that there are sentences of natural language which require more complex temporal and modal resources — resources whose increased expressive capacity amounts to such quantification. The purpose of this chapter is to describe these resources. In Chapter 8 we shall make the link with quantification over times and worlds. The resources we have described have been around for some time, although they have not usually been studied in a language which has *both* tense and modal operators. We will begin by considering tense, and will try to isolate what we regard as some important semantical features of tensed language. We will then go on to show that modal sentences share these same features. Look at

(1) One day everyone now miserable will be happy

(1) cannot be represented as

(2) $F\forall x(\textbf{\textit{miserable}}\ x \supset \textbf{\textit{happy}}\ x)$

for *that* says that one day all the people who are *then* miserable will be happy, and that is certainly not what (1) says. Does (1) mean

(3) $\forall x(\textbf{\textit{miserable}}\ x \supset F\ \textbf{\textit{happy}}\ x)$?

(3) is better. It says that for any miserable person (and that means anyone who is now miserable) there will be a time at which they are happy. But suppose today is Monday, and that one of the people miserable on Monday is happy on Tuesday, but then becomes miserable again. Another person miserable on Monday becomes happy on Wednesday and miserable thereafter. And so it goes on. They are never all happy at once. So even if everyone now miserable will one day be happy it does not follow that there will ever be a time at which *all* those now miserable will be happy. One solution (Kamp 1971[1]) is to introduce a sentential operator ***now***. Then we may express (1) by

(4) $F\forall x(\textbf{\textit{now miserable}}\ x \supset \textbf{\textit{happy}}\ x)$

The function of ***now*** is to take you back to today, so that *even in the future* (i.e., even in the scope of *F*) the people who then satisfy the complex predicate ***now miserable*** are those who are *now* miserable, i.e., those who are miserable today, and (4) says something about what will happen in the future to all of *them*. In this case, even though ***now*** occurs inside the scope of an *F*, its presence ensures that what is in its scope is evaluated at the time at which the original sentence is evaluated. In many sentences 'now' is redundant. If the train is departing then it is departing now. Where it is not redundant it is because, as in (4) it occurs in an embedded sentence in the scope of a temporal operator. Suppose we are assessing (4) for truth or falsity at a time $t_1$ in a world *w*. (Since *w* doesn't change in this example, reference to it will often be suppressed.) (4) will be true at $t_1$ iff

(5) $\forall x(\textbf{\textit{now miserable}}\ x \supset \textbf{\textit{happy}}\ x)$

is true at some $t_2$ where $t_1 < t_2$. And (5) will in turn be true at $t_2$ provided that for whatever value from the domain of individuals is assigned to *x*

(6) $\textbf{\textit{now miserable}}\ x \supset \textbf{\textit{happy}}\ x$

is true at $t_2$. And this will be so provided every *a* which satisfies ***now miserable*** at $t_2$ — i.e., every *a* who is miserable at $t_1$ — satisfies ***happy*** at $t_2$. In such situations as (6) ***now*** is not redundant, since it refers back to the time $t_1$ at which

[1]Kamp provides a proof that ordinary tense logic, without supplementation, is unable to express sentences like (1). Hazen 1976 also has examples of sentences which cannot be formalised in ordinary modal predicate logic, though his examples tend to concern the fact that an existence predicate cannot be defined in a language whose quantifiers only apply to the domain of what exists in the world of evaluation.

the original sentence (4) is being evaluated. The point of the ***now*** in ***now miserable*** is to ensure that

(7) ***now miserable*** $x$ is true at $t_2$ if ***miserable*** $x$ is true at $t_1$.

The standard way to solve this problem is to recognise that there are *two* temporal indices involved here. There is an index which changes from $t_1$ to $t_2$ because of the need to evaluate (5) at a time which is in the future of $t_1$. But there is also the index which remains the same, which is needed because ***now miserable*** $x$ doesn't change under the influence of $F$. The two temporal indices are sometimes called the *evaluation index* and the *reference index*.[2] We represent these as an ordered pair $\langle t,t' \rangle$, where $t$ is the evaluation index, and $t'$ is the reference index. We then amend the rules for $F$ and $P$ so that they work only on the evaluation index. I.e., for any $t$, $t'$, $t''$ and $w$

(8) $F\alpha$ is true at $\langle t,t' \rangle$ in $w$ iff for some $t''$ such that $t < t''$, $\alpha$ is true at $\langle t'',t' \rangle$ in $w$.

(9) $P\alpha$ is true at $\langle t,t' \rangle$ in $w$ iff for some $t''$ such that $t'' < t$, $\alpha$ is true at $\langle t'',t' \rangle$ in $w$.

In (8) and (9) the reference index $t'$ is unaffected by the tense operators $F$ and $P$. We also need the ***always*** operator $\oplus$ mentioned on p. 18. Put in English, $\oplus\alpha$ is true provided that $\alpha$ is always going to be true, and is true now, and always has been true. From the definition of $\oplus$ we have

(10) $\oplus\alpha$ is true at $\langle t,t' \rangle$ in $w$ iff for every $t''$, $\alpha$ is true at $\langle t'',t' \rangle$ in $w$.

In (10) as in (8) and (9) the reference index is idle, and is merely carried along, but the evaluation index shifts as we move through $\oplus$, or $F$ or $P$. next, we need a rule for evaluating ***now***. Instead of treating ***now*** as a propositional operator, we will add it as a propositional constant — i.e. as a simple sentence symbol with the following semantics. For any times $t$ and $t'$, and any world $w$

(11) ***now*** is true at $\langle t,t' \rangle$ in $w$ iff $t = t'$

What (11) states is simple. ***now*** is true at any time (in any world) iff that time is

[2]Such a semantics is sometimes called a 'two-dimensional' semantics.

the reference time. Given this, to say that α is true *now* is just to say that α is true *whenever it is now*, i.e.,

(12) $\oplus(\textbf{\textit{now}} \supset \alpha)$.

(12) says that it's always the case that whenever it is now then α is true. It is not hard to see that

(13) $\oplus(\textbf{\textit{now}} \supset \alpha)$ is true at $\langle t,t' \rangle$ (in $w$) iff α is true at $\langle t',t' \rangle$ (in $w$).

For $\oplus(\textbf{\textit{now}} \supset \alpha)$ is true at $\langle t,t' \rangle$ (in $w$) iff for every $t''$, if ***now*** is true at $\langle t'',t' \rangle$ then so is α. But from (11), ***now*** is true at $\langle t'',t' \rangle$ iff $t'' = t'$. So α is true at $\langle t'',t' \rangle$ iff α is true at $\langle t',t' \rangle$. (13) enables us to use ***now*** as an operator as in (4). We may define this as

[Def$_{\textbf{\textit{now}}^{\text{OP}}}$] $\quad \textbf{\textit{now}}\ \alpha =_{\text{df}} \oplus(\textbf{\textit{now}} \supset \alpha)$.

(13) therefore says that ***now*** α is true at $\langle t,t' \rangle$ iff α is true at $\langle t',t' \rangle$, and thus has the effect of turning the reference index into the evaluation index. In what follows when the context demands ***now*** to be a propositional operator we shall assume that it is defined as in [Def$_{\textbf{\textit{now}}^{\text{OP}}}$].

Begin the evaluation of (4) by assuming that the evaluation index and the reference index coincide. I.e., assume that (4) is to be evaluated at a pair $\langle t_1,t_1 \rangle$. (4) is true at $\langle t_1,t_1 \rangle$ (in $w$) iff (5) is true at $\langle t_2,t_1 \rangle$ for some $t_2$ such that $t_1 < t_2$, which will be so iff, for whatever value is assigned to $x$, if ***now miserable*** $x$ is true at $\langle t_2,t_1 \rangle$ then so is ***happy*** $x$. We may take an atomic formula like ***happy*** $x$ to ignore the reference index, so that ***happy*** $x$ will be true at $\langle t_2,t_1 \rangle$ iff the individual assigned to $x$ is happy at $t_2$. By (13), ***now miserable*** $x$ will be true at $\langle t_2,t_1 \rangle$ iff ***miserable*** $x$ is true at $\langle t_1,t_1 \rangle$, i.e. iff the individual assigned to $x$ is miserable at $t_1$, which gets us the meaning required.

But we are not done. Suppose (1) is not true now (on Monday), but used to be true. Perhaps it used to be the case (on Saturday) that all those then miserable (i.e., miserable on Saturday) were going to be happy (on Sunday), i.e.,

(14) Once everyone *then* miserable was going to be happy.

How do we formalise (14)?

(15) $P\forall x(\textbf{\textit{miserable}}\ x \supset F\ \textbf{\textit{happy}}\ x)$

will not do, for the same reasons as (3) would not do as a representation of (1). We need something like:

(16) $PF\forall x(\textbf{\textit{then miserable}}\ x \supset \textbf{\textit{happy}}\ x)$

In (16) ***then*** brings you to the past, and keeps you there, even in the scope of the embedded *F*. But when in the past? What we need is a mechanism for keeping track of these various times. In order to get the right result for (4) it was necessary to begin with the assumption that the reference world is the same as the evaluation world. While this may cause no harm if we are looking only at the completed sentence, complications arise when we recall that a sentence can itself be embedded in a more complex sentence. We need an operator which identifies the reference index with the current evaluation index. The need for such an operator seems first to have been noticed in Vlach 1973, and the operator required is similar to an operator that Quine 1960 calls 'Reflection'.[3]

(17) *Ref* α is true at $\langle t,t'\rangle$ in *w* iff α is true at $\langle t,t\rangle$ in *w*

We can then express (1) as (4) preceded by *Ref*, i.e., as

(18) $Ref\ F\forall x(\textbf{\textit{now miserable}}\ x \supset \textbf{\textit{happy}}\ x)$

For (18) will be true at any $\langle t_1,t_2\rangle$ iff (4) is true at $\langle t_1,t_1\rangle$, and then we can proceed as before. *Ref* makes the reference index the same as the evaluation index, so that the evaluation index is preserved as a reference index in the scope of temporal operators. In this way we can solve the problem of (16), but before we discuss (16) it is perhaps wise to compare the role played in English by 'now' and 'then'. In (18) *Ref* occurs at the beginning of the sentence, and that would also be appropriate for

(19) Once everyone *now* miserable was going to be happy.

Indeed one might say that the logical work done by the placement of *Ref* is signalled in English by the use of two different adverbs, with 'now' being used to signal the fact that *Ref* occurs at the beginning of the complete sentence. Since 'then' seems the more general word we shall use ***then*** from now on just as we

[3]The use of '*Ref*' for this operator is intended to suggest both Quine's term, 'reflection', and the fact that the operator turns the evaluation index into the reference index.

have been using ***now***, and so represent (14) as

(20) $P\ Ref\ F\forall x(\textbf{\textit{then miserable}}\ x \supset \textbf{\textit{happy}}\ x)$

(20) will be true at $\langle t_1,t_2\rangle$ iff there is some $t_3$ such that $t_3 < t_1$ and

(21) $Ref\ F\forall x(\textbf{\textit{then miserable}}\ x \supset \textbf{\textit{happy}}\ x)$

is true at $\langle t_3,t_2\rangle$, iff

(22) $F\forall x(\textbf{\textit{then miserable}}\ x \supset \textbf{\textit{happy}}\ x)$

is true at $\langle t_3,t_3\rangle$. (22) is just (4) with ***then*** for ***now***, and (4) will be true iff all who are miserable at $t_3$ will be happy at some later $t_4$.

Sometimes a sentence can contain more than one 'then'. Such cases are not completely natural English, yet here too the logical language we are developing can easily account for them:

(23) Once it was going to be that everyone then miserable was going to admire everyone who was then happy.

Look carefully at (23). It asks us (from 'once') to go back to a time, say $t_1$, such that it was going to be (say at some $t_2$ later than $t_1$) that everyone then miserable was going (say at $t_3$ later than $t_2$) to admire everyone then happy. Now what this amplification of (23) does *not* say is what times the two 'then's refer to. Suppose it is now Thursday, and that once (Monday) it was going to be (on Tuesday) that everyone then miserable was going to admire (on Wednesday) everyone who was then happy. It might mean that those miserable on Monday would admire (on Wednesday) those happy on Monday, or those happy on Tuesday, or Wednesday, or that those miserable on Tuesday might do this, or those miserable on Wednesday, or ... All we know is that, whatever day it is, the quantifier 'every' has refer to *everyone* who was miserable *on the same day* and everyone who was happy *on the same day*, where the day on which all are miserable must precede the day on which all are happy. (23) involves not only two ***then***s but also two *Ref*s. It can be represented as

(24) $P\ Ref_1\ F\ Ref_2\ F\forall x\forall y((\textbf{\textit{then}}_1\ \textbf{\textit{miserable}}\ x \wedge \textbf{\textit{then}}_2\ \textbf{\textit{happy}}\ y) \supset \textbf{\textit{admires}}\ xy)))$

In order to handle these indexed operators we need to evaluate sentences at triples of times. So, given a world $w$ we have for all times $t_0$, $t_1$ and $t_2$:

***then***$_1$ is true at $\langle t_0,t_1,t_2\rangle$ iff $t_0 = t_1$
***then***$_2$ is true at $\langle t_0,t_1,t_2\rangle$ iff $t_0 = t_2$
*Ref*$_1$ is true at $\langle t_0,t_1,t_2\rangle$ iff $\alpha$ is true at $\langle t_0,t_0,t_2\rangle$
*Ref*$_2$ is true at $\langle t_0,t_1,t_2\rangle$ iff $\alpha$ is true at $\langle t_0,t_1,t_0\rangle$

Obviously there is no reason in principle for stopping at triples, and in general we should think of evaluating wff at sequences of times. The appropriate generalisation of ***then*** becomes ***then***$_i$ where $1 \leq i$. Where $s$ is a sequence of times and $s_i$ denotes the i-th term in the sequence $s$,

(25) ***then***$_i$ is true at $s$ in $w$ iff $s_i = s_0$

The generalisation of *Ref* becomes

(26) *Ref*$_i\alpha$ is true at $s$ iff $\alpha$ is true at $s$[0/i], where $s$[0/i] is $s$ with $s_0$ in place of $s_i$. (I.e. for $j \neq i$, $(s[0/i])_j = s_j$, and $(s[0/i])_i = s_0$.)

When ***then***$_i$ is treated as an adverb and ***then***$_i\alpha$ is defined as $\oplus$(***then***$_i \supset \alpha$), (25) gives

(27) ***then***$_i\alpha$ is true at $\alpha$ in $w$ iff $\alpha$ is true at $s$[i/0], where $s$[i/0] is $s$ with $s_i$ in place of $s_0$.

One can think of these operations as a kind of recipe in which the 0 position in the sequence is where the action happens. The recipe says: get a value from a place in the temporal sequence, bring it to 0 (perhaps having 'stored' the original $s_0$ somewhere else while the operation is taking place) do something to the new $s_0$, and then put the result back somewhere, and if necessary, recover the original $s_0$.[4]

[4]For a fuller account of the devices natural language uses to express quantification, with particular application to times and worlds, see Forbes 1989, Cresswell 1990b and 1996, and the references to the linguistic literature contained in those works. Many of the objections to a tense-logical approach to the semantics of time focus on its inability to account for complex temporal anaphora. (See for instance Ludlow 1999, pp. 102-106.) While our book is not an account of any natural-language tense system, the introduction of operators like *Ref* and ***then***, as we illustrate in the text, expands the power of our tense-logical language in a way which remedies such defects. (Ludlow's own solution to these problems, pp. 111-135, is a little different from ours, and involves

Modal sentences also exhibit complexities analogous to those involving ***now*** and ***then***. We show that the range of modal sentences that need capturing is exactly parallel to the tensed sentences, and that the use of multiple indexing is sufficient to express this. The ***then*** and *Ref* operators have exact analogues in the modal case. Corresponding to (1) we have:

(28) It might have been that everyone actually miserable was happy.

Using *M* for 'it might have been that' the simplest formula for (28) is

(29) $M\forall x(\textbf{\textit{actually miserable}}\ x \supset \textbf{\textit{happy}}\ x)$

But now there is a problem about the meaning of ***actually***. On one account it is a redundant word — what is actually so is just what is so, the actually miserable are just the miserable. But that would make (28) equivalent to

(30) $M\forall x(\textbf{\textit{miserable}}\ x \supset \textbf{\textit{happy}}\ x)$

and (28) is not equivalent to (30). Of course

(31) Everyone actually miserable might have been happy

can be formalised as

(32) $\forall x(\textbf{\textit{miserable}}\ x \supset M\textbf{\textit{happy}}\ x)$

But (31) does not mean the same as (28) for the same reasons as (3) does not mean the same as (1). Think of the fact that every miserable lottery ticket buyer who didn't win might have won and been happy, but that it could not have been that *every* miserable buyer became a happy winner.

The solution to the problem of the temporal version of (28) was effected by *Ref* and ***then***, and required the omnitemporal operator $\oplus$. For the modal case

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what linguists call 'E-type' pronominal anaphora.) From the point of view of natural language tense theory a language with *Ref* and ***then*** could be criticised for 'over-generating' — in the sense that it may allow many formulae which do not correspond with sentences of English — or perhaps indeed of any human language. A study of natural language tenses may want to restrict the expressive power of the language. Thus for instance the theory of tense described in Hornstein 1990 is based on the three temporal indices E (event time), S (moment of speech) and R (reference point) found in Reichenbach 1947 (though note Hornstein's discussion in chapter 2 of complex tense structures.)

we use the operator □ of logical necessity, as described on p. 20, combined with a propositional constant ***actual***. □(***actual*** ⊃ ***miserable*** *x*) says that it is logically necessary that if we are in the actual world then *x* is miserable, which means that *x* is miserable in the actual world. To provide a full semantics we have to know which world that means, and that requires that a sentence now has a truth value at *two* sequences — a sequence *s* of times, and a sequence *u* of worlds, and that there are a family of ***actually***s, just as there are a family of ***then***s.

(33) $\boldsymbol{actual}_i$ is true at *s* and *u* iff $u_i = u_0$

We shall call the modal version of *Ref*, $Ref^{m}$, (26) then becomes,

(34) $Ref^{m}{}_i\alpha$ is true at *s* and *u* iff α is true at *s* and *u*[0/i], where *u*[0/i] is *u* with $u_0$ in place of $u_i$. (I.e. for j ≠ i, $(u[0/i])_j = u_j$, and $(u[0/i])_i = u_0$.)

By analogy with $\boldsymbol{then}_i$ we may define an operator $\boldsymbol{actually}_i$ as

$\boldsymbol{actually}_i\alpha =_{df} \Box(\boldsymbol{actual}_i \supset \alpha)$

When $\boldsymbol{actually}_i$ is treated as an adverb and $\boldsymbol{actually}_i\alpha$ is defined as $\Box(\boldsymbol{actual}_i \supset \alpha)$, then (33) gives

(35) $\boldsymbol{actually}_i\alpha$ is true at *s* and *u* iff α is true at *u*[i/0], where *u*[i/0] is *u* with $u_i$ in place of $u_0$.

(28) then becomes

(36) $Ref^{m}{}_1(M\forall x(\boldsymbol{actually}_1\ \boldsymbol{miserable}\ x \supset \boldsymbol{happy}\ x))$.

Possibilities are often introduced as the consequent of a conditional, since what is possible always needs constraining in one way or another:

(37) If it had been raining it would have been possible for everyone then miserable to have been happy.

Although an accurate account of (37) probably requires the □→ operator discussed in Chapter 5, we can, for the moment, take (37) to mean something like that it is possible for there to be rain along with the miserableness. (37) can then

be formalised, with ***rain*** as a sentence meaning that it is raining, as:

(38) $MRef^{m}_{1}$(***rain*** $\wedge$ $M\forall x$(***actually***$_1$ ***miserable*** $x \supset$ ***happy*** $x$)).

As in the temporal case, we may sometimes need to refer to several worlds:

(39) If it had been raining it might have been possible for everyone then miserable to have admired everyone who might then have been happy.

There are a number of ways in which, corresponding to (24) in the temporal case, (39) could be formalised. One is

(40) $MRef^{m}_{1}$(***rain*** $\wedge$ $MRef^{m}_{2}$($M\forall x\forall y$((***actually***$_1$ ***miserable*** $x \wedge$ ***actually***$_2$ ***happy*** $y$) $\supset$ ***admires*** $xy$))).

All this discussion may seem laboriously technical. We have included it because of philosophers like Prior who seem to have supposed that the use of temporal operators in the object language can avoid commitment to times. While such philosophers may have been in the minority, there are more philosophers who have supposed that the use of modal operators can avoid commitment to possible worlds. For that reason it is important to establish the power required by the range of temporal and modal sentences in natural language. We shall, in the next chapter, introduce explicit object-language quantification over times and worlds, and then show how that the language so obtained can be expressed in the $Ref^{m}$ and ***actually*** language we have presented. In this way it will emerge that an adequate semantics for temporal and modal sentences, even when formalised with the use of operators, already has the power of quantification over times and worlds.

# 8

# *Time and world quantifiers*

The first aim of the present chapter is to explain how the extensions to the modal and tense predicate languages that we presented in Chapter 7 amount in power to a language with explicit quantification over times and worlds. A second aim is to explain how such a language is in turn equivalent to a tenseless and non-modal language of the kind referred to on p. 13. As a result, when we make precise the distinction between presentism and eternalism, or the distinction between actualism and possibilism, as presented in Chapter 4, it will become clear why there can be no logical argument to decide the metaphysical debate between them. This chapter can be omitted by those who are prepared to take on trust that these languages do have such power.[1] Suppose that we have a language in which there are variables for moments of time. Let us use the boldface variables $\mathbf{t}_1$, $\mathbf{t}_2$ and so on to stand for the temporal variables, and $t_1$, $t_2$, etc to refer to moments of time. So when we say that **t** refers to $t$ we mean that the temporal variable in the object language refers to the moment of time $t$. **t** is a linguistic item, a temporal variable, while $t$ is a time, a part of the physical universe.[2] With this in mind use [**t**] to mean 'it is true at $t$ that', where **t** refers to $t$. Then for any sentence $\alpha$.

(1) [**t**]$\alpha$ is true at any $t'$ iff $\alpha$ is true at that $t$ which is the value of **t**.

That is to say [**t**]$\alpha$ has, at *any* time $t'$, the value that $\alpha$ itself has at the time $t$ which is the time assigned to the temporal variable **t**. The semantics of [**t**]$\alpha$

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[1]Readers who need a formal demonstration of the claims made in this chapter should read it in conjunction with Appendix 1 on pp. 209-223 below.

[2]We do of course, as here, often want to speak about what is going on in the metalanguage, as when we distinguish our use of '$t$' and our use of '**t**'. When we do this we are actually talking in a meta-metalanguage! However we hope that our practice is clear. In addition to the discussion by Prior in the passage mentioned in the text and elsewhere, the introduction of temporal variables into the object languages is found in other early works such as Rescher 1966, Rescher and Garson 1968, and Garson, 1973.

presupposes that it occurs in a tensed language — despite the fact that its truth value does not change from time to time. This is in contrast to the metalinguistic sentence (1), which, although it *mentions* [**t**] as part of a reference to the schematic sentence [**t**]α, is itself a sentence in the tenseless metalanguage, as presented in chapter 1, in which the truth conditions for tense operators are stated. Prior seems to have recognised this distinction when he says on p. 25 of 1957 that a follower of Quine should not analyse '*p* is true at *t*' as *Utp* (where for Prior *p* is a sentence and *Utp* means that it is true at *t*) but rather should analyse *p* as a predicate, and represent the claim that it is true at *t* as simply *pt* — 'or better still φ*t*.' (*loc cit.*) Therefore, when we speak of a 'tenseless' sentence we might mean one of two things. (a) We might mean a sentence in a tenseless language, or (b) we might mean a sentence in a tensed language, but which does not change its truth value with the passage of time. We shall continue to use the same word 'tenseless', and will allow the context to make clear whether (a) or (b) is the sense in question.

(1) is true or false at any time iff it is true or false at every time. Analogously, we may say that

(2) [**w**]α

is true in any world iff it is true in every world. (Notice that we are *not* claiming that α *itself* is true in any world iff it is true in every world. The claim is that 'α in *w*' is true in any world iff it is true in every world.) Sentences which include both time and modality:

(3) [**w**][**t**]α

will be true at any time or world iff they are true at every time and world. So the (a)-(b) distinction between the two kinds of tenselessness also applies to modality.

The use of operators involving variables like [**t**] and [**w**] may seem an addition to the predicate languages that we considered in Chapter 7, but in fact they are in a sense already there. For we shall see that what is achieved by mechanisms like *Ref* and ***then*** and ***actual*** is precisely what the use of world and time variables provides.[3] In order to establish this we extend the language of

[3]King 2003 (pp. 214-229), taking issue with Richard 1982, argues that tense in natural languages should not be treated by means of indexical (sentential) operators, but by quantifiers over times. But he does acknowledge that it can be treated by operators, provided that the indexicality is sufficiently complex. Our argument of course is that *any* adequate treatment of tense will give you

Chapter 5 by introducing quantifiers over the temporal and modal variables. For the temporal cases we add a 'now' predicate, so that $N\mathbf{t}$ means that it is now $t$, as when we might say, 'it is now 2011'. This 'now' predicate is related to, but is not the same as the ***now*** operator in (4) on p. 85. In particular it does *not* bring you back to the present in the scope of $P$ or $F$ — so that $PN\mathbf{t}$ means that $t$ used to be now, i.e., that $t$ is a past moment.

Now recall that we have available the 'always' operator, $\oplus$, discussed on p. 18. $\oplus\alpha$ means that $\alpha$ is true at every time, whether past, present or future, and is definable in standard tense logic when time is assumed to be linear. $\oplus\alpha$ is true at *any* time iff $\alpha$ itself is true at *every* time. This means that, using $N$ and $\oplus$ we can define the operator $[\mathbf{t}]$ in the following way:

$$[\text{Def } \mathbf{t}]\ [\mathbf{t}]\alpha =_{\text{df}} \oplus(N\mathbf{t} \supset \alpha)$$

$\oplus(N\mathbf{t} \supset \alpha)$ says that always has been, is now and always will be that if $N\mathbf{t}$ is true then so is $\alpha$. But the *only* time at which $N\mathbf{t}$ is true is the (unique) time assigned to $\mathbf{t}$. So that $\oplus(N\mathbf{t} \supset \alpha)$ will be true iff $\alpha$ is true at the time assigned to $\mathbf{t}$. [Def $\mathbf{t}$] enables (1) on p. 84 (One day everyone now miserable will be happy) to be formalised as:

$$(4) \qquad \exists\mathbf{t}(N\mathbf{t} \wedge F\forall x([\mathbf{t}]\ \textbf{\textit{miserable}}\ x \supset \textbf{\textit{happy}}\ x)).$$

The role of the initial quantifier in (4) together with $N\mathbf{t}$ is to fix the value of the variable $\mathbf{t}$ as a particular moment. $N\mathbf{t}$ ensures that $\mathbf{t}$ designates the 'now' of the complete sentence. It thus plays the role of *Ref*. Once the value of $\mathbf{t}$ is fixed with the aid of $N$, $[\mathbf{t}]$ always refers to that moment, even when inside the scope of $F$. $[\mathbf{t}]$ thus plays the role of ***then***. (14) on p. 87 (Once everyone then miserable was going to be happy) can be formalised as

$$(5) \qquad P\exists\mathbf{t}(N\mathbf{t} \wedge F\forall x([\mathbf{t}]\ \textbf{\textit{miserable}}\ x \supset \textbf{\textit{happy}}\ x)).$$

In (5) the quantifier is inside $P$ so that, together with $N\mathbf{t}$, it fixes the value of $\mathbf{t}$ as a past moment, and as before $[\mathbf{t}]$ still refers to that moment, even when inside the scope of another tense operator, in this case $F$. In (5) the moment fixed by $N\mathbf{t}$ is in English picked out by 'then', not by 'now' as it is in (4), but the logical process

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the power of quantification, whether or not you begin with explicit temporal variables or with operators, since these are logically equivalent. And in a similar way any adequate treatment of modality will have to have the power of quantification over worlds, whatever its natural-language syntax might look like.

is the same in both cases. (23) on p. 89 (Once it was going to be that everyone then miserable was going to admire everyone who was then happy) can be formalised as

(6) $P\exists\mathbf{t}_1(N\mathbf{t}_1 \wedge F\exists\mathbf{t}_2(N\mathbf{t}_2 \wedge F\forall x\forall y(([\mathbf{t}_1]\ \textbf{\textit{miserable}}\ x \wedge [\mathbf{t}_2]\ \textbf{\textit{happy}}\ y) \supset \textbf{\textit{admires}}\ xy)))$.

We can handle the modal case similarly, by using the variables $\mathbf{w}_1$, $\mathbf{w}_2$, etc., together with an 'actually' predicate $A$ by analogy with $N$ in the tense case. By analogy with [Def **t**] we may define [**w**] as

[Def **w**] $\quad [\mathbf{w}]\alpha =_{df} \Box(A\mathbf{w} \supset \alpha)$

$\Box(A\mathbf{w} \supset \alpha)$ says that $\alpha$ is true at every world at which $A\mathbf{w}$ is true. But $A\mathbf{w}$ is true only at the world assigned to the modal variable **w**. (28) on p. 91 (It might have been that everyone actually miserable was happy) then becomes

(7) $\exists\mathbf{w}(A\mathbf{w} \wedge M\forall x([\mathbf{w}]\ \textbf{\textit{miserable}}\ x \supset \textbf{\textit{happy}}\ x))$.

Temporal quantifiers will not do the job here because the miserable and the happy are supposed to be so at just the same time. Sentence (37) on p. 92 (If it had been raining it would have been possible for everyone then miserable to have been happy) can be formalised as:

(8) $M\exists\mathbf{w}(\textbf{\textit{rain}} \wedge A\mathbf{w} \wedge M\forall x([\mathbf{w}]\ \textbf{\textit{miserable}}\ x \supset \textbf{\textit{happy}}\ x))$.

As in the temporal case, we may sometimes need to refer to several worlds, as in (39) on p. 93 (If it had been raining it might have been possible for everyone then miserable to have admired everyone who might then have been happy). Corresponding to (40) on p. 93 we have

(9) $M\exists\mathbf{w}_1(\textbf{\textit{rain}} \wedge A\mathbf{w}_1 \wedge M\exists\mathbf{w}_2(A\mathbf{w}_2 \wedge M\forall x\forall y(([\mathbf{w}_1]\ \textbf{\textit{miserable}}\ x \wedge [\mathbf{w}_2]\ \textbf{\textit{happy}}\ y) \supset \textbf{\textit{admires}}\ xy)))$.

What these new quantifiers and variables do is provide a mechanism to keep track of where we are. We call what these quantifiers range over 'times' and 'possible worlds', though we must be careful to remember that *all* that the variables do is reveal the logical structure of temporal and modal sentences. Although we can think of $A\mathbf{w}$ as meaning that $w$ (the referent of the variable **w**)

is the actual world, this construal is for now purely heuristic. The introduction of variables for times is not motivated by the fact that you can name times, but by the fact that sentences like (1), (14) and (23) in Chapter 7 require a tracking mechanism of the kind provided by *Ref* and ***then***. Similarly, the motivation for world variables is not that there are nameable possible worlds — there probably aren't. It is that the analogous modal sentences require an analogous tracking mechanism.

We shall now show how to translate every sentence of the *N* and *A* language into the *Ref*/***then***/***actual*** language. Let $\mathcal{L}_{\text{multi}}$ denote a multiply-indexed language with *Ref* and ***then*** and ***actual*** of the kind presented in Chapter 7. So that $\mathcal{L}_{\text{multi}}$ contains no temporal variables or temporal quantification. Let $\mathcal{L}_{x\mathbf{tw}}$ denote a three-sorted language, i.e. a language with three separate kinds of variables, individual variables, $x, y, z$, etc, temporal variables $\mathbf{t}_1, \mathbf{t}_2, \mathbf{t}_3, ..$ etc., and world variables $\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, ...$ etc. The individual variables only occur in atomic wff of the form $\varphi x_1 ... x_n$, where $\varphi$ is an 'ordinary' n-place predicate, the temporal variables only occur in atomic wff of the form $N\mathbf{t}$, and world variables only occur in atomic wff of the form $A\mathbf{w}$. $\mathcal{L}_{x\mathbf{tw}}$ does not contain ***then*** or ***actual***. All formulae of $\mathcal{L}_{x\mathbf{tw}}$ are built up from atomic formulae by $\supset$, $G$, $H$, $L$, $K$, $\Box$, $\Box\!\!\rightarrow$ and $\forall$. (Assume the atomic wff include the constant false proposition $\perp$, so that $\sim\alpha$ can be defined as $\alpha \supset \perp$.)

We define what is called a 'translation function', $\tau$. I.e. for every wff $\alpha$ in $\mathcal{L}_{x\mathbf{tw}}$, $\tau(\alpha)$ is a wff of $\mathcal{L}_{\text{multi}}$. $\tau$ is defined inductively, first by specifying it for atomic wff, and then by showing how the translation of a complex wff is determined from the translations of its simpler parts:

For atomic $\alpha$, except for $N\mathbf{t}_i$ or $A\mathbf{w}_i$, $\tau(\alpha) = \alpha$
$\tau(N\mathbf{t}_i) = \boldsymbol{then}_i$
$\tau(A\mathbf{w}_i) = \boldsymbol{actual}_i$
$\tau(\alpha \supset \beta) = (\tau(\alpha) \supset \tau(\beta))$
$\tau(\alpha \Box\!\!\rightarrow \beta) = (\tau(\alpha) \Box\!\!\rightarrow \tau(\beta))$
$\tau(G\alpha) = G\tau(\alpha)$
$\tau(H\alpha) = H\tau(\alpha)$
$\tau(L\alpha) = L\tau(\alpha)$
$\tau(K\alpha) = K\tau(\alpha)$
$\tau(\Box\alpha) = \Box\tau(\alpha)$
$\tau(\forall x\alpha) = \forall x\tau(\alpha)$
$\tau(\forall \mathbf{t}_i\alpha) = Ref_j \oplus Ref_i \boldsymbol{then}_j \tau(\alpha)$, where j is such that $\mathbf{t}_j$ does not occur in $\alpha$. (And where $\boldsymbol{then}_j\tau(\alpha)$ is defined as $\oplus(\boldsymbol{then}_j \supset \tau(\alpha))$ as on p. 90).
$\tau(\forall \mathrm{w}_i\alpha) = Ref^{\mathbf{m}}{}_j \Box Ref^{\mathbf{m}}{}_i \boldsymbol{actually}_j \tau(\alpha)$, where j is such that $\mathbf{w}_j$ does not

occur in α. (And where $\textbf{\textit{actually}}_j\tau(\alpha)$ is defined as $\Box(\textbf{\textit{actual}}_j \supset \tau(\alpha))$ as on p. 92).[4]

In Appendix 1 we state and prove theorems which show that any wff α of $\mathcal{L}_{x\mathbf{tw}}$ is semantically equivalent to its translation τ(α).

The translation of $\mathcal{L}_{x\mathbf{tw}}$ into $\mathcal{L}_{\text{multi}}$ is in fact reversible, and in each direction a double translation gets you back to something equivalent to what you started with. Here is how the reverse translation function τ* is defined:

$\tau^*(\varphi x_1 \ldots x_n) = \varphi x_1 \ldots x_n$
$\tau^*(\bot) = \bot$
$\tau^*(\textbf{\textit{then}}_i) = N\mathbf{t}_i$
$\tau^*(\textbf{\textit{actual}}_i) = A\mathbf{w}_i$
$\tau^*(\alpha \supset \beta) = (\tau^*(\alpha) \supset \tau^*(\beta))$
$\tau^*(\alpha \mathrel{\Box\!\!\rightarrow} \beta) = (\tau^*(\alpha) \mathrel{\Box\!\!\rightarrow} \tau^*(\beta))$
$\tau^*(G\alpha) = G\tau^*(\alpha)$
$\tau^*(H\alpha) = H\tau^*(\alpha)$
$\tau^*(L\alpha) = L\tau^*(\alpha)$
$\tau^*(K\alpha) = K\tau^*(\alpha)$
$\tau^*(\Box\alpha) = \Box\tau^*(\alpha)$
$\tau^*(\forall x\alpha) = \forall x\tau^*(\alpha)$
$\tau^*(\textit{Ref}_i\alpha) = \exists \mathbf{t}_i(N\mathbf{t}_i \wedge \tau^*(\alpha))$
$\tau^*(\textit{Ref}^{\mathbf{m}}{}_i\alpha) = \exists \mathbf{w}_i(A\mathbf{w}_i \wedge \tau^*(\alpha))$

What τ and τ* show is that the use of object-language variables for times and worlds does no more than exhibit the same structure that the multiply-indexed semantics of Chapter 7 exhibits. If you look at these translation functions you will see that (4), (5), (6), (7), (8) and (9) can be translated by (1) on p. 84, (14) on p. 87, (23) on p. 89, (28) on p. 91, (37) on p. 92 and (40) on p. 93, and vice versa.

The formal languages introduced so far in this part of the book — the object languages of this chapter and the last — whether of the $\mathcal{L}_{\text{multi}}$ variety or of the $\mathcal{L}_{x\mathbf{tw}}$ variety, might be held to articulate the Prior/Adams/Plantinga conception of absolute truth described in Chapter 4, while the metalanguages in which the semantics of these languages are expressed might be held to articulate the indexical theory of the present and the actual. Since the semantics of formal tense/modal languages are typically given in untensed and non-modal

[4]Compare the definition of $\forall\mathbf{w}$ on p. 49 of Cresswell 1990b, and see Lemmas 8.2 and 8.3 in Appendix 1 on p. 211 below.

metalanguages it might be thought that tense and modality must ultimately be explained in an untensed and non-modal framework, and thus that tense and modality are not ultimately real. However, it turns out that there is an important sense in which a tense/modal language and an untensed/non-modal language are semantically equivalent.

On p. 13 we suggested that a genuinely tenseless and non-modal language could be expressed in a first order language — of the kind presented on pp. 59-63 above. Like $\mathfrak{L}_{x\mathbf{tw}}$ such a language would contain variables for times and worlds, but unlike $\mathfrak{L}_{x\mathbf{tw}}$ it would not contain tense and modal operators, and unlike $\mathfrak{L}_{x\mathbf{tw}}$ its formulae would be evaluated absolutely and not relative to a time or a world. Imagine two languages of first-order logic $\mathfrak{L}_{x\mathbf{tw}}$ and $\mathfrak{L}_i$, with a common set of individual, modal and temporal variables, truth functors, and quantifiers over individuals, worlds and times. $\mathfrak{L}_{x\mathbf{tw}}$ contains the tense operators $G$ and $H$, and the modal operators $L$, $\Box$, $K$ and $\Box\!\!\rightarrow$ together with the predicates $N$ and $A$. All predicates of $\mathfrak{L}_{x\mathbf{tw}}$ except $N$ and $A$ admit only the 'ordinary' individual variables as arguments. (Of course there is nothing to stop the ordinary individual variables from referring to times and worlds.) $\mathfrak{L}_i$ is an ordinary (three-sorted) first-order (extensional) language which must contain a restricted identity predicate which can apply to two modal variables or two temporal variables but may not mix categories, and the following extra predicates: a two-place predicate R, which applies only to the world variables, a two-place predicate <, which applies only to the temporal variables, a four-place predicate K, whose arguments must be a world variable, a temporal variable, a world variable and a temporal variable, and a four-place predicate C, whose arguments are a temporal variable and three world variables.[5] $\mathfrak{L}_i$ does not contain $G$, $H$, $L$, $\Box$, $K$, $\Box\!\!\rightarrow$, $N$ or $A$; and $\mathfrak{L}_{x\mathbf{tw}}$ does not contain R, <, K or C, but otherwise $\mathfrak{L}_{x\mathbf{tw}}$ and $\mathfrak{L}_i$ contain the same predicates. However, if $\varphi$ is an n-place predicate in $\mathfrak{L}_{x\mathbf{tw}}$ then it is an n+2-place predicate in $\mathfrak{L}_i$, with the last two arguments being a world variable and temporal variable, and the remaining arguments being individual variables. It is important to appreciate that although $\mathfrak{L}_i$ is tenseless and non-modal — although the truth or falsity of its formulae is not dependent on a time or a world — nevertheless, because it has time and world variables, it can express temporal and modal notions, and, as shown in Appendix 1, is equivalent in power to a tensed and modal language like $\mathfrak{L}_{x\mathbf{tw}}$. So if $\mathfrak{L}_{\text{multi}}$ and $\mathfrak{L}_{x\mathbf{tw}}$ formalise the presentist/actualist position, then $\mathfrak{L}_i$ is a way of formalising the eternalist/possibilist position. That is why the equivalence

[5] We use all these symbols ambiguously to designate predicates in $\mathfrak{L}_i$ and the corresponding relations in the model that they refer to. In the case of K and C we write $K(\mathbf{w}_1,\mathbf{t}_1,\mathbf{w}_2,\mathbf{t}_2)$ and $C(\mathbf{t},\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3)$ respectively.

results raise the question of whether there is content to this metaphysical dispute.

We can illustrate this equivalence, and at the same time see the strength of the world-time parallel, when we formalise metaphysical theses about time and modality in the tense/modal language $\mathcal{L}_{x\mathbf{tw}}$. Our first example is the claim that the actual world is unique. The following two formulae are both valid in $\mathcal{L}_{x\mathbf{tw}}$:

(10) $\exists \mathbf{w} A \mathbf{w}$

(11) $\forall \mathbf{w}_1 \forall \mathbf{w}_2 ((A\mathbf{w}_1 \wedge A\mathbf{w}_2) \supset \mathbf{w}_1 = \mathbf{w}_2)$

What these imply is the validity of $\exists_1 \mathbf{w} A \mathbf{w}$, which says that there is a unique actual world.[6] Further, everything that is actually true is true there:

(12) $\alpha \equiv \exists \mathbf{w} (A\mathbf{w} \wedge [\mathbf{w}]\alpha)$

The corresponding temporal claim is that the present is unique.

(13) $\exists \mathbf{t} N \mathbf{t}$

(14) $\forall \mathbf{t}_1 \forall \mathbf{t}_2 ((N\mathbf{t}_1 \wedge N\mathbf{t}_2) \supset \mathbf{t}_1 = \mathbf{t}_2)$

What these imply is the validity of $\exists_1 \mathbf{t} N \mathbf{t}$, which says that there is a unique moment which is now. Further, everything that is occurring is occurring at that moment:

(15) $\alpha \equiv \exists \mathbf{t} (N\mathbf{t} \wedge [\mathbf{t}]\alpha)$.

An eternalist may say that the validity of (15) simply points to the fact that at any $t$ a unique time ($t$ itself) satisfies $N$; and any $\alpha$ is true at $t$ iff there is a time which is identical with $t$, at which $\alpha$ is true. Then a possibilist can point out that $\mathcal{L}_i$ is non-modal as well as tenseless and that the translation of (12) into $\mathcal{L}_i$ is true because at any $w$ a unique world ($w$ itself) satisfies $\alpha$; and any $\alpha$ is true at $w$ iff there is a world which is identical with $w$, at which $\alpha$ is true.

A second example is the claim that the 'truth makers' of tensed sentences

[6] The expression $\exists_1$ is a 'uniqueness' quantifier, where, for any variable $x$ (whether an individual variable or a time or world variable) and any formula $\alpha$, $\exists_1 x \alpha$ can be defined either as the conjunction $\exists x \alpha \wedge \forall x \forall y ((\alpha \wedge \alpha[y/x]) \supset x = y)$, or else more simply by $\exists y \forall x (\alpha \equiv x = y)$, where $y$ is not free in $\alpha$.

are tenseless facts.[7] In a formal predicate language the truth or falsity of all sentences depends on the truth or falsity of atomic sentences. Any plausible 'truth maker' doctrine would seem to involve a claim that some facts are 'brute' and that all other facts depend on them. A 'logically perfect' language, of the kind sought by Russell 1918 and Wittgenstein 1921, which mirrors the structure of reality, is one which presumably ought to make its atomic sentences correspond to brute facts. So consider the sentence

(16) $\exists x(\textbf{\textit{philosopher}}\ x \wedge \textbf{\textit{ugly}}\ x)$.

In an untensed indexical language like $\mathfrak{L}_i$, ***philosopher*** and ***ugly*** are three-place predicates, having world and time arguments in addition to their individual argument. (16) corresponds to the sentence

(17) $\exists x(\textbf{\textit{philosopher}}\ x\mathbf{w^*t^*} \wedge \textbf{\textit{ugly}}\ x\mathbf{w^*t^*})$

in the sense that, where **t*** and **w*** are free variables representing the 'present' and the 'actual', (16) is true at *t* in *w* iff (17) is true when its free variable **t*** is assigned *t* and its free variable **w*** is assigned *w*.[8] ***philosopher*** and ***ugly*** are one-place predicates in $\mathfrak{L}_{x\mathbf{tw}}$ and three-place predicates in $\mathfrak{L}_i$.[9] This is not surprising. If you take truth in *w* at *t* as basic then it would be natural to acknowledge two extra argument places in sentences, and so treat an n-place predicate as (extensionally) an n+2-place relation. (Or an n+3-place relation if the person-relativity introduced on p. 28 is assumed.) If, on the other hand, you take an $\mathfrak{L}_{x\mathbf{tw}}$-style language then, as we have just seen, you don't need the extra argument places. So the 'brute' facts, which correspond to the atomic sentences in a tense/modal language are tensed and contingent facts, and are stated without reference to worlds or times, while the corresponding brute facts in the indexical language are tenseless and non-contingent, and do involve such relativity. One might be tempted to say that whether reality is indexical seems to depend on which language you use to describe it. You can either use an indexical language for tense and modality, or a language taking tensed and modal truth as basic. Of

[7]A principal theme of Mellor 1998 is to argue against tensed facts as truthmakers for tensed sentences.

[8]The proof of theorem 8.9 in Appendix 1 (on p.217) is a formal proof of results of this kind.

[9]So it is easy to see why, at least from the point of view of logic, the problem of what Lewis 1986a, pp 201-204, calls 'accidental intrinsics' and 'temporary intrinsics' is a pseudo problem. See the discussion of (1) on p. 165 below.

course you could take tense as basic and treat modality indexically, or you could take modality as basic and treat tense indexically, but there seems no principled reason for a differential semantic treatment. If you say that there *is* such a thing as absolute truth, semantics alone won't tell you whether it is tensed or tenseless or whether it is contingent or necessary.

An issue which is often held to divide philosophers of time is the question of whether time flows. The presentist believes that times begin in the future and move through the present to the more and more distant past. Recent followers of McTaggart[10] claim that the view that time flows carries with it a commitment to wff like,

(18) $\forall \mathbf{t}(PN\mathbf{t} \wedge N\mathbf{t} \wedge FN\mathbf{t})$

(18) is contradictory in tense logic, as indeed is the even weaker

(19) $\exists \mathbf{t}(PN\mathbf{t} \wedge N\mathbf{t} \wedge FN\mathbf{t})$

so whatever is meant by the flow of time it cannot be *that*. A wff which gets closer is

(20) $\forall \mathbf{t}(PFN\mathbf{t} \wedge (PN\mathbf{t} \vee N\mathbf{t} \vee FN\mathbf{t}) \wedge FPN\mathbf{t})$.

(20) says that every moment was once future, is, was or will be present, and will be past. It is valid in tense logic provided time has no first or last moment. (20) translates into the following wff of $\mathfrak{L}_i$ with a free temporal variable $\mathbf{t}^*$:

(21) $\forall \mathbf{t}(\exists \mathbf{t}_1(\mathbf{t}_1 < \mathbf{t}^* \wedge \exists \mathbf{t}_2(\mathbf{t}_1 < \mathbf{t}_2 \wedge \mathbf{t} = \mathbf{t}_2)) \wedge (\exists \mathbf{t}_1(\mathbf{t}_1 < \mathbf{t}^* \wedge \mathbf{t} = \mathbf{t}_1) \vee \mathbf{t} = \mathbf{t}^* \vee \exists \mathbf{t}_1(\mathbf{t}^* < \mathbf{t}_1 \wedge \mathbf{t} = \mathbf{t}_1)) \wedge \exists \mathbf{t}_1(\mathbf{t}^* < \mathbf{t}_1 \wedge \exists \mathbf{t}_2(\mathbf{t}_2 < \mathbf{t}_1 \wedge \mathbf{t} = \mathbf{t}_2)))$

(21) is a monster, but it *does* translate (20), and it *is* valid, even with its free $\mathbf{t}^*$, given the linearity of $<$ and that time has no first or last moment. Maybe (20) does not capture all there is to the flow of time, but it surely captures part of it.

The modal analogue of McTaggart's argument would claim that every

[10]We have in mind such writers as Mellor 1981 and Le Poidevin 1991. On p. 24 Le Poidevin says "each tense operator is applicable to every true proposition: $Pp$ & $Np$ & $Fp$." $N$ here is a sentential operator — not the predicate of times used in our text — and $Np$ is equivalent to $p$, and so Le Poidevin appears to be claiming that $p \supset (Pp \,\&\, p \,\&\, Fp)$ is valid in tense logic. Calling Mellor and Le Poidevin 'followers' of McTaggart needs qualification. Mellor and Le Poidevin hold that while McTaggart's arguments against the metaphysical reality of tense are valid, his argument that the contradiction is essential to time is not.

world is both possible and actual. If the actual includes the possible then indeed the actual must also be possible, but suppose, by analogy with talk about the past and the future, we use the operator $\Diamond^*$ to mean *properly* possible, in the sense of being true in at least one non-actual world — whether or not it is also true in the actual world. If we use $\Diamond^*$ in this way then

(22) $\forall \mathbf{w}(\Diamond^* A\mathbf{w} \wedge A\mathbf{w})$

is just as contradictory as its temporal analogue (18), since if any world is actual it cannot also be properly possible (This is because, if $A\mathbf{w}$ is true in any world, it is false at *every* other world.) The true claim is the claim corresponding to (20). And that is

(23) $\forall \mathbf{w}(\Diamond^* \Diamond^* A\mathbf{w} \wedge (\Diamond^* A\mathbf{w} \vee A\mathbf{w}))$

(23) says that any world could be properly possible and is either actual or is properly possible; and that is true. It too has a valid translation in $\mathfrak{L}_i$.

In the last few chapters we have attempted to show that natural language has the power of explicit quantification over times and worlds. We have done this by presenting a formal language of tense and modal predicate logic, and then considering how such a language might have to be extended in order to account for what is said by certain natural language sentences. This procedure invites two criticisms. (i) If you are interested in natural language why use formal languages at all? And (ii) if you do use formal languages to account for the phenomena of tense and mood in natural language you should surely pay attention to the additional complexities of tense and mood which are found there.

In answer to (i) we use formal languages for two reasons. First, because we want to be able to *prove* that they have the power of explicit quantification over times and worlds, and second, because we do not take ourselves to be making a claim about English or any other particular natural language. Our strategy is to exhibit things that people might want to say, in whatever language they happen to be using, and then show what the logical structure of these has to be.

In answer to (ii) our task is to argue that a formal language which enables the expression of certain things that people want to say must have at least the power of quantification over times and worlds. A formal account of other features of tense and mood in natural language, as any reader of journals like *Linguistics and Philosophy* will realise, will need to extend these languages in all sorts of ways. Some of the extra complexity is indicated in Appendix 4 on pp. 238-242

below.[11] All we are arguing here is that any such extensions must *at least* have the quantificational power we have discussed in this chapter, and for that purpose all we require is a formal language rich enough to handle the kinds of sentences introduced in Chapters 6 and 7.

[11]See also the remarks in footnote 5 on p. 15 above.

# Part III

# Times and Worlds, or Tense and Modality?

# 9

# *Primitive modality and primitive tense*

Do other possible worlds really *exist*? One way of approaching this question is to rephrase it, and ask whether possible worlds should be in the domain of quantification in a regimented formal language adequate for expressing truths about time and modality. The languages of formal logic contain the *universal* quantifier, $\forall$, and the *existential* quantifier, $\exists$. But how might we tell whether a construction is genuinely quantificational without already presupposing a domain? In fact there are a number of tests we can use. For a start we have the following equivalences:

(1) $\forall x{\sim}\alpha \equiv {\sim}\exists x\alpha$
(2) $\exists x{\sim}\alpha \equiv {\sim}\forall x\alpha$

This allows us to define $\exists x$ as ${\sim}\forall x{\sim}$ or $\forall x$ as ${\sim}\exists x{\sim}$, and permits a 'square of opposition' of the kind mentioned for modal and temporal operators on p. 19. But there are other principles that these quantifiers obey. For instance, we have the following equivalences:

(3) $(\forall x\alpha \wedge \forall x\beta) \equiv \forall x(\alpha \wedge \beta)$

and

(4) $(\exists x\alpha \vee \exists x\beta) \equiv \exists x(\alpha \vee \beta)$

And we have the implications

(5) $\exists x(\alpha \wedge \beta) \supset (\exists x\alpha \wedge \exists x\beta)$

(6) $(\forall x\alpha \vee \forall x\beta) \supset \forall x(\alpha \vee \beta)$

(7) $(\forall x\alpha \wedge \exists x\beta) \supset \exists x(\alpha \wedge \beta)$

(8) $\forall x(\alpha \supset \beta) \supset (\forall x\alpha \supset \forall x\beta)$

(9) $\forall x(\alpha \supset \beta) \supset (\exists x\alpha \supset \exists x\beta)$

but not their converses. We also have such principles as

(10) $\exists x\alpha \supset (\forall x(\alpha \supset \beta) \supset \exists x(\alpha \wedge \beta))$

(11) $\exists x\alpha \supset (\exists x(\alpha \wedge \beta) \vee \exists x(\alpha \wedge \sim\beta))$.

We can now see how to motivate the claim that tense and modality have quantificational structure. Suppose for instance we think of $\forall\alpha$ as meaning the same as $H\alpha$ — that $\alpha$ was always so — and therefore think of $\exists\alpha$ as $P\alpha$ — that $\alpha$ was once so. It is not difficult to see that $\forall$ and $\exists$, if interpreted in this way, then obey the quantificational principles listed above. Similarly if $\forall\alpha$ means that $\alpha$ will always be so, and that $\exists\alpha$ means that $\alpha$ will one day be so.

In chapter 1 we noted that there are many different senses of 'possible' and 'necessary'. This means that, when considering the quantificational principles just listed we must assume that in each principle it is the same sense of possibility or necessity which is involved. Given this caveat it is easy to see that the principles also hold for modality. Consider (5) and think of $\exists$ as expressing possibility. If it is possible that we are at Waitarere and that it is raining, then it is possible that we are at Waitarere, and it is possible that it is raining. But the converse does not hold. We could have spent the whole day in Palmerston and we could have spent it in Waitarere. Each of these is possible. But their conjunction is not. It is not possible that we both spend the whole day in Palmerston and also spend the whole day in Waitarere. That is, although two things which are possible together are possible separately, if two things are separately possible they need not be jointly possible. The other principles can be illustrated with similar examples.

(11) in particular plays a central role in obtaining worlds from possibilities. If we have the possibility that something is true, and consider any other possibility, the first possibility can be extended either to include the second possibility, or to include the *negation* of the second possibility. The possibility that we spent the day in Palmerston might be one in which it is raining, or might be one in which it is *not* raining. So it can be extended to two separate possibilities. Not all possibilities can be extended in *both* ways. The Palmerston

possibility cannot be extended to include our being all day in Waitarere. But, because of that, it *can* be extended to our *not* being all day in Waitarere. This process of extending possibilities can be carried out infinitely. Nevertheless, for reasons similar to that by which moments of time are the infinite limit of ever decreasing intervals, eventually — and it is an infinite 'eventually' — in the same way, successively more specific possibilities reach possibilities which are in a sense 'maximal' in that no more specific possibilities can be added except those that are already included. (2) on p. 1 asserts that rain is possible. What if it were really so? What if the rain were actual? The feature of possibility that we are relying on is that if something (say rain) is *possible* then it might be actual. Now reflect on what it would be like if it *were* actual. If rain is actual all sorts of other things must be so too. In fact if rain were actual think of any other possibility you like, think of *anything* that could be so or not so, anything that might be true or false. Either it would be *so* along with the rain, or it would be *not so* along with the rain. The totality of all these things — the either being so or not so of each one of them — is the kind of maximal possibility that theorists like Adams and Plantinga call a possible world. So if rain is possible at all it will be possible together with at least one of these maximal possible totalities, i.e. true in at least one possible world. And conversely if it is true in a world — possible together with all these other things — then it is obviously possible *tout court.*

A key assumption in the reasoning of the previous paragraph has a long history. It was used by Aristotle, and goes something like this: If a sentence could be true then imagine what it would be like if it *were* true.[1] Add to this the view that actuality is complete — that actual truth and falsity is determinate — that every sentence has just one of two truth values.[2] This use of the completeness of actuality is perhaps easiest to accept in the case of epistemic possibility. Learning might be held to consist in moving from $M\alpha$ and $M{\sim}\alpha$ to knowing which of $\alpha$ and $\sim\alpha$ is true. This process can never be completed, and yet, if we are classical about truth, the incompleteness does not stop us believing that a completion

[1]See Rini 2003. The assumption attributed to Aristotle is that whatever follows from assuming the truth of something possible is also possible together with the original possibility. This paper considers the claim (following Judson 1983) that Aristotle might be misapplying such reasoning in a way which can make trouble for his modal logic.

[2]At least that is the classical assumption we are working with in this book. If you deny *that* then we are having a debate about truth and falsity in general. That is a debate we should have, but it is not the problem of how to get from modal discourse (classically understood) to possible worlds.

exists.[3] Since we say this in advance of knowing *which* completion marks actual truth it seems reasonable to say of any α for which *M*α is true, that if α *were* true then it *would* be part of a completed actuality. Such intuitions may be questioned, but they are not intuitions which assume possible worlds from the beginning. They are intuitions about modality, and our aim is to investigate what sorts of intuitions they are which lead to worlds.

Notice something significant about our treatment of ∀ and ∃. (1)-(11) contain only one individual variable, *x*. Suppose we want to say that everyone rich is happy. In standard predicate logic you would formalise this as

(12) $\forall x(\textbf{\textit{rich}}\ x \supset \textbf{\textit{happy}}\ x)$.

But (12) could equally be expressed as:

(13) $\forall(\textbf{\textit{rich}} \supset \textbf{\textit{happy}})$

where no variables are used, as none were used in giving a temporal or modal interpretation of ∀ and ∃. (12) can be easily reconstructed from (13) because there is only one variable in the scope of the quantifier. The need for explicit variables comes in sentences like

(14) Every logician admires a linguist.

(14) can be formalised as

(15) $\forall x(\textbf{\textit{logician}}\ x \supset \exists y(\textbf{\textit{linguist}}\ y \wedge \textbf{\textit{admires}}\ x\ y))$

where the role of *x* and *y* in (15) is to keep track of the fact that in the natural interpretation of (14) it is the logicians who do the admiring, and the linguists who are admired. Without some tracking mechanism there is no way of distinguishing (15) from

(16) $\forall x(\textbf{\textit{logician}}\ x \supset \exists y(\textbf{\textit{linguist}}\ y \wedge \textbf{\textit{admires}}\ y\ x))$

[3]See Stalnaker 2003, p. 34. We might call this the 'completion principle'. Systems of logic which quantify over propositions, of the kind used by A.N. Prior, often need explicit principles to say that every proposition can be extended to a 'complete proposition'. See for instance Prior 2003, p. 129. The completion property for the 'states of affairs' used in Plantinga 1976 is given a more formal expression in chapter 16. See in particular formulae (3) and (4) on p. 235 below.

when we leave off variables, and while there *are* mechanisms for doing this by operators, such mechanisms can be shown to be exactly equivalent to quantification using variables; for instance, in order to express the meaning represented by (16), English could change (14) to 'every logician is admired by a linguist'.[4] In predicate logic the use of individual variables $x$, $y$, .. etc is often motivated by thinking of them as dummy names. And indeed if $a$ is a name, principles like

$$\forall x \varphi x \supset \varphi a$$

and

$$\varphi a \supset \exists x \varphi x$$

are standard principles of logic. One approach to quantification is to *begin* with a class of entities, think of them as having names, and then introduce quantifiers over them by treating the variables as names. That is a common practice in teaching logic. The other way of approaching quantification is by looking at the logical structure of sentences, and that is the approach we have been taking in this book. According to this approach the function of variables is to keep track of structure, and such keeping track may often be necessary even in cases of things we cannot name. Thus, although we may not be able to name atoms, we may well wish to say that if two atoms, $x$ and $y$, come together in certain circumstances where $x$ has one feature and $y$ another, then $x$ will come to stand to $y$ in a certain relation. What is more significant has been to show that temporal and modal discourse requires the tracking mechanisms supplied by variables in the manner referred to in the discussion of (15), and this of course was the burden of Chapter 8.

An area where questions are raised analogous to the question that began this chapter is in mathematics, as when it might be asked in the question 'Do numbers really *exist*?' Take

(17) 77 is Cresswell's weight in kilograms.

(17) is equivalent, in some strong sense, to

(18) Cresswell weighs 77 kilograms.

[4]Cresswell 1990b and 1996 explore such mechanisms. For a proof that first-order predicate logic can be analysed by operators in modal propositional logic see Kuhn 1980.

Further,

(19) 77 is less than the distance in kilometres between Wellington and Levin

is equivalent (in the same strong sense) to

(20) Levin is more than 77 kilometres from Wellington.

(18) and (20) are empirical facts, and 77 codifies a connection between them. If Cresswell reverts to his ideal weight of 74kg then a certain connection between (18) and (20) will be lost. The codification of connections between such sentences gives rise to arithmetic, and if you ask whether numbers really *exist*, then one might say that, according to the attitude to quantification just presented, you have missed the point. This attitude to quantification holds that questions of what our language quantifies over — questions of what there is — can only be addressed in the context of a language as a whole. Crispin Wright, in Chapter 1 of Wright 1983, calls this the 'context principle', and discusses its role in Frege's philosophy of mathematics. Notice that this attitude to quantification does not on its own require us to take sides on the realism/antirealism debate – for we have said nothing to suggest that sentences like (18) or (20) do not have a robust truth value because of the way the world is. All we have suggested is that questions concerning the domain of quantification are to be answered by considering their role in the structuring of truths.

In the case of ordinary perceptible objects it works like this. What is a Pohutukawa tree? Well, if you are in New Zealand, especially at Christmastime, you can point to something and say "That is a pohutukawa tree" — and the 'tree' represents whatever it is which makes your statement true. On p. 43f Wright reminds us (as perhaps Wittgenstein 1953 did before him) that the whole process of ostensive reference has to be understood in the context of a range of language use. Among the sentences about pohutukawa trees are those whose formal regimentation involves true sentences with an appropriate existential quantifier, where a quantifier is defined according to the criteria listed on pp, 107-108. The reason we add 'appropriate' is simply that not all true statements have existential force. While there may be true sentences about unicorns or fairies,

(21) There are fairies at the bottom of our garden

is not one of them, and the truth of

(22) There are no fairies at the bottom of our garden

gives no reason for believing in fairies. It is of course a non-trivial matter to determine what the appropriate sentences are — non-trivial in just the way that our regimentation of tensed and modal discourse in Part II is non-trivial. But, assuming that there are such sentences, and assuming that some of them are true, we can say that it is analytic that there are pohutukawa trees, given that the appropriate pohutukawa-tree sentences are true. One might call this primitivism about pohutukawa trees, provided we recognise that it is primitivism in a somewhat peculiar sense. Take someone who then goes on to ask what pohutukawa trees really *are*, and gives an answer in terms of arrangements of molecules, or atoms, or particles which are even more basic. Such a person might be held to be providing a reductive analysis of pohutukawa trees. The analysis may be wrong or may be right, but the test is whether it ascribes the correct truth conditions to pohutukawa-tree sentences. If the right kind of existential sentences about pohutukawa trees are true there can be no doubt *that* pohutukawa trees are. The reductive question can only be *what* they are. The point is important for our book because one of our principal claims has been that the structure of tense and modal discourse is quantificational. In the case of temporal discourse the matter plays out like this. Rather than talk about the 'existence' of times we might begin by taking tense as primitive, and then simply note three things:

(23) Temporal operators are quantifiers

(24) 'times' are whatever temporal operators quantify over

(25) Some appropriate temporal sentences are true[5]

As in the case of pohutukawa-tree discourse 'appropriate' is also required in temporal statements:

(26) It was raining yesterday

has existential force, but

[5]That enough appropriate temporal sentences are true to justify attributing quantificational structure may seem obvious, but Sider 1999a suggests that for a presentist many of these sentences are not true. Sider claims that a presentist should think of them as 'quasi-true'. (*Op cit*, p. 333.) Whatever may be said about this response, it would be equally available to an actualist.

(27) At no time was it raining

might be held true if there are no times. If you compare the sentences presented in Chapters 7 and 8, you will see that a sentence like (26) is an 'appropriate' sentence in the sense of (25). In this respect modal primitivism[6] seems exactly analogous, for we can understand the 'existence' of possible worlds to follow from

(28) Modal operators are quantifiers

(29) 'worlds' are whatever modal operators quantify over

(30) Some appropriate modal sentences are true

Justifying (23) and (28) was the principal task of chapters 7 and 8.[7] The arguments produced there are intended to show that while there may be debate about *what* times and worlds are, there can be no doubt *that* they are.[8] Lewis 1986a p.9 objects to such primitivism. But Lewis's objections are in essence that the languages of standard modal predicate logic are not powerful enough to express everything that quantification over worlds can handle. That is indeed true, and that is why the extra power of the languages of Chapter 7 is required.

Temporal or modal primitivism entails that it is *analytic* that times are whatever we quantify over in temporal discourse and that worlds are whatever we quantify over in modal discourse. We had indeed supposed this to be obvious, and yet in the modal case it has been denied. It is worth quoting from p. 14 of Nolan 2002:

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[6]Modal primitivism is similar to what Forbes 1989, p. 36, calls 'modalism', except that Forbes thinks that modalism involves a denial that modal operators are quantifiers over worlds. We discuss Forbes's view in the next chapter. This kind of modal primitivism could also be what is advocated by Prior in Prior and Fine, 1977, (See especially their remarks on p. 54.)

[7]There might seem more direct arguments for temporal and modal quantification, based on locutions like 'at that time' in the temporal case, and 'in that case' in the modal case. While we believe that there *are* such arguments, we are reluctant to rely on them since that might be thought question-begging.

[8]Thus there can be no distinction between whether other possible worlds really exist, or whether our modal discourse merely *pretends* they exist, and from the perspective of this chapter there is no content to the debate about what is called 'modal fictionalism' (See for instance Rosen 1990, 1993 and 1995.) More specifically, either fictionalism gives a good account of modal truth or it does not. If it does it has quantificational structure, and can be treated as in the text. If it does not then it is not adequate for the representation of modality.

> Even leaving aside the (doxastic) possibility that possible worlds and modal truth could be seen on a metaphysical par, with neither reducing to the other, it has always seemed clear to me that it is a substantial (rather than stipulatory) question whether possible worlds were the basis for modality, or whether it might be that possible worlds, and possible worlds discourse, were susceptible to reduction to modal discourse rather than *vice versa*.

Nolan here appears to us to amalgamate two questions which we feel should be kept distinct:

(31) Is it analytic that modal talk quantifies over possible worlds?

(32) Is it the case that the things that modal talk quantifies over can be analysed in terms of entities which are already needed for some *other* purpose?

(31) raises the possibility that although modal discourse does indeed have the structure of quantification over 'entities' of some kind, the only access we have to these 'entities' is through modal discourse itself; and indeed that even the sense in which they are 'entities' is up for debate. Many accounts of possible worlds are reductionist. One tries to construct worlds out of things which are considered to be required on independent grounds.[9] So let us see whether the temporal parallel helps here. Temporal operators — in the form of the natural language tense system — are also quantifiers. Suppose, as in (24), that we call what they quantify over times, and suppose that our only access to times is through tensed language. Need this show that tense cannot be analysed using times? Surely not. What it shows is that there is something that we are talking about when we use tensed language. Isn't all language like this? In the case of pohutukawa trees, if we found that a purported reduction didn't work we would say that whatever pohutukawa trees are it isn't *that*. One might even claim that

[9]Even Lewis's identification of worlds with maximal mereological sums of spatio-temporally related individuals might (Lewis 1986a, pp. 1f and 69) turn out to be of this kind — if such entities are indeed needed for some other purpose. (And of course a guiding principle of Lewis's modal realism is explicitly *not* any form of modal primitivism, since its purpose is to reduce modal notions to non-modal ones.) Lewis 1973a p. 85 rejects taking modal idioms as 'unanalysed primitives', but he seems to have in mind that such primitivism rejects the claim that modal operators are quantifiers. Be that as it may the 'primitivism' discussed in this chapter is a primitivism based on the view that because modal and tense operators have the structure of quantifiers they *are* quantifiers.

*no* reductive account is possible, and that we must take pohutukawa trees as primitive.

The question of what it is to take something as primitive exercises Stephen Schiffer in Chapter 3 of Schiffer 1987, where he considers the property of being a dog. On p.56 Schiffer poses the question "Exactly what property is this property of being a dog?" One answer is that the property of being a dog cannot be specified in other terms. In such a case, Schiffer says on p.58, it would have to be "primitive and irreducible", and he takes it that if being a dog is primitive and irreducible then it would be logically independent of, among other things, "every morphological and behavioral, every phenotypic and genetic fact about Gustav". But Schiffer needn't go so far — all that is required is that you cannot analyse being a dog in terms of the other simples in your favoured theory, and that does not entail that being a dog is logically independent of all other properties. We will discuss reduction briefly in Chapter 17. For now we point out that an affirmative answer to (31), i.e., given modal operators as quantifiers, that

(33) It is analytic that modal talk quantifies over possible worlds

leaves (32) still open. The sentences quoted above from Nolan 2002 suggest that Nolan is addressing (32), but he goes on to say (*loc cit.*)

> Furthermore, there is a lingering suspicion among many theorists that there are undischarged modal primitives (such as compossibility, co-satisfiability, entailment characterised as necessary truth preservation or suchlike) lurking within many standard accounts of possible worlds. These are often thought to be unattractive parts of the theories which they infest, but I have never seen it claimed that this meant that the theories could not really be talking about possible worlds. However one would expect this claim to have been made if it were *analytic* that possible worlds, if there be such things, would be the reductive basis of modal truth.

And he ends up by saying on p. 15

> ... I will not even be taking it that it is *true* that possible worlds are the metaphysical foundation of modality.

which strongly suggests (31). Whether Nolan is wrong or right about most philosophers *we* certainly take times and worlds to be whatever temporal and

modal operators quantify over, and so accept (33).[10] This is not just a terminological matter. On p. 50 we observed that an indexical semantics has sometimes been thought to provide a semantic reduction of tense and modality. The indexical theory of modality specifies that $M\alpha$ is true in $w$ (at some time $t$) iff $\alpha$ itself is true in some $w'$ accessible from $w$ — some $w'$ such that $w\mathrm{R}w'$. The indexical theory of tense assumes a temporal ordering and specifies that $P\alpha$ is true at $t$ (in $w$) iff $\alpha$ itself is true at some $t'$ earlier than $t$. (And analogously for $F$). This is often thought to show that tense is reducible to truth at a time.[11] However we have seen that you can equally define truth at $t$ in a tensed language like $\mathfrak{L}_{x\mathbf{tw}}$ in Chapter 8, and you can define $t < t'$ as $\oplus(PN\mathbf{t} \supset N\mathbf{t}')$ — i.e., $t$ precedes $t'$ iff whenever it was, is, or will be the case that $t$ *was* now then $t'$ *is* now. One could still maintain metaphysical neutrality even if you say that $<$ is a relation taken as primitive which is earlier/later only because it gives an analysis of tense. One might simply say that the world comes to us with a relation which we experience in terms of tense. And perhaps this is the *only* entry we have into making sense of this relation. That is to say our experience of tense and $<$ are *one and the same*. So that *it makes no sense* to ask which of tense or $<$ is primitive.

Similarly with modality. On p. 97 we introduced the operator $[\mathbf{w}]\alpha$ — meaning that $\alpha$ is true in world $w$ — as $\Box(A\mathbf{w} \supset \alpha)$. This operator uses an actuality predicate $A$ and the modal operator $\Box$. What this means is that, parallel with the temporal case, there is nothing to indicate whether modal operators or truth in a world is the more basic way of talking. Given an operator $M$ in a tense/modal language $\mathfrak{L}_{x\mathbf{tw}}$ we are able to define $w\mathrm{R}w'$ as $\Box(A\mathbf{w} \supset MA\mathbf{w}')$ — that it is logically necessary that whenever $w$ is actual then in the $M$ sense $w'$ is possibly actual.[12] For that reason it is difficult to argue that the indexical semantics has any metaphysical implications. Look at the objections brought against the use of world similarity in the analysis of counterfactual sentences in Lewis 1973a. Lewis's critics had supposed that intuitions about similarity do not support our intuitions about counterfactuals. According to Lewis 1979b we have intuitions about the truth of counterfactual sentences. Indeed the arguments of Lewis's critics depend on this. Also according to Lewis the semantics of

[10]It would perhaps be better to put our point less tendentiously by saying that, whether or not there is some *other* sense of ontological commitment, there is certainly the question of whether an area of discourse has quantificational structure, and as a consequence can be modelled in terms of domains and quantifiers.

[11]As for instance in Sider 2001, p. 13, though note also the more careful remarks in Dyke 2003. p. 381.

[12]Note that this does *not* mean that it is possible in *any* sense for $w$ and $w'$ to be *both* actual.

counterfactuals depends on a three-place relation among worlds — that $w_2$ is more similar to $w_1$ than $w_3$ is. There are however many different similarity relations. How do we know *which* similarity relation to choose? Well it's the one which gives the right truth conditions. How do we know there is such a relation? Because we have intuitions about counterfactuals, and our intuitions about counterfactuals just *are* our intuitions about the relation.

We can see some consequences of (33) by looking at arguments which have been brought against possible worlds. Richards 1975, pp. 110 and 112, asks how we can have epistemic access to other possible worlds. How could we *know* what happens in another world? The reply in our book is the one presupposed in (28)-(30): We know modal truths. Modal operators quantify over possible worlds. Therefore we know truths which involve quantification over possible worlds.[13] The temporal situation is analogous. Consider our knowledge of the past. We can no longer *inspect* the past, at least not without assuming time travel. Yet if we have knowledge of tensed facts, (23)-(25) assure us that we have knowledge of other times. When we consider the future the situation is more complex. Some things we may know about the future are as certain as our knowledge of the past. But what of others? We may know that rain is likely tomorrow, but we may not be able to know with certainty that it *will* rain tomorrow. How should this knowledge be expressed? While the content of our knowledge of the past may be structured in terms of a unique series of moments, our knowledge of tomorrow's weather may be structured in terms of alternative futures, of which some are more likely than others, and so involves modal knowledge.[14]

Forbes 1983, p. 127, suggests that an important difference between worlds and times is that we live through time but we stay in the same world. A first comment is that 'living through' is a temporal phenomenon. A person *p lives through* what happens in a world *w* between $t_1$ and $t_2$ if the person is alive in *w* at every moment between $t_1$ and $t_2$. In order to evaluate the alleged disanalogy consider a person who is alive (say today) in the actual world. This person will also be alive in many other possible worlds.[15] We don't call this 'living through',

[13]There is one bad argument: that we cannot know whether a proposition is true in a non-denumerably infinite set of worlds because we cannot inspect each in turn. (Kaplan 1978, p. 93, talks of a 'Jules-Verne-o-scope'.) But consider a similar objection to the analysis of motion as a continuous spatio-temporal path. Because we cannot inspect the non-denumerable points an object traverses, or cannot assume we have considered them all, it does not follow that this is not a good analysis of movement.

[14]See also the discussion of Skyrms 1976 in Yagisawa 2008.

[15]We discuss in Chapters 14 and 15 the views of those who believe that the same thing cannot appear in more than one world or at more than one time.

but, except for the obvious fact that worlds are not ordered by 'earlier/later' in the way times are, it is clearly the modal analogue of living through, and just as clearly is something which occurs. Notice that Forbes cannot simply be claiming that although there *are* moments of time to live through there *are* no other worlds to be in. Or if he is then we can re-express his claim, without mentioning moments of time, by using tensed language in the temporal case, and, without mentioning worlds, by using modal language in the modal case.[16]

An author who is bothered about our seeming inability to undertake interworld travel is Lycan 1979. On p. 295 Lycan complains that he doesn't see why he can't go to another world and meet himself. Look first at time travel. Bugsy goes back in time from noon on Sunday to noon on Saturday, meets himself and then returns to noon on Sunday. On one defence of the possibility of time travel what happens is this.[17] There are in essence *two time lines*, Bugsy's personal time line, and what we might call the 'regular' time line. In terms of the regular time line this is all that happens: Just before noon on Saturday there is a person, Bugsy. Suddenly a person Bugsy* appears who looks and talks like an older version of Bugsy. Just after noon on Saturday, Bugsy* disappears. At noon on Sunday there is only one person Bugsy, but just after noon on Sunday, Bugsy appears to 'remember' going to noon on Saturday and meeting himself. On Bugsy's 'personal' time line it's as if there was an interval around noon on Sunday at which Bugsy did the things that Bugsy* does in the 'regular' time line. Now consider the analogous modal situation. In the 'real' world $w$ Bugsy is perhaps in a trance or asleep throughout Sunday. In another possible world $w'$ there are Bugsy and Bugsy*, where Bugsy* has memories of times before Sunday which are just like Bugsy's in $w$. After Sunday Bugsy has 'memories' in $w$ of what happened to Bugsy* on Sunday in $w'$. According to (33) this claims no more than that it is possible that Bugsy and Bugsy* did certain things on Sunday, and that Bugsy (actually) has certain memories after Sunday. *That's* what it would take to do what Lycan assumes cannot be done, and it is not hard to see that while something like this is unlikely it is not impossible. We say 'unlikely' because for this to be genuine modal 'travel' the 'memories' that Bugsy actually has after Sunday cannot have been caused by anything that actually happened, just as in the temporal case Bugsy's memories at noon on Sunday have to be caused by what happened to Bugsy* at noon on Saturday, not by anything that happened at noon on Sunday. So there would seem little

[16]Some useful points about the modal equivalent of a temporal journey are made in Yourgrau 1986.

[17]See Lewis 1976

possibility of a technology for arranging the modal 'journey'.

Lycan may protest that this is not an *actual journey* to another possible world. Perhaps not, so let's look at how this would fare in the time-travel case. Lycan would have to protest that when Bugsy travelled back from noon on Sunday to noon on Saturday this is not a *present* journey to another time. For if it is real time travel Bugsy actually has to *go back* to noon on Saturday. Otherwise it would be like imagining that on another planet at noon on Sunday there is an individual who is currently (at noon on Sunday) doing exactly the things Bugsy did at noon on Saturday, and it is *this* individual that Bugsy at noon on Sunday goes to meet. Certainly this is possible but it is not time travel. And analogously the person who *actually* goes to visit a counterpart is not performing modal travel. Just as time travel is explained as a connection between what is happening now and what *has* happened, so modal travel is a connection between what *is* happening and what might have been happening. And even without talk of time travel we have to be clear that the modal correlate of 'change' is contingency. Where change involves something being so which was once not so, in the same way *contingency* involves something being so which might not have been so. Although this is the modal correlate of change it is not itself change, so that asking for a modal 'journey' is not asking for an actual journey. In the same way time-travel is not an actual journey in any ordinary sense. An ordinary journey is a path which associates a continuous series of spatial positions with a continuous series of temporal moments. A journey back through time does not do this, and a 'modal' journey is different again. So what precisely is the objection which is based on the observation that the past is something we *did* live through, and the future something we *will* live through, but that the possible is only something we *might* live through, or that we *might have* lived through? What more is it than stating the obvious that tense is tense and modality is modality?

# 10

# *'modalism' and 'tensism'*

In this chapter we shall examine further the suggestion made in the last chapter, according to which worlds and times are identified as the quantificational ranges of the modal and tense operators. In particular we shall look at the work of Graeme Forbes, since he is an author who is very well aware of the formal results which have played such a large part in our own book. Because Forbes's response to the formal results has been quite different from ours, a discussion of his position enables many of the issues at stake to be helpfully addressed. As noted in footnote 6 on p. 114 above, the view that modality is more basic than possible worlds is what Forbes 1989 p. 36 calls 'modalism', and in this chapter we shall look at what Forbes has to say about modalism, and will look at the temporal parallel. Although both Forbes 1985 and 1989 are about modality, and barely mention the temporal analogy (with the exception mentioned on p. 25 above), yet they provide a good source for examining parallel temporal arguments.[1] It's hard to get a word which corresponds to Forbes's 'modalism'. The temporal equivalent of the view in question is Prior's view that tense is more basic than temporal moments and that if we talk at all about moments we should make them up out of tense in much the way Forbes considers we should make up talk about worlds out of modal talk. (See Prior 1967, p. 189.) Prior's views are often spoken of as presentism, but the modal equivalent of that is normally called actualism. For want of a better word we shall speak of 'tensism'. It is not a word we like, in fact we don't like isms in general, but given that Forbes uses modalism we shall use tensism for the temporal equivalent of Forbes's 'modalism'.

In his defence of modalism Forbes agrees that standard modal predicate logic doesn't have the resources to express all the modal sentences that natural language requires, and on p. 27 says that

> we need an actualist language which exactly matches the expressive

[1] In footnote 20 on p. 35 of Forbes 1989 Forbes explains why he is ignoring time in his discussion.

> power of the corresponding possibilist language, and we have to construct one in some way that does not arbitrarily deprive the possibilist language of expressive resources, such as 'actually', which are clearly motivated by what we can say in natural language.

The bulk of Chapter 2 of Forbes 1989 is concerned with answering the claim of a possibilist that the domain of unrestricted quantification must include possibilia. Analogously, the defence of presentism in Crisp 2003 concentrates on the question of whether the domain of unrestricted quantification contains only entities which presently exist. Surprising as it may seem, in both cases it is easy to see that the issue of what quantifier to use turns out not to be a substantive one. In Chapter 6 the quantifiers $\forall$ and $\exists$ can range over things which do not now exist, even though they once did, and even over things which do not actually exist, though they might have. Presentists and actualists want their quantifiers to range only over things which now actually exist. Let us follow Prior 1957, p. 26, in using $\Pi$ and $\Sigma$ as the presentist/actualist universal and existential quantifiers. That is to say

(1) $\Pi x\alpha$ is true in *w* at *t* iff for every individual *a*, *provided a exists in w at t*, *a* satisfies $\alpha$ in *w* at *t*

and

(2) $\Sigma x\alpha$ is true in *w* at *t* iff there is an individual *a*, *which exists in w at t*, such that *a* satisfies $\alpha$ in *w* at *t*.

Suppose then that we want to define the eternalist/possibilist quantifiers $\forall$ and $\exists$ in terms of $\Pi$ and $\Sigma$. Since $\forall$ and $\exists$ are inter-definable, as are $\Pi$ and $\Sigma$, we need only consider one of them. We will show how to define $\exists$ in terms of $\Sigma$. We need the resources of Chapter 7, but these are resources that Forbes also has at his disposal. Then you can define $\exists$ in terms of $\Sigma$ as

(3) $\exists x\alpha =_{df} \mathit{Ref}_i\mathit{Ref}^{\mathbf{m}}{}_j\Diamond\otimes\Sigma x\boldsymbol{then}_i\boldsymbol{actually}_j\alpha$, provided neither $\boldsymbol{then}_i$ nor $\boldsymbol{actual}_j$ occur in $\alpha$.

While this definition may not wear its sense on its sleeve, it does work. What $\mathit{Ref}_i\mathit{Ref}^{\mathbf{m}}{}_j\Diamond\otimes\Sigma x\boldsymbol{then}_i\boldsymbol{actually}_j\alpha$ says is that it could have been the case that there exists, or existed or will exist, something that now actually satisfies $\alpha$. And this of course is just what possibilist quantification is, on the assumption that possibilia are just those things which exist in some world if not in the actual world. Here's how it goes. We want to say that some individual *a*, which doesn't

now actually exist, nevertheless now actually satisfies α, as it might be that Socrates is more famous now than when he existed. What you do is use $Ref_i$ and $Ref^m{}_j$ to 'hide' the present time and the actual world as the ith temporal index and the jth world index respectively. You then look for a time and a world where *a does* exist, and say of *a* at that world and time, where *a does* exist, that *a* now actually (i.e. in the actual world at the present moment) satisfies α. $\boldsymbol{then}_i$ and $\boldsymbol{actually}_j$ are what recover the present time and actual world, which have been hidden by the *Ref* and $Ref^m$ operators.

The consequence of this is that whether the quantifier is actualist or possibilist is not the significant question. So what *is* the significant question? Look at the following temporal example. On p. 214 Crisp 2003 considers the Roman empire, which no longer exists, and argues that where $t_\alpha$ is the present time then we can express the current non-existence of the Roman empire as (our numbering)

(4) *WAS*(something identical with the Roman empire will not exist in $t_\alpha$.)

Look at a formalisation of (4) expressed in the language of Chapter 7 (but with a presentist quantifier), using ***Roman empire*** as a one-place predicate. Then we have

(5) $Ref_1 P\Sigma x(\boldsymbol{Roman\ empire}\ x \wedge F\ \boldsymbol{then}_1 \sim \boldsymbol{exist}\ x)$

Although (5) only uses a presentist quantifier — modality is not involved here — its evaluation requires that the embedded sentence ~ ***exist*** *x* is evaluated for truth, and is indeed true, at a time (the present) when the Roman empire no longer exists. A modal case of the same phenomenon occurs in the analysis in (34) of (33) on p. 78 above, for that wff requires that ~***exist*** *x* and ~***famous*** *x* be true in a world in which *x* does not exist, but the quantifier in (34) only need range over things which exist in the world at which (34) is evaluated.

The real issue is whether there can be truths about an individual in a world or at a time at which that individual does not exist.[2] As we noted in footnote 3 on p. 3 above, Prior denies there can be facts about individuals which do not presently exist, and goes so far as to claim (1957, p. 31) that "'*x* exists'

[2]It is significant that one very influential account of the semantics of modal predicate logic, that given in Kripke 1963, explicitly allows (on p. 86n.) that a predicate can be true at a world of something which does not exist at that world, for all that Kripke's quantifiers only range over things which do exist at that world. (The reason for this is to preserve bivalence.)

must be logically equivalent to, and definable as, 'There are facts about $x$', $\Sigma x \varphi x$." These remarks occur in a context in which he is contrasting his views on time with those expressed in Quine 1953b, and Smart, 1955. And Forbes 1989 in Chapter 3 presents a defence of what he calls 'property actualism' that requires properties to hold only of things which exist. Prior appears to want to apply his doctrine to *any* truths about non-existent objects, and is thus led in Chapter 5 of 1957 to develop a many-valued logic in which a proposition may at a given time be 'unstatable'. As we explained in footnote 3 on p. 3, and on p. 36, and elsewhere, our attitude to truth is classical, so that while there is certainly room for debate about Prior's response, it is not our debate. Forbes is less radical, and only wants his property actualism to apply to 'unstructured properties', though, in the tradition inherited from Russell 1918 and Wittgenstein 1921, in their search for a logically perfect language, Forbes requires his atomic sentences to have as their values only unstructured properties. In this book we have taken properties to be functions from individuals to sets of indices — or equivalently, as explained on p. 32, functions from indices to extensions — and so make no distinction between 'structured' and 'unstructured' properties, and make no claims linking atomic sentences with the holding of any special kind of property. In any case nothing here gives any reason to make a distinction between the modal and the temporal.

Forbes 1989, p. 28, notes this definability of the possibilist existential quantifier in terms of an actualist existential quantifier. (Note that Forbes writes the *possibilist* existential quantifier as $(\Sigma x)$ and the *actualist* existential quantifier as $(\exists x)$ — the reverse of our usage.) He follows Prior and Fine 1977, p. 144, in using double indexed operators $\uparrow$ and $\downarrow$ equivalent to *Ref* and ***then***.[3] Forbes's modal equivalent of (3), in his notation, is

$$(\Sigma x)\varphi(x) =_{df} \uparrow \Diamond(\exists x)\downarrow \varphi(x)$$

Forbes appears to think that possibilists should have a problem with this definition, but it is hard for us to see any other significance than the fact that with

[3]Fine uses the symbols $\dagger$, and $\downarrow$ as modal analogues of Vlach's operators (see, p. 88 above) though he does not spell out the semantics of formal languages in which they occur. Forbes 1989 uses $\uparrow$ and $\downarrow$ on p. 27. In footnote 10 on that page he gives the impression that he thinks these are the same symbols as Fine's. Prior 1957, p. 52, had already recognised that if you allow reference to temporal distances you could say things like, "For some $n$ it will be the case $n$ days hence that there is an $x$ such that it was not the case $n$ days back that $x$ exists." But he denies that this is applicable in the modal case because "there is no modal analogy to the *order* in which past and future times are arranged about the present time." Kamp's and Vlach's operators, of course *do* have corresponding modal operators, which Forbes is exploiting.

enough of the other resources which are needed, the precise nature of the quantifier becomes unimportant. Pollock in fact, on p. 130 of Pollock 1985, interprets this logical fact quite differently from Forbes — Pollock claims that actualism is false because we *can* define a possibilist quantifier in terms of an actualist quantifier.

On p. 91 of Forbes 1989, Forbes proposes that his indexed actuality operators are 'scope-indicating devices'. We need not quarrel with that, but the formal theorems he goes on to prove establish that such devices are *exactly* equivalent to quantification, in the sense that all and only what can be expressed in one way can be expressed in the other way. One might in fact respond that *all* quantification is a 'scope-indicating device'. So the only issue is whether there is any metaphysical sense to be attached to the claim that although these sentences contain quantifiers, and although many of them are true, yet this does not yet tell us whether the things which the true sentences say exist really do exist. It will not do to distinguish between those sentences which naturally correspond to surface English quantificational sentences and those which naturally correspond to sentences containing operators, since it is plausible (See Cresswell 1990b and 1996) that ordinary quantification can just as naturally be treated indexically in the structure of English, and therefore presumably in other natural languages.[4]

Melia 2003, p. 93, puts the point using the following diagram, in which the top line is a standard quantificational way of expressing the matter, and the second line is in terms of Forbes's indexed operators:

$$\underline{\exists w_1}(\underline{\forall w}\ \underline{\forall x}(\underline{Exw^*}\ \underline{\rightarrow}\ \underline{Exw_1})\ \underline{\&}\ \underline{\exists y}(\neg\underline{Eyw^*}))$$

$$\overline{\Diamond_1}\ \{[\overline{\Box}\ \overline{\forall x}(\mathbf{A}\overline{Ex}\ \overline{\rightarrow}\mathbf{A}_1\overline{Ex})]\ \overline{\&}\ \overline{\exists y}\ \neg\mathbf{A}\overline{Ey}\}$$

Melia's comment is this:

[4]The situation here is rather like that which faced Alonzo Church and others in the 1930s. The problem was how to capture the informal notion of an effective — mechanical — procedure. Since the notion is informal there seemed no hope of giving a *proof* that any precisely defined notion is equivalent to it. But what emerged was that whenever the notion was made precise it was a notion that could be proved equivalent to every other precise notion which purported to capture effectiveness. What then became known as *Church's Thesis* was simply the claim that the intuitive notion should be formalised in one of these ways. The fact that it doesn't matter which carries no threat of circularity. (For an account see Copeland 1997.)

> Were I a linguist who came across an unknown tribe who used the subscripted boxes and diamonds in this way, and were I to notice such close grammatical and structural similarities between the sentences of a first-order language that quantified over worlds, I would be strongly tempted to conclude that what we had was not a totally new way of thinking about modality, but merely just a slightly different notation for making the same old claims about modal reality.

However Melia appears to regard this as an argument against modalism. Neither he nor Forbes appears to acknowledge that modalism could be compatible with the claim that modal talk just is quantificational, and that maybe the only access we have to what it is that we are quantifying over is via modal sentences. On p. 33f Forbes alludes to Pollock's claim that the whole matter is purely terminological. Forbes's reply is "I think there is more substance than this to the issue." But what he goes on to argue is the superiority of modalism over world quantification, on the basis that the dispute *is* more than terminology.

Forbes takes it as given that the defender of possible worlds claims not to be able to understand modality, but claims to be able to understand quantification over worlds, and only by that means, can to come to an understanding of modality; while the advocate of modalism claims to understand modality but not to understand possible worlds. In particular he assumes that both he and his metaphysical opponent know what 'explain' means, that both are using it in the same way, but that one claims that the explanation goes in one direction while the other claims that it goes in the other direction. This ignores the possibility that it *is the same thing* that each understands. Surely Pollock's claim has to be that understanding quantification over worlds just *is* understanding modality, and that there is no asymmetry. If so, an argument is still required that the issue is more than terminological. Forbes's strategy seems to be to portray the possibilist as someone who thinks that, in the light of the translational equivalences of the kind presented in his own book and in Appendix 1 and Appendix 2 of the present work, something *more* has to be said to make possibilism different from actualism. On p. 34 Forbes puts the nature of the dispute as follows

> ... the crucial difference between $\Sigma$ and $\uparrow\Diamond\exists\downarrow$ according to the possibilist is that the former is meant to express a *primitive* notion of being which is *neutral* between actual and merely possible existence, and this neutrality is lost by the actualist analysis in terms of $\uparrow\Diamond\exists\downarrow$. For $\exists$ is supposed to express a primitive notion of existence enjoyed only by

> actuals and which is applicable only to non-actuals under modal modification: the non-actuals enjoy the same primitive existence property as actuals in fact enjoy. But what the actualist calls the primitive existence property is seen by the possibilist as a special case of the real existence property (a special case only definable with extra resources): a restriction of primitive existence, or *being*, to the things which happen *actually* to exist. ... Since an actualist does not regard his notion of existence as any kind of restriction of some more general, more basic, idea of being, we should not think of the actualist's $\ulcorner(\exists x)\phi(x)\urcorner$ and the possibilist's $\ulcorner(\Sigma x)(E(x) \,\&\, \phi(x))\urcorner$ as having the same meaning.

(Recall that $\Sigma$ and $\exists$ in this quotation are our $\exists$ and $\Sigma$ respectively.)

We suggested above that Forbes seems to be assuming that there is a clear difference between the modalist and the believer in world quantification. If so then there should be an exactly parallel difference between the tensist and the believer in temporal quantification. Look at how the last quotation would go if we expressed it as a contribution to the presentist/eternalist debate:

> ... the crucial difference between $\Sigma$ and $\uparrow\Diamond\exists\downarrow$ according to the eternalist is that the former is meant to express a *primitive* notion of being which is *neutral* between present and past or future existence, and this neutrality is lost by the presentist analysis in terms of $\uparrow\Diamond\exists\downarrow$. For $\exists$ is supposed to express a primitive notion of existence enjoyed only by present individuals and which is applicable only to past or future individuals under temporal modification: the past and future individuals enjoy the same primitive existence property as present individuals now enjoy. But what the presentist calls the primitive existence property is seen by the eternalist as a special case of the real existence property (a special case only definable with extra resources): a restriction of primitive existence, or *being*, to the things which happen *presently* to exist. ... Since a presentist does not regard his notion of existence as any kind of restriction of some more general, more basic, idea of being, we should not think of the presentist's $\ulcorner(\exists x)\phi(x)\urcorner$ and the eternalist's $\ulcorner(\Sigma x)(E(x) \,\&\, \phi(x))\urcorner$ as having the same meaning.

Here's a story about time. We all experience the passage of time. We look forward with anticipation to something which is going to happen. We then experience it, and then we recall that it once happened. This is so general and so universal that we attempt to formalise it, and eventually we discover that it can

be formalised by sentences which have the structure of quantification over a linear sequence governed by a 'before and after' relation, and we discover how to give an indexical semantics for these sentences. Does the fact that this process *began* with experiences described in tensed terms cast any doubt on the quantificational model? The defender of tense may well point out, and with some reason, that if the ordering *didn't* satisfy the requirement of tense then it wouldn't be a *temporal* ordering. The defender of the primacy of the ordering will similarly reply that the indexical account does not deny the experience of tense but gives a structure to it. Such theorists may or may not choose to have a Forbes-like debate about the order of *explanation* or of *understanding*, but it is hard to see why the temporal situation should be significantly different from the modal situation.

On p. 93 Forbes 1989 notes that it is possible to get the effect of the simple 'actually' operator by adding plural quantification in the sense of Boolos 1984, which is understood as having no ontological commitment to anything beyond the individual objects thought of together. Bricker 1989 makes a similar claim. The procedure can also be applied to the tense case, and can handle sentences like those discussed in Chapter 7 above without temporal quantification. Thus (1) on p. 84 above could be represented with a predicate variable $Q$ as

(6) $\exists Q(\forall x(Qx \equiv \textbf{\textit{miserable}}\ x) \wedge F\forall x(Qx \supset \textbf{\textit{happy}}\ x))$

However, as pointed out on p. 90f of Cresswell 1990b, Bricker's procedure does not appear to take care of all cases. Consider

(7) $\exists \mathbf{t}_1(N\mathbf{t}_1 \wedge G\exists \mathbf{t}_2(N\mathbf{t}_2 \wedge \exists x(\textbf{\textit{happy}}\ x \wedge [\mathbf{t}_1](P\forall y([\mathbf{t}_2]\ \textbf{\textit{miserable}}\ y \supset \textbf{\textit{admire}}\ xy))))$

(6) can be read as

(8) It is always going to be the case that there is a happy person for whom there is someone then miserable who used (before now) to admire them.

Meyer 2009 argues for a variation of the Forbes/Boolos/Bricker suggestion which uses full set theory with the membership predicate $\in$. Thus, sets can be the values of individual variables. We can use $s_1$ and $s_2$ for set variables, bearing in mind that syntactically they are just individual variables. In addition, set-theoretical

symbols like $\in$, and those definable in terms of it, will be ambiguously used for the set-theoretical notions themselves, and the symbols which represent them in Meyer's first-order tense logic. With Meyer's recipe (p. 243) to eliminate $\mathbf{t}_2$, (7) becomes:

(9) $\exists \mathbf{t}_1(N\mathbf{t}_1 \wedge G\exists s_2(\forall y(y \in s_2 \equiv \textbf{\textit{miserable}}\ y) \wedge \exists x(\textbf{\textit{happy}}\ x \wedge [\mathbf{t}_1](P\forall y(y \in s_2 \supset \textbf{\textit{admire}}\ xy))))$

Now apply the recipe to $\mathbf{t}_1$. Since both $x$ and $s_2$ are free in $[\mathbf{t}_1](P\forall y(y \in s_2 \supset \textbf{\textit{admire}}\ xy)$ we get:

(10) $\exists s_1(\forall x\forall s_2(\langle x,s_2\rangle \in s_1 \equiv P\forall y(y \in s_2 \supset \textbf{\textit{admire}}\ xy)) \wedge G\exists s_2(\forall y(y \in s_2 \equiv \textbf{\textit{miserable}}\ y) \wedge \exists x(\textbf{\textit{happy}}\ x \wedge (\langle x,s_2\rangle \in s_1 \supset \textbf{\textit{admire}}\ xy))))$

To handle (7) this recipe requires reference to sets of sets, and would require even higher-order reference if there are further embedded operators. For that reason Meyer's procedure cannot be construed in terms of plural quantification as Bricker advocates.[5] Meyer argues against applying the procedure to the modal case because he rejects possibilia (p. 246). But, metaphysics aside, whatever may be said about his set-theoretic way of dealing with sentences like (6) or (10), it is applicable equally in the tense or the modal case, and does not in itself challenge any arguments for the world-time parallel.

[5]Forbes (1898, pp. 93-102) is more tentative about the generality of the procedure.

# 11

# *The present and the actual*

What happens when we apply the parallel to metaphysicians' arguments about the present and the actual? In his essay 'On what there is' (Quine 1948) Quine is concerned to argue that no sense can be attached to the admission of 'possibilia' into one's ontology. On the other hand A.N.Prior is one, who, we shall claim, should be understood as holding the same attitude to past or future individuals which do not currently exist. Our concern in this chapter will be to compare these attitudes.[1] Quine introduces a philosopher he calls 'Wyman' — the believer in unactualised possibles. (See p. 75 above.) On p. 3 he says that

> Wyman, by the way, is one of those philosophers who have united in ruining the good old word 'exist'

Prior's response will be to say that Quine himself is a philosopher who has succeeded in ruining this good old word. For Prior will say that Socrates does not exist. Of course he used to but he does not now. Prior will say that he can attach no sense to the claim that there could be a sense of 'exists' which does not mean 'exists now'. Much as Prior stipulates that by 'exists' he means 'exists now', Quine stipulates that by 'exists' he means exists somewhere in space and time. For Prior, when Quine talks of 'what there is', he has to mean anything that either once existed somewhere in space or presently exists somewhere in space, or will exist somewhere in space. Apply this to Wyman. Wyman says that he means to include possibilia — i.e., Wyman claims to be talking about things which might

[1]In using Prior in this way we should point out that, although we do not think we are being unfaithful to him — and indeed we shall make use of quotations from his views — our purpose is not primarily Prior scholarship, and there may be places where it could be argued that we have not got him exactly right. There is one aspect of Prior's views that we are ignoring in this book. We shall concentrate on past individuals, since Prior's attitude to the future involves the claim that there is a close connection with future truth and determinism. Such issues are important in a study of Prior's work, but are issues we wish to avoid, and will. So we shall not, when discussing Prior, assume a difference between the past and the future in terms of the assignment of truth values, except for a brief discussion of the 'sea battle', on p. 159 below.)

have existed as well as things which do. And Wyman may say that this is quite ordinary talk. We say that there *is* a philosopher who wrote a book called *Real Time*, that there *was* a philosopher who was executed by drinking hemlock, and that there *might have been* a 20th century philosopher who wrote a better article than 'On denoting'.

Prior will acknowledge that *of course* he can understand that by 'is' Quine has to mean 'is, was or will be'. There are, to be sure, problems about quantification in such contexts — problems raised by sentences like 'there have been two kings of England called "James"' — but aside from these expressibility problems Quine's stipulation can easily be understood by Prior. Prior might then go on to ask Quine how he would reply to someone like David Lewis, who stipulates that by 'is' *he* means 'exists somewhere in some world and some place at some time'. Quine would presumably say that he simply cannot understand such language, much as Lycan 1979 p. 290 says of the Meinongian[2]:

> I have to take my place among those who find *Relentlessly* (i.e., *genuinely* or *primitively*) Meinongian quantification simply unintelligible.

Lycan is talking of quantification here, not of truth, but in fact any claim that something exists is equivalent to the claim that an assertion of existence is true, so if you find a stipulative definition of truth unintelligible then of course an account of quantification which depends on it will be equally unintelligible. And Prior will say the same about untensed truth. But more about Lycan later. We imagine, as others writing on this topic have said[3] that we come to philosophy with a body of uncontroversial truths. Or perhaps better to say truths or falsehoods. Thus for instance someone might tell you that Auckland is the capital of New Zealand, perhaps on the ground that they know that Auckland is New Zealand's largest city, or that they know it was the capital before 1865, and didn't realise that the capital had then been shifted to Wellington. Although people may disagree about whether this is or is not a truth it is not in any sense a *philosophical* disagreement, and any conceptual framework must make room for such disagreement in a way which does not take sides on the metaphysics of truth.

But how do we have access to what it is we hold to be true? It is hard to

[2]This seems to us to be a somewhat Pickwickian sense of the term 'Meinongian' since, on p. 277n, Lewis is classified as a defender of a strong form of it

[3]See for instance Loux 2002, p 16.

suppose that we have access in any way other than in language, and it is for this reason, in our view, that philosophy (or at least metaphysics) has to begin with an account of the logical structure of everyday claims. Of course we are not undertaking a linguistic study of natural language. Although this view of metaphysics is widely acknowledged it is more common to treat metaphysics as consisting of making a list of what we do or don't believe in[4] and may perhaps be at the root of the 'incredulous stare' objections to Lewis's metaphysics of possible worlds. The dispute between David Lewis and the incredulous starers is often portrayed as the view that there are many things that ordinary folk do not believe in but that Lewis does[5] — for instance talking donkeys. While Lewis may himself be guilty of putting the dispute in this way,[6] we believe that there is an important sense in which this is to misconceive the nature of the dispute. Here is how we think it should go:

INCREDULOUS STARER: I don't believe in talking donkeys.

LEWIS: Oh but you do — for you believe that talking donkeys are possible. According to the semantics I am defending, to believe, in the ordinary, non-philosophical, everyday sense that there are no talking donkeys is to believe that talking donkeys are absent from our world, and I too believe that. To believe, in my *metaphysical* sense, that there are talking donkeys is just to believe, in the ordinary sense, that talking donkeys are *possible*. It is to believe that talking donkeys exist in another world. You do believe they are possible, so you do believe they exist in other worlds. When you say that you don't believe in talking donkeys I construe you simply to be denying that they are actual, and I too believe that they are not actual — that there are none in our world. When you repudiate talking donkeys then I do so too.

---

[4]Russell 1918 famously claimed that when you made a list of what there was in the world you would include facts but not propositions, rather as you might include elephants but not unicorns.

[5]Thus Melia 2003, p. 111: "Consider the radical damage Lewis's theory does to our commonsense beliefs.". And see also Brock and Mares p. 174, though they are more careful in acknowledging that the matter is more complex than that.

[6]See for instance p. 133f of Lewis 1986a. But note that after appearing to admit that common sense does disagree with modal realism, Lewis at least raises the question of the precise sense in which he disagrees with common sense. It seems to us that his remarks on the 'spokesman for common sense' who 'thinks actuality is all there is' indicate that this spokesman has begun to be a philosopher, even if a 'common sense' philosopher.

There may indeed be a dispute about the correctness of Lewis's semantics, i.e. one may legitimately take issue with whether or not to accept that Lewis is correct in his *analysis* of the modal beliefs that we share, but that is quite different from disagreeing about what there is, at least it is quite different if the dispute is between Lewis and ordinary folk, rather than a dispute between Lewis and various other metaphysicians and semanticists. The *hard* questions are not whether or not you believe in talking donkeys. The hard questions are about what this claim *means* — that is why we must begin with semantics. In fact it seems to us highly likely that we may be able to translate most competing metaphysical theories in such a way as to respect equally the pre-philosophically agreed 'facts'.

The temporal parallel should make this clear. Prior believes that Socrates doesn't exist, although he used to exist. Hugh Mellor believes that Socrates exists. Does this mean that Mellor believes in things that Prior doesn't? Perhaps in some sense yes, but in another sense no. One could imagine someone claiming that Socrates escaped from the prison, discovered the secret of immortality, and is currently living with Elvis in Oamaru. We could no doubt agree that that is not what Mellor believes. Prior and Mellor no doubt agree that Socrates was executed in 399 BC, and that in some sense he no longer exists. But Mellor admits Socrates into his ontology in a sense in which Prior does not. Prior would say that although Socrates doesn't exist he *used to* exist, that although there are now no facts about Socrates there used to be facts about him, in a sense in which it is not now a fact about him that there used to be facts about him. Mellor would say that in some tenseless sense Socrates exists in 403 but not in 2011, and that this is the true account of the situation. So that although Mellor would take issue with Prior's analysis of the facts, and would say that in an important sense Prior is wrong, he, like Lewis would not take issue with Prior about what things they each believe, in a non-philosophical sense, to exist.

In discussing the temporal parallel Yagisawa 2008 introduces what he calls 'modal tense' as an option for a modal realist. Although we do not use this term our attitude is similar to Yagisawa's, but we prefer to think of it as an option for an actualist. In just the way that Prior uses tense to articulate presentism we suggest that an actualist should use modal tense to articulate actualism. An *actualist* as we understand the term is a philosopher who believes that the only truth is actual truth, just as we understand a *presentist* to be someone who believes that the only truth is present truth. Of course for a presentist there can be present truths about the past or the future — that is precisely what Prior invented tense logic to express — just as an actualist can believe that there are truths about what might have been.

In what is sometimes called the 'old B-theory of time' it was supposed

that every tensed sentence could (and should) be translated into an untensed sentence. Thus

(1) Today is sunny at Waitarere

means something like

(2) It is sunny at Waitarere at 11.26am NZDT on 24 January 2010

where the 'is' is supposed to be what we have been speaking of as 'tenseless'. But of course, if it is wet tomorrow (1) will be false, but (2) will remain true. Those who call themselves the followers of the 'new B-theory of time' do not deny this.[7] They say that there are (and must be) tensed sentences, which can change their truth values with the passage of time, but that the proper explanation of this makes truth relative to a temporal index. So, as on p. 12,

(3) (1) is true at *t* iff it is raining at Waitarere at *t*

while

(4) (2) is true at *t* iff it is raining at Waitarere at 11.26am NZDT on 24 January 2010.

In the indexical treatment we use a tenseless metalanguage to account for a tensed object language, and a non-contingent metalanguage to account for a contingent object language. For that reason we have to be very careful about which language we are using in our metaphysical theorising.[8] For a start if we use a metalanguage in which words like 'past' and 'present' occur then we have to recognise that we are theorising in a tensed language, in which a 'tenseless "is"' has to mean 'is, was or will be', and if we are theorising in a non-contingent metalanguage we have to recognise that 'is' has to mean 'is or might have been'. In the remainder of this chapter we shall illustrate the problems which can arise

[7]See for instance Mellor 1981, who is concerned to distance himself from those like Russell 1915 who seem to have supposed that every tensed sentence could be translated into a semantically equivalent untensed sentence. The eternalism that we have just been describing makes no such claim.

[8]On the importance of this see pp. 320-324 of Hazen 1979. Hazen points out how a failure to distinguish between the object language being studied and the metalanguage used in studying it affects the argument in Kripke 1972a against the counterpart theory of Lewis 1968.

when theorists forget this.

Our first illustration of the importance of tense comes from Mellor's *Real Time*:

> So, does tense affect actual tenseless facts? On some views it does, for example the view ... that only what is past or present is real. The same goes obviously for the even more extreme view that only what is present is real. In these views, incidentally, existence must of course be conceived tenselessly: if 'exists' *meant* 'exists now', it would be the merest tautology that only the present exists. So existence must not here be considered in a present tense way: proponents of these views are not just peddling tautologies. They are saying for example that before the nineteenth century things and events were present, they had no reality at all, i.e. there were not even any tenseless facts about them. (Mellor 1981, p. 30.)

Now for the modal analogue:

> So, does contingency affect actual non-contingent facts? On some views it does, for example the view that only what is actual is real. In this view, incidentally, existence must of course be conceived non-contingently: if 'exists' *meant* 'actually exists' it would be the merest tautology that only the actual exists. So existence must not here be considered in a contingent way: proponents of these views are not just peddling tautologies. They are saying for example that even if a 19th century detective really had lived in Baker Street, truths about such an individual would have had no reality at all, i.e. there would not have been even any non-contingent facts about that individual.

Note here that because in the modal analogue there is nothing comparable with the difference between the past and the future we have not talked about more or less extreme versions of actualism in any sense corresponding to that distinction. With regard to the first paragraph we take it that Prior might well imagine that he was 'peddling tautologies', for the only kind of truth he imagines is present truth. That is why he says you don't need a present tense operator (1957, p. 9f). The problem for actualism raised by the modal analogue of Mellor's paragraph is that if 'exists' does *not* mean 'actually exits', how *does* an actualist understand it? It is easy to see how Lewis understands it — it means exists somewhere, though not necessarily in our world. But that can hardly be how an actualist understands it,

and it seems difficult to see how an actualist *could* understand it without 'peddling tautologies'.

At least one actualist appears to argue that it *is* 'the merest tautology' that the only truth is actual truth. On what Robert Stalnaker (2003, p. 30) calls the 'Augustinian theory of time' there *is* only the way things are. He claims that the Augustinian theory "says something substantive about what alone is real", whereas on Stalnaker's view 'actual' makes no analogous substantive claim. This seems puzzling. For a start there is a perfectly innocuous way of taking both theses. An Augustinian who says that 'there is no real philosopher who once drank hemlock' could happily admit that 'there was once a real philosopher who drank hemlock'. What *was* real is not now real. This seems no more restrictive than the analogous claim that while there never was a real detective called 'Sherlock Holmes' there might have been a real detective with that name. Stalnaker claims that the Augustinian thesis 'says that the actual world is a moment of time', but that would imply that the Augustinian does not admit a difference between the claim that there actually was a philosopher who drank hemlock and the claim that there might have been such a philosopher. The clue to the Augustinian's defence of privileging the present is found in Stalnaker's question on p. 28f of how one can look at all possible worlds as equally real. His answer is

> Only if one identifies the objective or absolute standpoint with a neutral standpoint outside of all possible worlds. But there is no such standpoint. The objective, absolute point of view is the view from within the actual world, and it is part of the concept of actuality that it should be so.

The Augustinian will point out that in the same way there is no standpoint outside the present, and that although past and future individuals *were* right or *will be* right to "affirm their full-blooded reality" — the phrase is from Stalnaker p. 29 — they are not *now* right to do so. (See Prior 1967 p. 198f.) This does not mean that they are fictional, because 'fictional' is a modal notion not a temporal notion, and, as we have just pointed out, the Augustinian is not obliged to identify the past or future with the fictional. It does though mean that the past and the future are in a sense inaccessible. The future is accessible by waiting, but you can't have it *now*.

Lycan 1979 also seems to assume that actualism is a 'merest tautology' in that he suggests that the denial of it is contradictory. But some care is needed in dealing with this important and influential article. Lycan surveys work on theories of possible worlds. He first describes the 'relentlessly Meinongian'

approach. We shall refer to its temporal counterpart simply as the Quinean approach. We understand by this the view that all times are equally real, that quantifiers range over things existing at any time, and that understanding the quantifiers this way is primitive. For Quine there would be a predicate 'present' but it would be indexical in just the way that 'actual' is for Lewis. Lycan's criticism that the Meinongian needs a predicate 'actual' becomes the criticism that Quine needs a predicate 'present'. For a certain type of presentist 'present' is not relational, and some of the 'impasse' that Lycan talks about may involve some cross purposes. Lycan treats Lewis and Meinong almost as advocating the same metaphysics. This is unfortunate as there is a sense in which Meinong's view *can* be regarded as contradictory. For it might be interpreted in the sense discussed on p. 81 above, according to which the truth that there is no present king of France supports the inclusion in the domain of quantification of a non-existent king of France. It was pointed out there that that is no part of the argument for possibilia. For that reason, in the 'parody' which follows we shall take Lycan's remarks about Meinong to be addressed to a Lewis-style modal realist. What we shall do is give a passage from Lycan's paper arguing in favour of actualism against possibilism, and will then present the parallel argument in favour of presentism against eternalism. We will take the eternalist 'culprit' to be Quine, and the 'forces of decency' that Lycan speaks of on p. 275 will be represented by Prior. (See Prior's remarks about the 'Quine/Smart thesis' on pp. 25-28 of Prior 1957.) The way to read the quotations which follow and their temporal versions is this. Assume that Lycan is committed to a basic notion of truth which is contingent — assume that for Lycan when we speak of something's being true, all we can *mean* is that it is actually true, where actual truth is contingent. Then take Prior's view that the basic notion of truth is present truth — that all we can *mean* when we say that something is true is that it is now true. Lycan will profess not to understand any notion of truth which is not actual truth, and Prior will profess not to understand any notion of truth which is not present truth. If you read the extracts in this way you will, we hope, see how the temporal version of what Lycan claims will be just as obvious to Prior as the original version is to Lycan. In the case of the temporal version we have to imagine Prior arguing against Quine in the same way as Lycan is arguing against Meinong. Here is what Lycan 1979 says on p. 283:

> A first try at quantifying over "things that do not exist" yields
>
> (A) $\exists x \sim \exists y(y = x)$

> ... And this formula is a *contradiction*. The crux is that, unlike the notion of a property or proposition, the notion of a *non-existent* thing or world is not merely queer or obscure or marginally intelligible, but is an apparent and overt self-inconsistency.

The temporal equivalent is

> A first try at quantifying over "things that no longer exist" yields
>
> (A) $\exists x \sim \exists y(y = x)$
>
> and this formula is a *contradiction*. The crux is that, unlike the notion of a property or proposition, the notion of a *no longer existing* thing or time is not merely queer or obscure or marginally intelligible, but is an apparent and overt self-inconsistency.[9]

Lycan, *loc cit*, puts a response into the mouth of the Meinongian. We need to consider how such a response would fare in the mouth of a Quinean:

> The obvious move in this direction is to disambiguate (A)'s quantifier. Quine might therefore distinguish two different operators, one continuing to indicate present existence, and the other indicating some so far mysterious *secundum quid*. The former would be given the usual model-theoretic semantics; the latter would remain to be explained.

Here's how Lycan fills out the response:

> Meinong and other apologists for possibilia point out that this "mysterious" second operator already has a perfectly intelligible and straightforward English counterpart, viz that which occurs in "there are things which don't exist", "There is a character in *Hamlet* who is smarter than anyone in our department".

The Quinean response might go like this:

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[9]We hope it does not need stressing that our treatment of Lycan in this fashion should in no way diminish our respect for his article. As with many of the authors we criticise, Lycan is a philosopher for whom we have the highest regard, and it is in fact because he is so clear about the modal case here we single him out to illustrate the temporal analogy.

> Quine, and other apologists for erstwhilia point out that this "mysterious" second operator already has a perfectly intelligible and straightforward English counterpart, viz that which occurs in "there are things which no longer exist", "There is a philosopher in ancient Greece who is smarter than anyone in our department".

Lycan is not impressed by the Meinongian version of this. He goes on (p. 283f):

> "Actual", and "existent" figure as predicates in Meinongian English, and it is simple just to take this usage over into our logical theory. Thus, "There are things that don't exist" would be translated into Meinongian not, as (A) but as
>
> $\exists_M x{\sim}\text{Actual } x$
>
> or as
>
> $\exists_M x{\sim}(\exists y{:}\text{Actual } y)(y = x)$
>
> neither of which is formally contradictory. And our original standard quantifier can now be introduced as a defined sign.
>
> $\exists_A x \ldots x \ldots =_{df} \exists_M x(\ldots x \ldots \wedge \text{Actual } x)$
>
> or
>
> $\exists_M(x : \text{Actual } x)\ldots x\ldots$
>
> (A), disambiguated accordingly, would be ruled satisfiable by whatever semantics the Meinongian intends to provide.
>
> This last unspecific referring phrase was tokened in a smug and deprecating tone. It presupposes a demand on our part that the Meinongian search for, find, and offer "a semantics for" the quantifier $\exists_M$, and we have some fairly specific constraints in mind as to what is to be counted as success in this. In particular, what I am implicitly demanding is a *model-theoretic* semantics, done entirely in terms of actual objects and their properties for *what else is there really*? I am allowing the Meinongian his funny operator only on the condition that he

explain it to me in non-Meinongian terms.

(We have used $\exists_M x$ for Lycan's $(\exists x)_M$, and so on.) The temporal version would look like this:

> "Present", and "currently existing" figure as predicates in Quinean English, and it is simple just to take this usage over into our logical theory. Thus, "There are things that no longer exist" would be translated into Quinean, not as (A) but as
>
> $\exists_Q x \sim$Present $x$
>
> or as
>
> $\exists_Q x \sim (\exists y$:Present $y)(y = x)$
>
> neither of which is formally contradictory. And our original standard quantifier can now be introduced as a defined sign.
>
> $\exists_P x$ ...$x$... $=_{df} \exists_Q x$(...$x$... $\wedge$ Present $x$)
>
> or
>
> $\exists_Q(x$ : Present $x)$...$x$...
>
> (A), disambiguated accordingly, would be ruled satisfiable by whatever semantics the Quinean intends to provide.
>
> This last unspecific referring phrase was tokened in a smug and deprecating tone. It presupposes a demand on our part that the Quinean search for, find, and offer "a semantics for" the quantifier $\exists_Q$, and we have some fairly specific constraints in mind as to what is to be counted as success in this. In particular, what I am implicitly demanding is a *model-theoretic* semantics, done entirely in terms of present objects and their properties for *what else is there really*? I am allowing the Quinean his funny operator only on the condition that he explain it to me in non-Quinean terms.

p. 285

> It should be noticed that the impasse is slightly lopsided: Though each of the participants has so far run up just one unexplained primitive, the Meinongian is in fact stuck with a second, viz, the predicate "Actual". I see no way of explicating "Actual" in terms of $\exists_M$ plus notions accepted by both sides (that is not without the use of further new primitives). It is true that, so far as has been shown, attempts to explicate the Meinongian "there is" in terms of the actualistic notions already in play in reality-oriented semantical metalanguages may just fail, and that we *may* therefore be forced to introduce a new primitive into the metalanguage amounting to a Meinongian quantifier.

The temporal version in fact suggests an important lesson. We get the following:

> It should be noticed that the impasse is slightly lopsided: Though each of the participants has so far run up just one unexplained primitive, the Quinean is in fact stuck with a second, viz, the predicate "present". I see no way of explicating "present" in terms of $\exists_Q$ plus notions accepted by both sides (that is not without the use of further new primitives). It is true that, so far as has been shown, attempts to explicate the Quinean "there is" in terms of the presentist notions already in play in reality-oriented semantical metalanguages may just fail, and that we *may* therefore be forced to introduce a new primitive into the metalanguage amounting to a Quinean quantifier.

Ultimately what Lycan seems to be suggesting is a debate in which neither side can win. Why? Well the presentist like Prior claims not to understand any notion of truth which doesn't mean present truth, in the same way that Lycan claims not to understand any notion of truth which doesn't mean actual truth, and if so there is no hope of explicating the situation in terms acceptable to both sides. And if that is right then, in the light of the formal results of Part II, we have a metaphysical dispute which, if there is such a thing as the right or wrong of it, is too deep to be addressed by logical argument. As we have said our role is not to make any judgement about ultimate metaphysical truth. Our purpose is solely to assess metaphysical arguments, and when argument stops, so do we.

# 12

# *Utterances*

In preceding chapters we have taken it as obvious that the sentences of natural language — in our case English — are tensed. On the indexical account presented in chapter 1,

(1) It rained yesterday at Waitarere

is true at a time because there was rain on the previous day. I.e. (1) is true at $t_1$ (in a world $w$) because

(2) It is raining at Waitarere

is true at a time $t_2$ (in $w$) on the day preceding $t_1$. This account takes as evident the relation '$\alpha$ is true at $t$ in $w$'. That is to say we have taken it that (2) is something which can, in this or that possible world, be true at one time and false at another. We call this account 'indexical' because worlds and times are what on p. 18 are called, *indices of truth*, meaning that truth is truth at a time in a world.[1]

Some philosophers have wanted to make a difference between the sense or proposition expressed by a *sentence*, and the sense or proposition expressed by an *utterance*. Thus Mellor 1981, p. 36, argues that while tensed *sentences* cannot always be translated without loss of content into untensed sentences yet the content of *utterances*, or 'tokens' as he calls them, is untensed. This latter account is often referred to as the 'token-reflexive' theory of tense, and, as we shall explained, it is easily confused with the indexical theory. The principal

[1]We have already noted on p. 8 that some authors use 'indexical' in a different sense from ours. For instance Smith 1993, p. 110, speaks of 'A-indexicals', like 'now', 'currently', and so on. The use of such words is to 'protect' an index so that it still applies even when a modifier has changed the evaluation index. The 'now' in (1) on p. 84 above ensures that the adjective ***miserable*** is evaluated at the original index even though it is inside the scope of a past tense operator. Smith distinguishes such words from tense operators which he does not regard as indexical. Also see Percival 1990 for still other ways of using 'indexical'.

reason for distinguishing between the indexical and the token-reflexive theories is to stress that the indexical theory permits utterances to have tensed content.[2] The relation '$\alpha$ is true at $t$ in $w$' makes no reference to any *utterance* of $\alpha$. Still less does it suggest that an utterance expresses a temporally constant proposition. In fact we have already noted that 'temporal propositions', sets of world-time pairs, are necessary to handle the two executives from p. 39, and for sentences like Prior's celebrated:

(3) Thank goodness that's over

discussed on p. 45. Furthermore, (3) can be turned into a counterexample to the token-reflexive theory in the following way. Imagine two utterances:

$u_1$: That's over
$u_2$: Thank goodness for that.[3]

Presumably the 'that' in $u_2$ refers to the *content* of $u_1$, but the content of $u_1$ is tensed, since the utterer of $u_2$ is not thanking goodness that the event in question is finished before a certain time, or that its occurrence precedes $u_1$. Giving (3) tensed content does not of course provide any support for a privileged present, since the process can be mirrored by *de* se attitudes of the kind discussed by Lewis 1979a. If Heimson says that he is Hume, his utterance of the sentence 'I am Hume' need not express the (logically contradictory) proposition that

[2]For instance Bigelow 1991, p. 2, discusses what he calls the 'indexical theory' even though the theory he discusses is the token-reflexive theory. Le Poidevin 2003, p. 307, presents what he calls "two versions of the tenseless theory of time" — the token-reflexive theory and the date theory. He does not mention the indexical theory so it is not clear which he would regard as better fitting with a tensed or a tenseless theory of time. On p. 229 of Smart 2008, Smart refers to the indexical theory as Donald Davidson's, citing Davidson 1967. Although they explicitly do not consider tense, the indexical theory therefore seems at least compatible in this respect with the 'semantic minimalism' of Cappelen and Lepore 2005. This compatibility is noted by MacFarlane 2007, p. 247, bearing in mind that MacFarlane means by 'non-indexical' what we mean by 'indexical'.

[3]Smith 1993 and Craig 2000a; 2000b mount strong arguments against the token-reflexive theory but none of these touch the indexical theory. Smith 1993, in Chapters 2 and 3 (pp. 27-93), provides a comprehensive discussion and criticism of the many attempts to provide untensed contents to utterances of tensed sentences. Smith distinguishes on p. 94 between A-propositions and B-propositions. There is a sense in which, semantically at least, you can regard a set of world-time pairs as being (or representing if you prefer) an A-proposition; and so in that sense even a B-theorist could agree with Smith's claims in these chapters of his book. On this interpretation the indexical theory is compatible with the falsity of Smith's (4) on p. 95. Smith 1993, p. 44f cites Kaplan 1979 who identifies 'object of thought with content — cognitive significance with character'. If Kaplan means that an utterance can express a character then this is a variant of the indexical theory rather than the token-reflexive theory.

Heimson is Hume. What Heimson says can be just the meaning of his sentence: the set of $\langle w,t,p \rangle$ triples in which $p$ is Hume, and nothing follows from that which gives Heimson or Hume any particular ontological privilege.

We have been assuming that a language is an abstract structure whose syntax is generated by a set of formal rules, and that the meanings of sentences are recursively derived from the meanings assigned to the simple symbols. This assumption has the immediate consequence that sentences take semantic priority over utterances. Given that most sentences generated by a grammar are never uttered, this seems the only plausible approach to semantics. Evans 1985[4] mounts an attack on the semantic parallel between modal and tense logic, which makes crucial use of the content of utterances. We shall look in some detail at what Evans says, and we shall then look at an argument in Mellor's later work on what he thinks of as an alternative version of McTaggart's argument against the reality of tense. Evans (p. 344) calls the relation 'true at $t$', $\text{true}_t$, and rightly points out (p. 345) that it is the central notion in understanding tense logic. However, he goes on to say that he does not believe "that a theory without implications for the correctness or incorrectness of utterances can lay claim to the title of a semantic theory". There is a sense of course in which he is quite right — it is the linguistic behaviour of a population which determines what the sentences of the language of that population mean, but the connection is we believe a holistic one, whereby the truth conditions of a particular utterance depend on the truth conditions of the sentence it expresses, according to the syntactic and semantic framework which best fits the linguistic behaviour of the population using that language. Evans takes his claim to be that we must be able somehow to define the truth at $t$ of a particular sentence in terms of the correctness of utterances of that very sentence — or at least that the purpose of an adequate semantic theory of tense is to generate equivalences which connect the correctness of an utterance $u$ with the truth of $\alpha$ where $\alpha$ is a sentence of which $u$ is an utterance.[5] Part of Evans's concern is whether the correctness of $u$ is or is not relative to a time $t$. Evans does not mention relativity to a world here, and the fact that he does not is instructive. Evans advocates what he calls a Fregean theory of meaning, and (p. 350) cites Frege approvingly when Frege says

[4]Evans's article was actually written in 1979, presumably before the importance of the work on indexicality in Lewis 1979a and Perry 1979 was appreciated. Though, as we observed on pp. 46-46 Perry himself appears not to have recognised how it is that times and persons can figure in the content of beliefs.

[5]We are of course assuming a distinction between the proposition expressed by the utterance and the speech act performed by the utterance. This raises issues in the philosophy of language which we do not discuss in this book.

> A thought is not true at one time and false at another, but it is either true or false, *tertium non datur*. The false appearance that a thought can be true at one time and false at another arises from an incomplete expression. A complete proposition or expression of a thought must also contain the time datum. (Frege 1956)[6]

This means that a thought or sense is not time-relative. But what about worlds? Can we suppose that a thought (the same thought) cannot be true in one world but false in another? If we are wondering whether it will rain tomorrow it is surely the case that the content of our thought is not affected by it will or will not rain.[7] If so Evans has already built in the difference between times and worlds. He appears to take this as 'obvious' (p. 351) and uses its obviousness to protest against those who are "blinded by the structural parallels between time and modality." (p. 352) Evans's argument against the world-time parallel amounts to this: the truth at *t* of a sentence α is to be understood in terms of the content of an utterance *u* of α at *t*. But the content of *u* at *t* is a Fregean thought which, although its truth may vary from world to world, does not, in the same world, vary in truth value from time to time. So the content of α cannot be tensed, i.e.

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[6]Davies 1981, p. 203, follows Evans and Frege in this regard, and what we say about Evans is also applicable to Davies's presentation of Evans's argument, and to the exchange between Forbes 1983 and Davies 1983. In particular Davies holds that the basic semantic truth predicate, which he writes as TRUE, applies to a content which is not time-dependent. The argument then turns on the fact that utterances of *tensed* sentences can express different (untensed) propositions depending on when they occur, while utterances of modal propositions (barring perhaps some which contain words like ***actual*** in some of the senses of such words) express the same proposition in whichever world they occur. Metaphysically these authors would offer an indexical account of tense but, presumably, some other account of modality. While these assumptions may well allow them to discern "a conceptual difference between the modal and the temporal cases" (Davies 1981, p. 205) the disanalogy can only be extracted if it is put in in the first place. Davies may in fact agree since on p. 130 of Davies 1983 he describes the main claim of Forbes 1983 as being that "a deep disanalogy cannot be demonstrated without appeal to metaphysical differences between times and possible worlds." Davies continues, "I believe Evans would have agreed." Percival 2002, p. 108f, discusses Evans's argument in the context of an extended discussion of Mellor's work. See also p. 113 of Salmon 2003.

[7]Evans and Davies consider the supposition that just as (on their view) the time of an utterance fixes *what was said*, i.e. fixes the proposition — Fregean sense or set of worlds — expressed by the utterance, so one might say that the time *and world* of the utterance fixes what was said. But then, as they point out, to know what was said one has to know what world the utterance is in, and that means being omniscient, which is absurd. Further, what is said then becomes just a truth value. (And indeed if one is omniscient and knows what is said then one knows its truth value.) This much may be said of course. Since in hearing an utterance we frequently do know what time it is, we can use this knowledge together with the (perhaps tensed) content of what is said to work out a Fregean sense. But that need not commit us to the claim that this sense is what the utterance says, and there may well be cases, say in a perjury trial, where the facts are known, but where it is the time of utterance which is in question.

α cannot have the same tensed content at all times in the way it can have the same content in different worlds. Neither of Evans's assumptions seems self-evident. It seems easy to deny that the content of tensed sentences must be explained in terms of the content of their utterances; and it seems easy to deny that the content of an utterance requires its truth value to be constant over time in a way in which it need not be constant over worlds. These disanalogies between worlds and times can only be supported if the assumptions put them there in the first place.

Even so, there is still a problem about the content of utterances, and it emerges in the following way: On Monday we perform an utterance of (2). Call this utterance *u*. Since it is, let us say, raining on Monday at Waitarere, *u* is a correct utterance. Is *u* still correct on Tuesday when it is sunny? The answer seems to be 'yes'. This means that if *u* is an utterance of (2) in a world *w* there is a mismatch between

(4) *u* is correct at *t* in *w*

and

(5) α is true at *t* in *w*.

For if *u* takes place on Monday and *t* is Tuesday then (4) is true but (5) is not, given that it is raining on Monday but not on Tuesday. So far there need be no problem. We simply say that *u* is correct at *any* time *t* in *w* iff, when $t_u$ is the time at which *u* occurs in *w* and *u* is an utterance in *w* of α, α is true in *w* at $t_u$ (whether or not α is also true at *t*.)[8] But if we say this in the case of times do we still say it in the case of worlds? Consider an utterance *u* today (Tuesday) of (2) in a world *w* in which it is raining. That utterance is correct in *w*. But is it correct in the real world where it is not raining on Tuesday? Surely there is no sense in which it is (actually) correct given that the sun is shining. Yet we say that yesterdays's utterance is correct today. So if we take the parallel seriously why isn't the possible utterance equally correct in the actual world?

There are two cases to consider. One case is when *u* occurs not only in *w*,

[8]Evans considers Prior's endorsement (Prior 1967, p. 15f) of Geach's view that 'complete' propositions can be tensed, in the sense of being able to be true at one time yet false at another. Evans finds this 'not coherent'. (Geach 1949.) He points out that the same historical act should not be held to be the assertion of a truth at one time but not at another. This seems too swift. On the Geach/Prior view, to say that *u* is a correct assertion could be simply to say that the proposition which is the meaning of the sentence α of which *u* is an utterance, is true at the time at which *u* occurs. Of course one could not then explain truth at a time in terms of a correct utterance, but we feel no need to do so. The intuition that utterances are to be assessed for truth or falsity only at the time of their occurrence is not universally shared, at least in the case of statements about the future. See for instance Macfarlane 2003 p. 328. We discuss this briefly on pp. 159-160 below.

but in the actual world as well. If *u* is an utterance of (2) which is actual as well as possible it seems impossible to describe *u* as correct in the actual world on Tuesday. Does the same apply in the temporal case? Well, we have spoken of the time of an utterance as if it were a single instant. An utterance is, presumably, an event, and it is at least *prima facie* plausible that an event can occur in more than one world as in 'If the war had ended earlier much suffering would have been avoided.' But an utterance is also extended in time, so take an utterance *u* of

(6) The front of the train has passed the signal.

*u* might take time, say from $t_1$ to $t_2$, and it might happen that (6) is false at $t_1$ but true at $t_2$. So that, at least for times at which *u* is taking place, correctness does seem a relative notion, just as it is for worlds. The argument in the case of (2) of course considered a time at which the utterance was no longer occurring, and the idea seems something like this. Barring utterances like (6), an utterance does not change its truth value with the passage of time, and we can simply categorise it as correct or incorrect. If we are proceeding analogously in the modal case, we would say that if *u* is an utterance whose truth can change from world to world then, as with (6), we relativise, but if *u* is an utterance whose truth is constant from world to world we don't. Is this troublesome? We are considering an utterance *u* which has a fixed truth value, say true, in every world in which it occurs. What are we to say about the truth of *u* in a world in which it does not occur? Well, presumably we say that it is true, on the ground that *u* cannot exist without being true. That is to say it is *logically impossible* for *u* to occur without being correct. The reason we may be reluctant to say this might be that we think of an utterance *u* of (2) and ask what would happen if *u* were to occur in a world in which it is not raining. But that is to take *u* as something which *does* occur in at least one non-rain world, and so is a case like (6). One might of course counter the problem of (6) by following Mellor 1981, p. 37, and stipulating an instant during the utterance as the instant of utterance. If one takes this line then the parallel is maintained, since one can equally stipulate a particular world as the world of utterance, and if the same event occurs in different worlds, it would count as a different utterance in each world. Each utterance would therefore have a truth value which is neither time-dependent nor world-dependent. So there is nothing here which need impugn the world-time parallel.

In comparing the truth of an utterance *u* of (2) on Tuesday in a rain world with *u*'s occurrence (on Tuesday) in a non-rain world (say the actual world) we are talking about the *same utterance* in a different world. There is no argument for the disanalogy if we are comparing an utterance $u_1$ of (2) in a rain world with

an utterance $u_2$ of the same sentence (2) at the same time but in a different (non-rain) world. For that would be like comparing an utterance $u_1$ of (2) in the actual world on Monday with an utterance $u_2$ of (2) in the same world on Tuesday, and there is agreement that $u_1$ is evaluated according to whether or not (2) is true on Monday, while $u_2$ is evaluated according to whether or not (2) is true on Tuesday. It is also important, in appreciating what has just been said, not to be misled by introducing David Lewis's counterpart theory. Lewis (1968 and 1986a) holds that the very same thing cannot exist in more than one world or at more than one time. Look carefully at (6). For Lewis an utterance $u$ of (6) is in fact an infinite family of utterances, some of which, those occurring at instants before the train has passed the signal, are false, and others of which, those occurring at instants after the train has passed the signal, are true. Take one of these utterances, say $u_2$, occurring at $t_2$ when (6) is true. Assume $t_1$ is a time at which the train has not passed the signal, and ask

(7) Was $u_2$ true at $t_1$?

In Lewisian terms, the answer to (7) depends on whether $u_2$ has a counterpart at $t_1$, $u_1$ say, such that $u_1$ is true. Now $u_1$ occurred before the train had passed the signal, so that if $u_2$ has a counterpart at $t_1$ and given that (6) is false at $t_1$ this means that the answer to (7) is 'no', even though $u_2$ is true when it occurs. So Lewis's counterpart theory will not help us if we wish to argue that (6) is not really a case of a temporally variable utterance.

We have rejected Evans's assumption that truth at $t$ must be analysed in terms of utterances, and perhaps it is necessary to say something in justification of this. The reason lies in the truth of sentences like

(8) Once there were no utterances.[9]

(8) is true, but you can't analyse its truth by saying that there was once a time at which an utterance of

[9]See e.g., Smith 1993, pp. 72-78. Mellor 1998, pp. 32-34 agrees that sentences like (8) require the index to be a time rather than an utterance. Dyke 2002b, p. 340, speaks of 'unuttered tokens' as a way of solving problems like this. Montague 1974, p. 230, refers to "the useful idea of construing a token as a pair consisting of a type and a point of reference". With some such definition as this one can certainly admit a 'token' at which no utterance of the sentence in question occurs. This does not affect the points made in the text of course, since it merely reflects a way (slightly odd perhaps) of using the words 'utterance' or 'token'. What is crucial is that the semantics of a sentence should not be at the mercy of which utterances of that sentence actually occur.

(9) There are no utterances

expressed (or even would have expressed) a truth. The indexical theory allows (9) to be true in *w* at a time *t* even though no true utterance of (9) can occur in *w* at *t*. We have already pointed out (p. 29) that an index of truth need not literally be a context of utterance. There can of course be true utterances of (8). That is not the problem. Nor is there a problem about the *fact* reported by (9). The problem is that a recursive specification of the truth conditions of (8) will treat it as having the form

(10) *P*(There are no utterances)

in which (9) occurs as an embedded sentence; and the semantics of the operator *P* must be able to determine the truth conditions of *P*α, for any sentence α, on the basis of the truth conditions of α. This can be done when α is a sentence with a truth value at each world and time, but it cannot be done if the possibility of a value for α is contingent on the occurrence of an utterance of α.

In Mellor 1998, Hugh Mellor realises that McTaggart's argument (McTaggart 1927) against the tensed view of time has not carried universal conviction, and on pp. 78-81 uses utterances in an alternative version.[10] We want to look at what the modal parallel of this version would be. The reason we do so is this. If Mellor is right that there is a contradictory version of the tensed view of time, then, given the structural parallel between modal and temporal discourse, there had better be an analogous contradictory version of modality. And if there is we had better be aware of it. On p. 78 Mellor writes:

> All parties also agree however that, for any event *e*, '*e* is past' is true at all *B*-times later than *e* and false at all other times. So if the *A*-fact that *e* is past is to be the truthmaker for '*e* is past', that is when this *A*-fact must exist. And when it does so, it must make true not only this proposition but also all its tokens, thus requiring all tokens of '*e* is past' to have the same truth value at any one time, namely to be true if the time is after *e* and otherwise false. For if at any time different tokens of '*e* is past' differed in truth value, they would need different truthmakers, and '*e* is

[10] An earlier version of the argument occurs on pp. 98-102 of Mellor 1981. The parallel between the argument in Mellor 1981 and its modal counterpart has been commented on in Percival 1989 (pp. 193-195). The points made in the present chapter are elaborated in more detail in Cresswell and Rini 2010, with a response in Mellor 2011 and a comment on the response in Cresswell and Rini 2011.

past' and its tokens would not be made true by a single fact.

Mellor's example is Jim, who races at 4.30. Given that he does, an utterance of the sentence

(11) Jim is racing

made at 4.30, can be held to be made true by the fact that Jim is racing at 4.30. The fact that Jim is racing at 4.30 is a tenseless fact. Given that Jim *is* racing then, it always was and always will be that he races then. To avoid problems of the kind raised by (6) we may need, as mentioned on p. 147, to nominate a single time as *the* time of utterance. The modal parallel is clear. Take the sentence

(12) Jim races at 4.30 on 2 June 1998

and consider an utterance *u* of (12) made (at any time) in a world *w*. (If *u* occurs in other worlds as well then, by analogy with a Mellor-style treatment of (6), we nominate *w* as the world of utterance, and treat *u* in another world as another utterance.) What makes *u* true, on this account, would be the fact that (12) is true in *w* — and this fact is not only tenseless, but is also non-contingent, since if *u could* be true in *w* then it *must* be true in *w*, for the reasons pointed out in the discussion of (35) on p. 25. Thus, in the sense in which tenseless facts provide the truthmakers for utterances of tensed sentences, so non-contingent facts can provide the truthmakers for utterances of contingent sentences.

What we have just said only shows that, in the sense in which tenseless truthmakers are available for tensed sentences, so non-contingent truthmakers are available for contingent sentences. But Mellor's argument is intended to establish that *tensed* truthmakers are *not* available for tensed sentences. The argument is that if (11) had a single (tensed) truthmaker then *all* utterances of (11), *whenever they occurred*, would be true at 4.30 — and whenever else Jim was racing — and false at other times. So that someone who uttered (11) at 3.30 would have produced an utterance which is true at 4.30, even though Jim was *not* racing at 3.30, when the utterance occurred. Given that utterances are evaluated for truth at the time of their occurrence, this is absurd, and so is its modal analogue. If (12) has a single truthmaker, in the sense that Mellor requires, this would mean that, given that it is actually true, we would have to say that an utterance of (12) would still have been a true utterance *even if* Jim had not raced at 4.30. So that, in whatever sense Mellor has shown that tensed facts are a myth, his argument seems to show that contingent facts are a myth.

Although Mellor's argument is put in terms of utterances, it is in fact an argument about the nature of time; and it is not at all clear why the status of utterances should be relevant. Consider an utterance *u* of a sentence α, as it might be (11), which occurs at a time $t_u$ in a world $w_u$. As we have noted above, you can stipulate that *u* is to count as an utterance which is (absolutely) true or correct, i.e. true at any time in any world, iff α is true at $t_u$ in $w_u$. The question is then what makes α true at a time *t* in a world *w*, and that turns on the metaphysics of facts. In the case of (11) you can either say that there is a single 'fact' of Jim's racing, which exists at some times and in some worlds, and makes α true at *t* in *w* if it exists at *t* in *w*, or you can say that, for each *w* and *t*, there is the separate (untensed and non-contingent) fact that Jim is racing at *t* in *w*; and in the case of (12) you can either say that there is a single contingent fact of Jim's racing at 4.30, which exists in some worlds and not in others, or that, for each world in which Jim races at 4.30, there is the non-contingent fact that Jim races at 4.30 in that world. Both ways of talking are semantically equivalent. One could of course equally say that facts are tensed but world-bound, or are untensed but contingent, and these options too are semantically equivalent. More importantly, nothing about the truth or falsity of utterances gives any reason for treating worlds and times differently. As before the lesson for us is that you can only deduce a metaphysical difference between times and worlds if your *metaphysics* puts it in to begin with.[11] There is no argument here from *semantics* that they are not exactly parallel.

[11]Percival 2002, p. 98, characterises Mellor's 'tenseless' metaphysics as embodying the claim that facts don't change over time. If that is right, and if Mellor holds that facts *do* change from world to world, then once more we see how the ontological difference between times and worlds is there because a metaphysician has introduced it.

# 13

# *Relativity*

In this chapter we shall examine the claim that presentism is incompatible with the special theory of relativity. We shall not so much be concerned with just how good it is as an argument against presentism, but merely to consider how it might impact on the world-time parallel. In discussions of this objection writers on both sides spend a lot of time setting out what they take the special theory of relativity to be.[1] While this is important, it is in a sense subsidiary to our claim, which is not to take a stand on whether presentism is true or not, or even whether or not it is compatible with the special theory of relativity, but solely to investigate what a modal parallel would look like. We are not scientists, and are not making any claims about the physical structure of space and time. In fact we had better not be. The issue for semantics is not whether the special theory of relativity is correct or not, but rather what it is about the special theory of relativity which might contradict presentism, and how it might do so, and what an analogous modal example would look like. Some in the dispute seem to think it might be a contingent matter whether or not presentism is true (Crisp 2003, p. 215, Rea 2003, p. 248). At first sight this might seem an obvious disanalogy with actualism, since it hardly seems a contingent matter whether actualism is true. However the parallel is not whether presentism, if true, *could* be false — the correct parallel is whether presentism, if true, *was once* false, or *will be* false, and we shall take it that no one in the debate supposes that *that* is possible. But even if presentism is contingent, the philosophical task is to isolate what kind of empirical scientific facts could be relevant, and why these facts are so relevant. It is then up to the scientists to establish whether or not they obtain. Semantics cannot suppose the truth or falsity of any scientific theory, since semantics must allow in advance the articulation of any theory which can be subjected to empirical test.

Whatever the details of the special theory of relativity the important consequence for presentism is that whether or not two events occur

[1]See for instance the discussions in Crisp 2003, pp. 232-235, and Rea 2003 pp. 269-276.

simultaneously can be relative to a frame of reference — sometimes called an 'observer'. On p. 29 we introduced what we called a 'person' index, $p$. The sentences used to motivate the introduction of the person co-ordinate in Chapter 2 used either pronouns like 'I', 'you' and the like — or used phrases involving proper names like 'Palmerston' or descriptive phrases which could also pick out locations. By contrast sentences about what happened in the past or the future do not seem to involve this kind of explicit marking of the dependence. But dependence on a person need not always be syntactically marked. There is a theory of morality according to which

(1) That was a kind action

means something like

(2) I approve of that action

so that in (1) the reference to the source of the approval, which is made explicit in (2), is not syntactically indicated.

In our discussion of (8) on p. 29 we pointed out that from a formal point of view the nature of $p$ doesn't really matter, and so there is no block to taking it as a frame of reference, although, for convenience, we shall often continue to speak of it as the person index. One thing such an index does is completely uncontroversial. As we showed on p. 28, using the person index to evaluate the word 'I' can easily explain how in a given world at a given time

(3) I am hungry

can be true at some $\langle w,t,p\rangle$ and false at $\langle w,t,q\rangle$ where $p \neq q$. In fact presentism is quite compatible with much observer relativity. None of this causes presentism trouble. It is the claim that $<$ is observer relative that seems to be the problem. One way to model the situation is to assume that, for a given observer $p$ and world $w$, there is temporal relation $<_{\langle w,p\rangle}$, where $<_{\langle w,p\rangle}$ could be a linear ordering. One would then say that $P\alpha$ is true at $\langle w,t,p\rangle$ iff $\alpha$ is true at $\langle w,t',p\rangle$ for some $t'$ such that $t' <_{\langle w,p\rangle} t$. A first payoff is that the temporal ordering could vary from world to world, but more important for our purposes is that it could also vary from person (observer, or framework) to person. For obviously you could have $p \neq q$ and $t_1 <_{\langle w,p\rangle} t_2$ but not $t_1 <_{\langle w,q\rangle} t_2$. We have already noticed (p. 119) that one way to make sense of time travel is to assume that there can be a timeline which is relative to a person, and differs from the 'ordinary' time line. But even without

science fiction this might be what our actual world demands. Of course this would mean that communication between $p$ and $q$ might suffer from a concealed relativity. But, if we have understood things aright, science would come to the rescue and say that if $w$ is like the actual world, and $p$ and $q$ are people communicating in ordinary circumstances, then the difference between $<_{\langle w,p\rangle}$ and $<_{\langle w,q\rangle}$ is too small to matter.

Whether this accurately mirrors the situation of relativity theory is not our concern. What it *does* do is mirror the fact that two events might occur simultaneously from the point of view of one observer, but not from the point of view of another. In describing matters in this fashion we have spoken of the ordering of times themselves as depending on the observer. But this situation could equally be stated, and usually is stated, in terms of events,[2] where two events might be simultaneous with respect to one observer but not with respect to another. Stated in this way $e_1 < e_2$ is intended to mean that $e_1$ occurs before $e_2$. We have not presupposed an ontology of events[3] but it is not hard to see that you could equally have $p \neq q$ and $e_1 <_{\langle w,p\rangle} e_2$ but not $e_1 <_{\langle w,q\rangle} e_2$.

Truth at $\langle w,t,p\rangle$ is actual present truth relative to a particular observer, and there seems no reason in logic to take the relativity as an argument against presentism rather than an argument against actualism. What if we put it in the following way? If, in world $w$, $e$ occurs at $t$ according to $p$ but not according to $q$ then assume that, according to $q$, $e$ occurs at, say, some other time $t'$. Suppose that the argument against presentism goes like this: In $w$, $e$ occurs at $t$ according to $p$ but at $t'$ according to $q$, where $t \neq t'$ and $p \neq q$. If this is the argument then imagine the following argument against actualism: At $t$, $e$ occurs in $w$ though not in $w'$ according to $p$, but in $w'$ though not in $w$ according to $q$, where $w \neq w'$ and $p \neq q$. One might claim that the contents of a time can change depending on an observer — what this means is that whether $e$ occurs at $t$ depends on $p$. Well, actually it's that whether $e$ occurs at $t$ in $w$ depends on $p$. But if that is the way it is with times you could just as easily say that the content of the *world* changes depending on $p$ — for in $w$, $e_1$ may occur before $e_2$ according to $p$ but after $e_2$ according to $q$. Formally all that can be said is that for propositions like this you don't get definite truth until you supply a world *and* a time *and* an observer.

Relativity is a threat to the temporal ordering of times because the relation $<$ used in interpreting $P$ and $F$ is not absolute but is observer-dependent, and perhaps world-dependent as well. An analogous modal claim would be that the

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[2]As for instance in Rea 2003, pp. 269-276

[3]See Prior 1967 p. 18 for some sceptical remarks about the role of events in tense logic.

similarity relation between worlds, $<_{\langle w,t\rangle}$ used in the interpretation of $\Box\!\!\rightarrow$ might also turn out to be observer (or framework) dependent. $w_2 <_{\langle w_1,t\rangle} w_3$ is intended to mean that, at $t$, $w_2$ is closer to $w_1$ than $w_3$ is. Suppose it were to be claimed that the relation needed is not just $<_{\langle w,t\rangle}$, but a relation $<_{\langle w,t,p\rangle}$ which depends on an observer in addition to $w$ and $t$, and that this is required, not just in the trivial sense that it is an extra parameter, carried along like $t$; but in the stronger sense that even when $t$ is kept constant, which worlds are accessible, or which worlds are close to ours can depend on $p$. Whether there is *need* for such a relation depends on what happens to the truth of counterfactuals in a relativistic setting, and how an actualist might respond is not a matter about which we have anything to say. But if this is the parallel it seems mysterious to see how the fact that the counterfactual nearness relation might be observer dependent need have any implications for actualism. Indeed, whether or not the truth of counterfactuals is ever observer dependent, there are certainly some relations involving worlds which are. In the semantics for *K* given on p. 66 we noted that it was the knowledge of some fixed observer. But we also pointed out that in the indexical semantics of Chapter 3 this was a relation between $\langle w,t,p\rangle$ triples, a relation which depended on the person co-ordinate, and has to do so to account for *de se* knowledge.

One could perhaps imagine an even more radical difference between how things are with $p$ and how things are with $q$ by supposing that there is an event $e$ which occurs at a certain time for $p$ but does not occur *at all* for $q$. Using Lewis's terminology from 1986a one might say that $p$ is spatio-temporally connected with $e$ but $q$ is not, and therefore that while $e$ is in $p$'s world $e$ is not in $q$'s world. This means that if an actualist describes the actual as everything spatio-temporally connected with some actual event it seems that $e$ will have to be actual for $p$ but not actual for $q$, and so we would appear to have a difficulty for actualism parallel to that raised against presentism. We don't wish to make too much of these last remarks, or indeed make too much of our whole discussion of relativity, except to suggest that the debate about relativity and presentism might be closer to a debate involving actualism than theorists have supposed.

Notice that if something like this is the way things are it will only make sense to attribute absolute truth to the utterance of a sentence if it is specified not only at what time and in what world it occurs, but also with respect to which 'observer'. Of course if you take utterances to be physical things then an utterance can supply this information, and indeed for a sentence like (5) on p. 28 we need such information. We might get some help here from an unlikely quarter. Hugh Mellor is no friend of presentism, but, as we saw in Chapter 12, he does want to be able to assign an absolute truth value to an utterance. However,

he also realises that there are sentences like our (9) on p. 149, for which we have to speak of truth at a time, and here too you need a notion of absolute truth. Mellor is aware of the demands of relativity theory in such a case, and on p. 34 of Mellor 1998 we read:

> Nor does it matter if we agree with Einstein that it takes an arbitrary choice of reference frame to fix which events throughout space are located at a local time *t* (and which events throughout time are located at a present place *s*). For as we saw [earlier] it remains a fact in any one frame which events are located at *t*, and so whether 'There are no tokens now' is true at *t* in that frame; and similarly for 'There are no tokens here' at *s*. So we can again ignore special relativity by letting any one reference frame stand in for all the rest.

So at least for some purposes it seems acceptable to get absolute temporal truth by nominating a privileged frame of reference. Should this kind of solution worry a presentist? One worry might be that it makes the presentist's primitive notion of truth a notion of *my* present truth. This certainly worried Prior who speaks on p. 137f of Prior 2003 about a 'points of view' logic, and Prior worries about its 'solipsistic ring', which he finds 'as hard to take as Platonism about instants'. Is this solipsism? Well, one might say 'I am the only person present at the *me-here-now*' in the same way as one might say 'now is the only present time'. To argue that it is not genuine solipsism we can consider an extreme version of presentism which we might call 'solipsistic presentism'. Solipsistic presentism would claim that not only *is* there no philosopher drinking hemlock, there never *was* one. Ordinary presentism, by contrast, says that although there *is* no such philosopher yet there *was* one. Now solipsism is surely the view that there is no one at all apart from me. But the analogue to presentism would simply be the claim that although only *I* exist at *me*, you exist at *you*, and everyone else does at *them*. Some philosophers of time, like William Craig for instance, and perhaps Lynne Baker in Baker 2007, might actually find *support* from such a construal of relativity, since in a sense it establishes the ultimate primacy of the 'me here now'.[4]

Suppose you think that the indispensability of contingent language is an

[4]Craig 2000a on pp. 117-122 links the reality of the present with the reality of a transcendent self. Craig's own attitude to relativity theory seems, however, a little different. Craig 2001 mounts a case that the best framework for special relativity may in fact be one in which time and space have an absolute structure. Whether or not this is scientifically correct does not affect anything we say in this book.

argument for actualism. If so you may be tempted to think that the indispensability of tensed language is an argument for presentism. Actualists like Adams and Plantinga claim to avoid having to take other worlds as equally as real as the actual by assuming primitive contingent truth together with modal operators. Prior claims to avoid taking other times as equally as real as the present by assuming primitive present truth and using tense operators. In order to appreciate what is going on let us revisit the reasons for indexicality considered in Chapter 2. J.M.Barrie's play 'The Admirable Crichton', is about an aristocratic family marooned on a desert island in the Pacific. At the beginning of Act III we read "It is a fine summer evening." Consider a performance on a wet afternoon, and ask whether

(4) It is raining

is true. Well it's false in the play, but it's true outside the theatre. So it seems that (4) and

(5) It is not raining

are both true. How do we resolve the apparent contradiction between (4) and (5)? One way is to contrast the actual world with other possible worlds — worlds in which the things portrayed on the stage are actually taking place. In the actual world (4) is true, while in the worlds of the play (5) is true. The contradiction is resolved by relativising to two different indices. The alternative — in the spirit of the actualism of Adams and Plantinga — is to take actual truth as primitive and make use of (equally primitive) modal operators. You can think of 'in the play' as a kind of modal operator, like $L$ or $M$, and then you can have both (4) and

(6) [In the play] it is not raining

true.[5] So much for one kind of contradiction. But suppose we are concerned only with the actual world. (4) might be true on Tuesday, while (5) is true on Wednesday. You can resolve this kind of contradiction by relativising to truth at a time, or you can follow Prior by taking tensed truth as primitive and using tense

[5]There is of course a third response. If like Quine you don't like modal operators at all you might say that *no* sense can be attached to (6), except perhaps in being shorthand for a description that certain words are written or are being spoken and certain other activities are taking place.

operators. So on Wednesday you can have both (4) and

(7) *P*(it is not raining)

as simultaneously true, and thus the apparent contradiction is resolved. What we notice here is that tensed truth is introduced as a way of removing contradictions, even when we are restricted to actuality. In the case of relativity theory the contradiction seems to re-appear in a sentence like

(8) The earthquake occurred before the fire

where (8) and

(9) The fire occurred before the earthquake

are both true, but with reference to different observers. Now suppose that we are faced with the facts of relativity — that you can only speak of earlier and later relative to a frame of reference. If you don't want to make temporal truth relative then you can extend the primitive notion of simple truth so that it is *my* truth. One reply to this suggestion might well be that no one supposes that there is such a notion of primitive truth. While ordinary language may well be argued to operate with a simple notion of tensed truth, analogous to the actualist's notion of simple contingent truth it never operates with a notion of 'my truth'. The reason is not difficult to see. The possibility of (8) and (9) both being true is not a possibility which arises in the experience of ordinary folk, and ordinary language has no need of operators to account for it. On the other hand, the passage of time, and the uncertainties of life demand that any notion of truth take account of tense and modality. What can be said however is that there is no *logical* bar to beginning with a notion of my truth, and one could certainly have a language in which there were operators like tense and modal operators but which depend on the observer or 'person' index in the way in which temporal operators depend on the temporal index and modal operators depend on the world index.[6] In fact phenomena like those involved in the discussion of (4) on p. 40 might well require just such operators. And the introduction of such operators would seem no more a threat to presentism than the introduction of temporal operators is a threat to actualism.

[6]For a study of Prior's attitude to a logic of this kind, and of Prior's attitude to relativity in general see Müller 2007, pp. 244-247, especially p. 247f. Müller gives a helpful bibliography of works connecting logic and relativity.

Our discussion in this chapter has been in line with our assumption of a classical bivalent notion of truth, and before we leave the topic of relativity we should perhaps comment on the view that statements about the future lack a truth value. The best known illustration is the famous 'sea battle' passage discussed by Aristotle (*De Int*, Ch 9). The suggestion is that if the future is not determined we cannot say that either

(10) There will be a sea battle tomorrow

is true, or that

(11) There will not be a sea battle tomorrow

is true. However, as Aristotle reminds us, we *can* say that

(12) Either there will be a sea battle tomorrow or there will not be a sea battle tomorrow

is true. Yet the classical semantics for ***or*** requires that $\alpha$ ***or*** $\beta$ is true iff either $\alpha$ is true or $\beta$ is true. There is controversy about Aristotle's own attitude on this matter, and one can certainly articulate non-classical logics which provide a semantics which denies truth values to statements about the future.[7] But the classical reply goes like this. The mistake, it says, is to confuse being determined with being determinate. Perhaps, depending on your views about the future, the facts as of now do not *determine*, in any robust sense, whether there will be a sea battle tomorrow. It could plausibly be argued to be a consequence of a classical attitude to truth that the indefiniteness of the future can be represented by alternative world histories. Of the world histories which coincide up to today there is a sea battle tomorrow in some but not in others. So that if you use 'true' to mean that $\alpha$ is true today iff it is true in *all* world histories which coincide up to today, then indeed neither (10) nor (11) need be true. On the classical view of disjunction (12) is true in all futures. From the classical viewpoint such a use of 'true' actually means 'already determined to be true', and this is a modal notion. The fact that (12) can be true while both (10) and (11) are false simply reflects the fact that where $L$ is the dual of $M$ and means 'it is already determined that', ie., true in all world histories which coincide up to the present, the modal formula

[7]See, for instance, chapter 7 of Prior 1967, pp. 113-136; and Thomason 1970.

(13) $L(p \vee q) \supset (Lp \vee Lq)$

is not valid, and in particular, while $L(Fp \vee F{\sim}p)$ is always true $LFp \vee LF{\sim}p$ is not.[8]

The use of the phrase 'world history' needs some clarification. Those who speak of a 'branching futures' model of time often take that to mean that $<$ is not a linear relation. But if we take a sentence to be true or false at both a time and a world, it should be clear why $<$ can be linear. Suppose Tallulah is debating at $t_1$ what to do tomorrow, at $t_2$. In world $w_1$ she takes a holiday, in world $w_2$ she goes to work. As of today, $w_1$ and $w_2$ are indistinguishable, but on the classical viewpoint even today they contain different truths. In $w_1$ the future tensed sentence

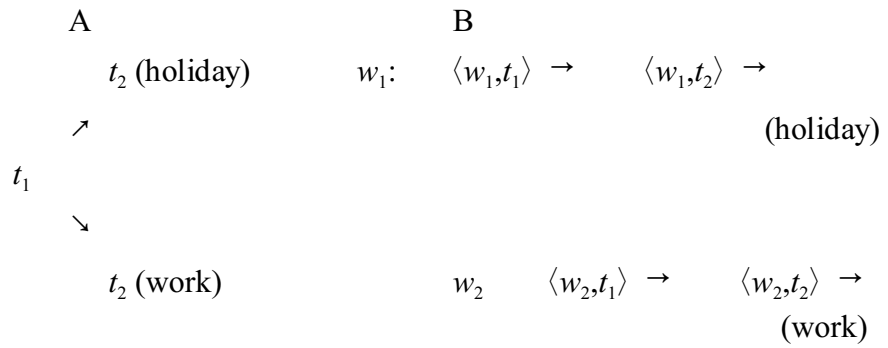
(14) Tallulah will take a holiday tomorrow

is true at $t_1$, while in $w_2$ it is false. In $w_1$

(15) Tallulah will work tomorrow

is false at $t_1$, while in $w_2$ it is true. The debate is about what these worlds will be like tomorrow, at $t_2$. Those who speak of a branching structure are treating each of its points as a world at a time. The phrase 'world history' makes that clear, since the 'world history' model actually constructs worlds in this way. The world/time approach models branching with the aid of a time-dependent accessibility relation $R_t$, taking both worlds and times as basic and as separate indices, and has nothing to say about the nature of worlds. And this is compatible with the relation $<$ on times being linear. The following picture might help to portray the situation. A represents the branching model while B represents the world-and-time model.

[8]The world index here plays some of the role played by the 'context of assessment' index in the approach to future contingents in MacFarlane 2003. (See especially p. 327f.) Suppose that $L$ represents historical necessity in the sense described in the text. If we agree to call an *utterance* of $\alpha$ at $t$ *true* in a world $w$ only if $L\alpha$ is true at $t$ in $w$, then we respect the intuition that an utterance need not count as true at $t$ in a world $w$ when the future is not yet determined, even if $\alpha$ should turn out to be true in $w$. One might say that a world $w$ is a deterministic world if at any $t$ if any wff $\alpha$ is true at $t$ in $w$ then, $L\alpha$ is also true at $t$ in $w$ — or, what comes to the same thing, that the only world accessible from $w$ (in the sense of 'accessible' which corresponds to historical necessity) is $w$ itself. At any rate the framework of part I of this book certainly permits many more indices than we have been considering, and could easily accommodate the view of truth defended in MacFarlane 2003, 2005 and elsewhere.




The reason why there is only one $t_1$ in A is that, although (as B makes clear) there are two worlds involved, yet there are no present facts which distinguish between them. They do of course differ in their future facts. While the sea-battle problem may seem different from the problems posed by relativity, it illustrates how the introduction of a world index can be used to save a linear temporal ordering against one kind of argument.

We will not pretend that in this chapter we have done more than make some remarks about how the *semantic* implications of relativity theory could be understood. It may of course be that relativity calls into question the classical nature of truth itself, whether absolute — as in the Prior/Adams/Plantinga variety discussed on p, 54 — or indexical — as favoured by David Lewis. A debate on whether truth is classical, however, raises questions beyond anything discussed in the present book.

# Part IV

# De Rerum Natura

# 14

# *Individuals and stages*

At least one philosopher, David Lewis, has argued that no ordinary individual — as it might be no aspidistra, donkey or lampshade — can exist in more than one world. We will begin by quoting the first five sentences of Lewis 1986a.

> The world we live in is a very inclusive thing. Every stick and every stone you have ever seen is part of it. And so are you and I. And so are the planet Earth, the solar system, the entire Milky Way, the remote galaxies we see through telescopes, and (if there are such things) all the bits of empty space between the stars and the galaxies. There is nothing so far away from us as not to be part of our world. Anything at any distance at all is to be included. (*op cit*, p. 1)

By a possible world Lewis understands the following. Begin with a 'thing' $x$. Now consider something $y$ such that $x$ and $y$ are both members of the same space-time system. Call $x$ and $y$ *world-mates*. A 'world' $w$ is a *maximal* entity such that all $w$'s parts are world-mates, and any world-mate of any part of $w$ is also a part of $w$. This is sometimes put by saying that worlds are *mereological sums* of world-mates — mereology being simply the formal theory of the part-whole relation. Lewis's philosophy has met with what he calls 'incredulous stares' (Lewis 1973a, p. 86, Lewis 1986a, p. 133) because of his view that other possible worlds are just like that except for being spatio-temporally disconnected from ours. Whether or not the stares are justified it is important to see that a perfectly innocuous looking opening deserves closer scrutiny. The operative word here is 'part'. Later in the book Lewis uses the fact that a world is something an individual can be a part of, in an argument to establish that nothing can be in more than one possible world. The argument goes like this: there are some properties that a thing might have, say its shape, which it might have lacked, but which are *intrinsic* to it. To say that $F$ is *intrinsic* to $x$ is to say that if $x$ is $F$ then adding new things to the world cannot change this — cannot make $x$ not-$F$. If $F$ is accidental to $x$ then there must be a world in which $x$ is not $F$. So suppose that

$x$ were to exist in two worlds $w$ and $w'$, where $x$ is $F$ in $w$ but not in $w'$. Then $x$ is a *part* of $w$ and is $F$ (in $w$). What this means is that you get to $w$ by adding things to $x$, and you get to $w'$ by adding *other* things to $x$. But if $F$ is *intrinsic* to $x$ and $x$ is $F$, then *nothing* you add to $x$ could make it not-$F$ and so you could never get to $w'$. But then $F$ is not accidental to $x$, contradicting the supposition that it is what Lewis calls (p. 201) an 'accidental intrinsic'.[1] Consider the following 'temporal' version of Lewis's opening paragraph:

> The present moment is a very inclusive thing. Every stick and every stone is part of it. And so are you and I. And so are the planet Earth, the solar system, the entire Milky Way, the remote galaxies we see through telescopes, and (if there are such things) all the bits of empty space between the stars and the galaxies. There is nothing so far away from us as not to be part of the present moment. Anything at any distance at all is to be included.

Ignoring questions raised by the special theory of relativity, let us look at this temporal version of Lewis's paragraph. First note that, whether or not we think it is true, it has lost the apparent obviousness of the original. Of course all that that may show is that the world index *is* different from the time index, but at least it should make us pause.

Now look at the parallel argument about what Lewis calls 'temporary intrinsics' (p. 203). Suppose that you begin with an entity $x$. The claim that $x$'s shape is a temporary intrinsic is just like the claim that it is an accidental intrinsic — it is the claim that you can't change $x$'s shape by adding new things to the universe. So suppose $x$ is square. Then if yesterday and today are formed by adding different things to $x$, and if adding new things doesn't change $x$'s shape, it would seem that $x$ cannot have a different shape yesterday from the shape it has today. Clearly something has gone wrong here, though it is difficult to see just what. It is sometimes assumed (see for instance Stalnaker 2003 p. 114, and Mackie 2006 p. 79) that if you accept Lewis's view of other worlds you cannot accept trans-world individuals.[2] One might then expect that those who think that

[1] It is not our intention to make a detailed reply to this argument, though one premise is that there are such properties as shape which remain unchanged by adding new things to the world. Suppose that the new 'things' were new spatial points. Might that not change an object's shape?

[2] Stalnaker (*loc cit*) argues that this follows from the principle that you cannot be in different places at the same time. But all that that principle delivers is that there is no world in which you are in different places at the same time. To get the conclusion Stalnaker needs, the principle must be modified to read that you could not have been in any place at any time except the place that you

other times are just as real as the present would take it as obvious that the same individual cannot exist at more than one instant of time. And indeed Lewis has argued that this is so. Lewis's way of preserving the analogy between worlds and times insists that 'genuine' things only exist at an instant. Arthur Prior on the other hand is unequivocal in his rejection of such entities. "One thing that tense-logic is designed precisely to facilitate is talk of persisting objects, and one thing that it is designed precisely to avoid is the introduction of pseudo-entities like 'me at $t$' and 'me at $t'$'". (Prior 1967, p. 169) The problem with these momentary things is that their identification appears to require times and 'ordinary' continuing things in the first place.

If we look at the semantics for modal and tense languages presented in Chapter 5 we notice an interesting fact. Take the sentence

(1) $\exists x(\boldsymbol{philosopher}\ x \wedge \boldsymbol{ugly}\ x)$.

In an indexical semantics (1) is true at $t$ in $w$ iff there is someone who is an ugly philosopher at $t$ in $w$, and nothing stops the very same philosopher being ugly at one time and not at another. In effect this semantics assumes that the solution to the problem of how things can be different in different worlds or at different times is that properties represented by such words as ***ugly*** (or even by ***exists***) are really relations between individuals, worlds and times, as set out on p. 31 above. Lewis (1986a, p. 201) dismisses this solution by saying simply 'I know better', though, as we have indicated, his reasons depend on his mereological construction of worlds and times. How then does Lewis treat a sentence like

(2) That sideboard would have looked better in the alcove

(where the sideboard is not in fact in the alcove) or

(3) That sideboard used to be green

(where the sideboard is now mauve)? (2) asks us to envisage a world in which the sideboard is in a different place from where it is in the actual world; but if the sideboard can only exist in the actual world, there would not seem to be a world in which it is in the alcove. And if the now mauve sideboard did not exist in the past it never was green. It is here that the notion of a *counterpart*, introduced in Lewis 1968, comes in. If (2) is true in a world $w_1$ this is because in a world $w_2$

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*are* in at that time; and this principle, is, at the very least, question-begging.

sufficiently like $w_1$ there is a sideboard which in $w_2$ is the *counterpart* of the sideboard in $w_1$, and which looks better than does the sideboard in $w_1$. And (3) claims that a past counterpart, often called a past *stage*, of the sideboard is green.

There are still problems however. To say that Socrates is bald in 404 is just to say that his 404 stage is bald. But then to say that Socrates is famous in 2004 would be to say that his 2004 stage is famous. But surely Socrates *has* no 2004 stage in the actual world? One way of proceeding would be to introduce 'courtesy' temporal stages of a continuing individual. Even those committed to continuing individuals can accept such things, since courtesy stages can unproblematically be regarded as $\langle w,t,p\rangle$ triples. Then the claim that the sideboard $p$ is mauve today and green tomorrow (in world $w$) is just the claim that $\langle w,\text{today},p\rangle$ satisfies ***mauve*** and $\langle w,\text{tomorrow},p\rangle$ satisfies ***green***, where ***mauve*** is stipulated to be a predicate satisfied by $\langle w,t,p\rangle$ iff $p$ is mauve at $t$ in $w$, and ***green*** is stipulated to be a predicate satisfied by $\langle w,t,p\rangle$ iff $p$ is green at $t$ in $w$. Using Lewis's terminology we can say that $\langle w,\text{tomorrow},p\rangle$ is tomorrow's *counterpart* of $\langle w,\text{today},p\rangle$. What this trick does enable us to do is provide a formal notion of 'part' even for those who are sceptical about whether 'modal part' or 'temporal part' even make sense.[3] Notice however that the predicates ***green*** and ***mauve*** become predicates of ordered triples. It is now recognised that the most significant advance in our understanding of the nature of relations took place in the 19th century, when it was realised that with perfect generality we may reduce any n-place relational expression to a one-place predicate by treating its relata as single ordered n-tuples.[4] It is easy to see that non-relational predicates of parts in *this* sense are only terminologically different from relational predicates of continuants.

In discussing the arguments presented by Richards and Lycan on pp. 118-120 we answered them using (33) on p. 116, i.e., as the principle that

(4) Worlds are *by definition* just what modal operators quantify over.

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[3]Perhaps most questions about whether things have such parts are terminological. For instance the question of whether Socrates 'exists as a whole' in 404 can have the answer 'no' if by that you mean to deny that he exists at other times as well, but can be answered 'yes' if you mean that all his spatial parts at 404 themselves exist then. One might be tempted here to adapt the remarks on p. 200f of Lewis 1986a and say: "We know what to say if we want to make believe formally that things have temporal parts, but we know better."

[4]In set theory a (two-place) relation is *defined* as a set of ordered pairs, and the ordered pair $\langle a,b\rangle$ is defined in terms of ordinary (unordered) sets by some such trick as $\{\{a\},\{a,b\}\}$. Relations in this sense are understood extensionally.

But they were in fact concerned to address the particular treatment of possible worlds advocated by David Lewis. Richards and Lycan were using Lewis 1973a as their target, but his metaphysical views have subsequently been articulated and defended in Lewis 1986a. Lewis's theory has in fact been attacked on the ground that it does not satisfy (4), or rather, to be precise, because

> Moreover, and more incredibly, the states of some otherworldly cats — those whose similarity to our cats qualifies them as *counterparts* of our cats — underwrite *de re* possibilities for our cats. A host of questions arise: Why aren't cats that are spatio-temporally unrelated to us and our pets still parts of actuality? What qualifies spatio-temporal relatedness as the worldmate-determining relation? How does the mere existence of a three-legged cat similar, but spatio-temporally unrelated to my cat, ground the *de re* possibility that my cat is three-legged? (McGrath 1998, p. 590)

What is the temporal parallel here? As we have seen Lewis holds that worlds are (maximal) mereological sums of 'word-mates'. Although he has nothing comparable to say about times his view that individuals exist at only a single time would enable a time to be a (maximal) mereological sum of contemporaries.[5] ('contemporary' may have to be observer relative in a more scientifically plausible temporal framework.) If you accept (4) then it will follow analytically that there will be a world in which $\alpha$ iff it is logically possible that $\alpha$; and if you accept the temporal analogue of (4) it will follow that there is a time at which $\alpha$ iff it is, was or will be that $\alpha$. Suppose then that the question is asked: Why should the fact that there is a sum of contemporaries containing Charles I's execution have anything to do with the fact that Charles I was executed? The problem is to get a handle on what such a question amounts to. But this we *can* say. If we accept a temporal analogue of (4) we will argue somewhat as follows: Assume that times are what temporal operators quantify over. What features must they have? In particular do they have to be just like the present in that everything that is happening *then* is happening *then* in just the same way as everything that

[5]Such a view is not confined to Lewis. On p. 212 of Crisp 2003 we find

> Secondly, we shall think of *the present time* as follows. Say that an object $x$ is *slim* if, for any $y$ and $z$, if $y$ and $z$ are parts of $x$, then there is either no temporal distance or a temporal distance of zero between $y$, and $z$. A *time*, let us say, is a maximal slim object: an object such that the mereological sum of it and anything which isn't a part of it is not slim. The present time, intuitively, is the maximal slim object that includes as a part every events that occurs *now*.

Though Crisp is using this in the aid of a defence of presentism.

is happening *now* is happening *now*? If it turns out that this is so then other times will be like the present, *and* will explain tense. This alone of course will not get you to a temporal analogue of *Lewis's* metaphysics. In any particular world, a time, in the temporal analogue of Lewis's 'modal realism', is a maximal mereological sum of everything in that world that was once *then* contemporaneous, or of everything that is now contemporaneous, or of everything that one day will then be contemporaneous. So, to get Lewis's metaphysics one has to assume that the totality of what is happening now is a maximal mereological sum of contemporaries; and it might well be that this is where the *real* objections to Lewis should come from. In the same way one might be sceptical of the opening sentences of Lewis 1986a, as quoted on p. 163, according to which the actual world is a mereological sum of us and all our world-mates.

Objections to Lewis sometimes take the form of pointing out that he needs a 'principle of plenitude' that there must be *enough* worlds for all possibilities, and that there can be no metaphysical guarantee that this will be so.[6] The temporal parallel helps out here too. At first it might be thought that times lack this kind of plenitude. Certainly it does not follow that everything that might happen will happen, or that everything that might have happened has happened. But, as we pointed out on p. 25, times explain tense while worlds explain modality. In that sense temporal plenitude encompasses all that was, is or will be. However, whether or not you accept Lewis's particular construction of worlds, or an analogous construction of times, a metaphysics which accepts (4) will *begin* by assuming that times analyse tense, and worlds analyse modality; and will then go on to ask whether the 'times' and 'worlds' which do so are just like ours. If the answer is yes one can *then* ask what the present time, or what the actual world is like. Only then do we ask whether the present is a sum of contemporaries, or the actual is a sum of world-mates, and the answer to *that* need not be dictated by the fact, if it should turn out to be a fact, that neither the present nor the actual are metaphysically special.[7]

Many years ago Roderick Chisholm (Chisholm 1967) taught us that questions of identity across possible worlds are hard. He imagines Adam slowly changing into Noah in such a way that it seems we have to imagine a world just like ours except that Adam is Noah and Noah is Adam. Although Chisholm was not thinking so much of change over time it is not hard to tell a story in which

[6]See van Inwagen 1986, p. 197.

[7]This in fact is the argument of Lewis 1973a, p. 85f. The 'reductive' analysis of worlds, which is such an important part of Lewis 1986a, seems a later development.

there is going to be a time just like the present, except that I am then just as you are now and you are then just as I am now.[8] Of course a world like this is not our world. In our world the changes that are involved in most ordinary talk are about moderately stable 'things'. If there is a problem here it seems to us a problem in the notion of a thing, and only appears to arise as a particular problem for modality because talk about what could or might happen is not constrained in the way talk about what has happened or what will happen is constrained. In fact, suppose that unbeknownst to us we actually *are* in a world in which such things are going to happen. Then what Chisholm says about modality would seem to apply equally to tense.

In the rest of this chapter we shall look at questions of identity which arise in a purely spatial context. What we shall see is that while there may be serious problems about the identity of ordinary individual things, these problems arise quite apart from questions about times and worlds, and do not tell against the parallel between worlds and times. Many things are composed of parts. A table has legs and a top, an aspidistra has leaves and a stem. A long time ago David Kaplan (Kaplan 1978) used the example of a highway. His was in California; ours is in New Zealand. In New Zealand Highway 1 and Highway 5 coincide between Taupo and Wairakei for a distance of about 15 kilometres, but before and after that stretch they diverge. Highway 1 goes all the way from Wellington to Cape Reinga in the North Island (and from Picton to Bluff in the South Island) while Highway 5 goes from Tirau to Napier. In some sense they share a common part. The idea of a spatial part of a road seems fairly straightforward. Some philosophers think of the part/whole relation as of metaphysical importance, but if we look at roads it's not really obvious that a part of a road is more basic ontologically than the whole road. Perhaps we may think of the pebbles and the tarsealing and the painted centre lines, and perhaps the signposts, as the basic parts, or perhaps we must think of *their* parts as more basic. Perhaps indeed we must go down to some metaphysically basic atoms, or perhaps not. Luckily metaphysics is not our concern in this book so let's take it for the moment that we can think of the road as being divided into parts without having to take a stand on what is the more basic.

Some properties of the road are certainly properties of its parts. There is

[8]Chisholm's particular story involved properties like living for 930 years. Such properties are tenseless, so are not appropriate in the temporal parallel. The temporal parallel would involve properties like being bald, or living in College Station. Even earlier, Wilson 1959, p 522, had asked the question: 'What would the world be like if Julius Caesar had all the properties of Mark Antony, and Mark Antony had all the properties of Julius Caesar?', and had answered that it would 'look exactly the same.' (p. 523) Prior 1960 (p. 81 of 2003) responds that 'It *would* look exactly the same to him or to me; but would it have looked the same to Caesar or to Antony?'

a right-angled bend in Highway 1 in the township of Taupo — where it meets the lake if you are driving south, or leaves the lake if you are driving north. Because that part coincides with Highway 5 there is a right-angled bend there on Highway 5. But Highway 1 goes from Wellington to Auckland, while Highway 5 does not. The point is a simple one. While both the properties of having a right-angled bend and going from A to B are properties of a road, one of these properties holds of the road in virtue of holding of a part of the road while the other does not. A property, like having a bend, holds of the whole road because it holds of that part of the road so that if that property holds of a road then it holds of any road which shares that part, while the property of going from Wellington to Auckland holds of the road as a whole. In speaking of a *property* of the road we simply mean to refer to something that enables us to say true or false things about it. Here are some of the things we may want to say:

(5) H1 (Highway 1) has a bend in Taupo

(6) H5 (Highway 5) has a bend in Taupo

As with times and worlds, solutions to the semantic problem divide into two classes — those which take the predicates to attach directly to the roads H1 and H5 as a whole, and those which take the predicates to attach directly to their parts. Both (5) and (6) are qualified by 'in Taupo' and their truth depends upon this. (H1 contains no right-angled bends between Himatangi and Foxton.) We can look at three preliminary rough parsings of the sentences with variables replacing the proper names.

(7) $x$ has a bend in $y$

(8) [$x$ has a bend] in $y$

(9) [$x$ in $y$] has a bend.

(7) is the simplest analysis. It simply claims that having a right-angled bend is a relation between an entity such as a road and another entity, a region of space. In this case $x$ can be H1 (or H5) itself, and $y$ can be Taupo. Suppose though that you think it odd to say that having a bend is a *relation* between a road and a region. Take ***Taupo*** to be the proper name of a spatial region. Suppose you claim that ***in Taupo*** is an *adverbial* phrase which modifies the sentence

(10) *x* has a bend.

Although some philosophers speak as though adverbial theories make fewer ontological commitments than relational theories it is easy to see that an adequate semantics for (8) differs from (7) only in the mechanics of its evaluation. If ***in Taupo*** is to be treated as a sentential modifier a sentence like (10) must be true or false at a place. This means that spatial regions are *semantical indices* in the sense of pp. 18 and 29.[9] (10) is true at a spatial region *r* iff *x* contains a bend in *r*. And

(11) α ***in Taupo***

is true at any *r* iff α is true at *r′* when *r′* = Taupo — assuming that Taupo is a spatial region. Whichever of (7) or (8) you favour the basis is the same. You only get a truth value out when you plug in values for *x* and *y* such as H1 or H5, and values for *r* or *r′* such as Taupo. So both these solutions are of the first kind. The second kind of solution — the one represented in (9) — works a little differently. In this solution some roads have bends and some do not. Having a bend is a yes or no property. You can't have a bend in one place but not in another. It is not at all difficult to think of a complete road which runs from one end of Taupo to the other in just the way that H1 and H5, do. And that part of H1 does not differ at all from that part of H5, so that (5) and (6) can both be true. If you want to say that H1 has a bend in Taupo what you say is that the Taupo part of H1 has a bend simpliciter, while the Foxton to Himatangi part does not.

There are, however, a range of problems which neither of these approaches handles uncontroversially, and it is to these we turn. Suppose we are in Taupo and are pointing to the highway. We might say

(12) That road is H1

(13) That road is H5

(14) H1 is H5 in Taupo

(15) That is only one road

(16) That is two roads.

[9]For this discussion we shall ignore other indices such as times, worlds and the like.

The problem is not to decide whether or not (12)-(16) are true. It is plausible that there is a sense in which they all are. Perhaps (15) and (16) might seem a little controversial. They seem to contradict each other. Yet don't we want to say that *in a sense* each is true, and the problem is making clear what that sense is. Even (12) and (13) might begin to seem problematic. For when two motorists point to the road outside the Taupo 'superloo' and one utters (12) while the other utters (13) it seems that they are pointing to the same road. From this it would seem that we may infer (14). (14) appears to be a statement of identity. Is it true? One wants to say 'yes and no', as witness the temptation to say that in a sense (though not in the same sense) both (15) and (16) are true. If (14) is genuine identity it seems false, for there are truths about H1 which are not truths about H5:

(17) H5 goes to Napier but H1 does not

Solutions of the first kind (those which take the predicates to apply directly to H1 and H5) would claim that the ***is*** in (14) is not a genuine identity but is a predicate of *coincidence*. H1 and H5 *coincide* in Taupo though they are not identical there or anywhere. The second kind of solution analyses (14) as

(18) [H1 in Taupo] is [H5 in Taupo]

In (18) ***is*** is a straightforward identity, and (18) is true because the Taupo parts of H1 and H5 just *are* identical.[10] If the coincidence approach is adopted, there are two roads which coincide in Taupo. That gives a sense in which (16) is true, but doesn't help account for the sense in which (15) is true.

Similar problems appear in the temporal case. Here is an example. In New Zealand you can buy sets of wooden trivets for putting on a table under hot dishes. These are cunningly made of curved pieces of wood (as it might be in the shape of an S) in such a way that they can be put together as one trivet, or taken apart to form two or three. Name the three pieces of wood *a*, *b* and *c*. Then Monday's trivet is *abc*, Tuesday's are *ab* and *c*, Wednesday's are *a* and *bc*, while on Thursday we are having a dinner party which requires the three separate trivets *a*, *b* and *c*. Let's ask on Monday where today's trivet will be on Thursday. If (on Thursday) *a* is on the dining table, *b* on the sideboard while *c* is in the kitchen you could say on Monday

[10]The account of temporal identity presented in Gallois 1998 is intended to permit contingent identity while yet maintaining that his interpretation of the predicate = is straightforward identity. His account can be modelled by requiring that the *quantifiers* range over functions from times to objects, but that all the predicates denote relations between ordinary objects.

(19) It will be on the table, on the sideboard and in the kitchen

meaning that at least one part will be in each place.[11]

We do not have answers to the problems just raised. But providing such answers is not one of our aims. What we have tried to do is show that there is no uncontroversial use of sentences involving individuals which forces a decision either way on the question of whether predicates apply to wholes in virtue of applying to their parts, or apply to their parts in virtue of applying to the wholes. And that seems to us to remain true whether the parts are legs, roads, spatial regions, times, worlds, or n-tuples of these or other things. Most importantly of all for the purposes of this book, the easy cases are still easy and the hard cases are still hard, whether or not we think of the predicates as applying primarily to parts or to wholes. And this remains true *even when* we are talking about objects which, in a perfectly ordinary and everyday sense do seem to be composed of parts. In the case of worlds and times, speaking of 'parts' could seem somewhat contrived; so it is good that questions about parts and wholes are not specific to questions about worlds and times. At the very least such issues seem of little help in looking at the difference between worlds and times.[12]

[11]The trivet case is a real life case of splitting individuals. Splitting is often discussed in terms of persons and that raises complications — they are complications to do with consciousness in a way which makes it tempting to think there must be a fact of the matter whether we survive this or that eventuality. If the problem is about splitting, one might defuse it by saying that the things we talk about in natural language are normally presumed not to split, though the case of the trivets may make this questionable. Lewis protests that that option is not available in the modal case, since any splitting which *could* take place *does* take place in some possible world.

[12]One can of course *make* a metaphysical difference between times and worlds in respect of the nature of genuine objects. Thus, for Heller 1990 no metaphysically real object could be in a different place at a given time from the place it actually is in then — so that metaphysically real objects cannot have genuine modal properties. In chapter 2 Heller bites the bullet and describes 'ordinary' objects like Descartes as just 'conventional', which don't really exist.

# 15

# *Predicate wormism*[1]

In Quine 1976, W.V. Quine attempts to show that, even if you have them, worlds, unlike times, cannot provide robust continuity conditions for the sort of ordinary objects that we want to talk about. Quine begins his paper by explicitly invoking the world-time parallel:

> Identifying an object from world to possible world is analogous, it has been suggested, to identifying an object from moment to moment in our world. I agree, and I want now to develop the analogy. (1976, p. 859)

Like Quine we will restrict our discussion to physical objects. Quine's conception of a physical object is given in the second paragraph of his paper:

> Consider my broad conception of a physical object: the material content of any portion of space-time, however scattered and discontinuous. Equivalently, any sum or aggregate of point-events. The world's water is for me a physical object, comprising all the molecules of $H_2O$ anywhere ever. There is a physical object part of which is a momentary stage of the silver dollar in my pocket and the rest of which is a temporal segment of the Eiffel Tower through its third decade. (1976, p. 859)

So Quine's metaphysics is in the mereological tradition examined in the last chapter. We have stressed that this book does not take sides on metaphysical issues, so we make no claims about whether this is a viable metaphysics, or even how precisely it is to be understood. Quine, as we present him, thinks of individuals as 'worms' construed as strings of events over times. In this modelling of Quine's framework we have to take the 'point-events' as primitive. A good way to think of a point-event is as a 'pixel' on a television or computer screen. Of course, as when a television set is malfunctioning, not all successions of pixel patterns will be structured in a way which makes sense to human viewers, but they will all be possible successions nonetheless. To ease the discussion we shall temporarily distinguish six views by the following labels:

[1] A longer version of this chapter is found in Cresswell 2010.

Four-dimensional stageism
Four-dimensional counterpart wormism
Four-dimensional predicate wormism

Five-dimensional stageism
Five-dimensional counterpart wormism
Five-dimensional predicate wormism

These are all based on a metaphysics, in which the domain of 'what there is', the domain of (unrestricted) quantification, contains all mereological sums of 'point-events'. Four-dimensionalism is the view that things have *temporal* parts — things like 'me at time $t_1$' or 'Mt Cook at time $t_2$, and so on. Five-dimensionalism is the view that things also have *modal* parts. So that you can have 'Mt Cook at time $t_2$ in world $w_3$', and so on. We might say that a *strong* four-dimensionalist is a four-dimensionalist who holds that although things have temporal parts they do not have modal parts. A weak four-dimensionalist holds that they have temporal parts and also might have modal parts. For our purposes we shall take Sider 1996, 2000 and 2001 to be a strong four-dimensionalist. (If we haven't understood him correctly it will not affect any of our claims.) We take Lewis 1986a to be a five-dimensionalist, and therefore a weak four dimensionalist.

Quine is therefore a four-dimensionalist. It may well be that *in propria persona* he is a strong four-dimensionalist, but the burden of Quine 1976 is to play along with assuming five-dimensionalism for the purposes of argument, since he is trying to show that the problem with worlds arises even if you grant a mereological metaphysics which admits them. On p. 860 Quine describes the extension of his ontology to worlds by saying that trans-world individuals are "sums of physical objects of the various worlds, combining denizens of different worlds indiscriminately." The difference between four and five-dimensionalism arises in the context of the correct explanation of modality. A criticism of strong four dimensionalism is that, while it may allow for an explanation of temporal properties in terms of temporal parts, it does not allow for an analogous explanation of modal properties. By allowing modal parts a five-dimensionalist is able to explain modal properties in a way exactly analogous to the way a four-dimensionalist explains temporal properties.

Assume that there are metaphysically simple 'point-events' which exist at just one time. A four-dimensional *worm* is a collection of these. Some of them might all exist at the same time, as say the many parts of Mt Cook at $t_2$. The collection of all of Mt Cook's parts at $t_2$ would be a *stage* (or *manifestation*) of Mt Cook. Mt Cook would have manifestations which exist at different times. The

difference between stageism and wormism (though not always discussed under those names) is acknowledged by both Sider (2001, p. 209f) and Lewis (1986, p. 217) to be a matter of semantics not of metaphysics — that is to say it is not a difference about what there is. While both stageists and wormists agree about what there is, stageism, whether four-dimensional or five-dimensional, holds that the predicates of our language apply to stages. Wormism, whether four or five-dimensional, holds that the predicates apply to the worms themselves.[2] Quine is a wormist, while both Lewis and Sider are stageists. Sider is unsure whether or not the distinction is important. Lewis is more sympathetic to temporal (four-dimensional) wormism than to modal (five-dimensional) wormism.

For a four-dimensional stageist the problem is what to say about temporal predicates. To say that Mt Cook at $t_2$ has been climbed many times seems odd, because it is not *that* stage which has been climbed many times. Stageists solve this problem by the method of temporal counterparts. The $t_2$ stage of Mt Cook has been climbed many times because there are many previous stages – many of its previous temporal counterparts – which are (in some absolute sense) climbed. On the worm theory, the theory that Quine advocates, the predicate applies to the worm. It is the whole Mt Cook worm which, at $t_2$, has been climbed many times.[3] Counterpart theory and the worm theory are different in the case of stages which are stages of many worms. What we say about modal and temporal properties of such stages depends on the counterpart relation, where a stage may have more than one counterpart at another time or in another world. On the worm view there are different worms which *coincide* in the sense that they share the same stage.

Counterpart wormism says that the only worms that are eligible for predicates to apply to are worms whose stages are linked by a counterpart relation.[4] Counterpart wormism, as we are understanding it, does not deny that

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[2]Linguists have sometimes distinguished between 'stage-level predicates' and 'object-level predicates'. See Carlson 1978, Dowty 1979 p. 177 and Kratzer 1995. Some philosophers have followed these linguists. Thus Parsons 2005 argues that sortals apply to worms while non-sortals apply to stages. We take Quine to be claiming that *all* predicates apply to worms. Obviously any stage predicate can be construed as applying to a worm at a time in a world solely in virtue of features of the manifestation of that world at that time — predicates like 'is in Wellington'.

[3]In set-theoretical terms 'having been climbed many times' can be modelled by the function which assigns to any time *t* the class of worms which have been climbed many times before *t*. Note that this is different from 'having been climbed many times before $t_2$'. *That* would be modelled by the function which assigns to any *t* the class of worms which have been climbed many times before $t_2$. Of course any worm will *at* $t_2$ satisfy the one function iff it satisfies the other.

[4]Sider 2001, p. 225, suggests that for a wormist this is a relation of 'genidentity'. 'Sortal wormism', of the kind espoused by Parsons (see footnote 2) is a compromise in which the sortals provide the genidentity relation. But it shares with stageism the claim that since some predicates

there are worms which are not linked by any counterpart relation, and admits such worms into its ontology. But it does deny that predicates can apply to such worms, or at least that our ordinary predicates can. For counterpart wormists, as for stageists, the counterpart relations are somehow 'there' antecedently to the application of the predicates. It might be supposed that four or five-dimensionalism require counterpart theory. Certainly stageism requires counterpart theory, and so does counterpart wormism, by definition. But predicate wormism does not require counterparts. In the passage quoted above, Quine is explicit that any temporal worm, however discontinuous, counts as an object, and on p. 860 he extends this to worlds:

> Thus quantification over objects across possible worlds does not require us to make any sense of 'counterpart'.

He then insists that all these objects are in the range of the quantifiers. The real work is done by the predicates. On p. 861 Quine writes

> We saw that in our own world the identification of a physical object from moment to moment makes sense only relative to the principle of individuation of one or another particular predicate – usually, though not necessarily, the predicate 'body' or one of its subordinates. Such cross-moment groupings are indifferent to the actual quantification over physical objects, since the quantification respects all cross-moment groupings, however random. But they matter to the predicates. ... Since all sentences contain predicates, cross-moment identification of one sort or another is a crucial matter in its proper place.
>
> Similarly, if one quantifies over objects across possible worlds, one needs cross-world identification relative to whatever predicate one uses in such sentences. Typically these predicates, again, will be subordinates of 'body'.

It is the role of predicates in Quine's account that we wish to concentrate on, since its importance still seems underappreciated in the literature. For instance, Markosian, 2008 (p. 343), speaking of the combination of the 'quark on your nose' and a 'quark near Alpha Centauri' says "But such an object, if it exists, is

– the non-sortals – apply to stages, *these* predicates need something — in fact sortals — to specify which stage the predicate applies to at another time or in another world.

certainly not recognized by common sense." So not only do we have the predicate 'object' but also the predicate 'recognized by common sense'. From the Quinean perspective such items are undeniably in your ontology, provided that ontology satisfies the principles of mereology. But from that *nothing* follows about which worms are recognised by the predicates of common sense.[5] Here is a simple illustration. We will assume that there are two worlds, $w_1$ and $w_2$. We have said 'worlds' but it could be times, or whatever other indices are being used. Think of ♥ and ♣ as two different pixel patterns in $w_1$; and think of ◘ and ☼ as two different pixel patterns in $w_2$. So there are four worms, *a*, *b*, *c* and *d*:

**Domain Items**

| | $w_1$ | $w_2$ |
|---|---|---|
| *a* | ♥ | ◘ |
| *b* | ♣ | ☼ |
| *c* | ♥ | ☼ |
| *d* | ♣ | ◘ |

Here is an example of how things should go on the wormist view that we are attributing to Quine. Imagine that *a* and *b* are both people, and suppose that ***wins*** is a predicate of people, and suppose that *a* wins in $w_1$, while *b* wins in $w_2$.

***wins***

| $w_1$ | $w_2$ |
|---|---|
| *a* | *b* |

The predicate ***wins*** is true of *a* (and only *a*) in $w_1$, and of *b* (and only *b*) in $w_2$, where *a* and *b* are the worms described in the list of domain items. On the wormist view adopted here it is *worms*, specifically *a* and *b*, which satisfy the predicate ***wins***. By contrast, on the stageist view, it is not worms but pixel

[5]So, while such disputes as that between van Cleve 2008 and Markosian 2008 may have *semantical* significance they need not involve any disagreement about what exists.

patterns which satisfy the predicates. On the stageist view, one would say that ♥ wins in $w_1$ and ☼ wins in $w_2$, and one might therefore be tempted to refer to *c* as 'the winner', and say that by necessity *c* satisfies the predicate ***wins***, since in $w_1$ its manifestation, ♥, wins in $w_1$, and in $w_2$ its manifestation, ☼, wins in $w_2$. However this would be a mistake. For *c* is a worm, and what wins on the wormist account is not a manifestation, not a pixel pattern, but a worm, and in this case neither of the worms which win at any world is *c*. For only *a* wins in $w_1$, and only *b* wins in $w_2$, and $c \neq a$, and $c \neq b$. On the wormist view the predicate ***wins*** does not apply to the *stages* but only to whole worms, and *c* is not a worm which wins in *any* world, and so *a fortiori* it is not true that *c* wins by necessity.

Contrast ***wins*** with what is sometimes called an 'essential' predicate like ***human***.

| ***human*** | |
|---|---|
| $w_1$ | $w_2$ |
| *a* | *a* |
| *b* | *b* |

It is often supposed that the use of predicates like ***human*** involves a metaphysical claim about an individual rather than merely claiming a logical relation between predicates. That at least seems part of what was driving Quine's early criticisms of quantified modal logic. (See for instance his dissatisfaction with 'Aristotelian essentialism', expressed on p. 154f of Quine 1953a.) So it is important to recognise that, on the 'Worlds Away' model, the difference is also a fact about the meaning of the predicates. Neither *c* nor *d* are human in any world, but both *a* and *b* are human in both worlds. Note that this difference between humans and winners is solely determined by the particular meanings of ***wins*** and ***human***, and has nothing to do with the domain of quantification. All that the *metaphysics* can tell you is that *a* and *b* and *c* and *d* (and many other worms) are equally good examples of 'what there is'. Note also that in each world the manifestation of both *c* and *d* is the manifestation of a human, for all that neither *c* nor *d* is itself human. This is parallel to what Quine tells us on p. 859 – that a time slice can be part of many time-extended physical objects.

In the example so far *a* and *b* exist in both $w_1$ and $w_2$, and essential predicates are usually held to entail only that they apply to things in all worlds in which those things exist. From the worm perspective one may say that a worm

exists in a world or at a time iff it has a non-empty manifestation in that world or at that time. So we could have the following model:

| | $w_1$ | $w_2$ | $w_3$ |
|---|---|---|---|
| *a* | ♥ | ◘ | – |
| *b* | ♣ | – | ♠ |

In this model there are three worlds (or times) $w_1$, $w_2$ and $w_3$. *a* exists in $w_1$ and $w_2$, while *b* exists in $w_1$ and $w_3$. The addition of $w_3$ obviously increases the number of worms and *a*, *b*, *c* and *d* as previously defined in the two-world model no longer exhaust the possibilities. In the three-world model, in respect of these particular worms ***human*** would have to look like:

***human***

| $w_1$ | $w_2$ | $w_3$ |
|---|---|---|
| *a* | *a* | – |
| *b* | – | *b* |

Both *a* and *b* satisfy ***human*** in every world in which they exist. Of course, on the worm view nothing prohibits a predicate from applying at a time or in a world where the worm has an empty manifestation. It is solely the meaning of ***human*** which imposes this constraint. The following would be an equally legitimate predicate:

***human****

| $w_1$ | $w_2$ | $w_3$ |
|---|---|---|
| *a* | *a* | *a* |
| *b* | *b* | *b* |

Note that ***human**** applies to *a* in $w_3$ *even though a* has an empty manifestation

in $w_3$ and applies to $b$ in $w_2$ even though that $b$ has an empty manifestation in $w_2$, and so does not entail the necessary existence of any human*. All it does entail is that you can be a human* when you don't exist, though you can't be a human when you don't exist. Or to put the point in another way, the issue about whether you have to exist to be human is no more than whether 'human' means ***human*** or ***human****.

The Quinean view has an easy way of showing that the problem of 'trans-world identity' is a pseudo problem.[6] Consider an individual (i.e. a worm) which is manifested by ♥ in $w_1$, and an individual which is manifested by ◘ in $w_2$. Are these the same individual? From the 'worm' point of view there are at least three distinct worms:

| | $w_1$ | $w_2$ |
|---|---|---|
| $a$ | ♥ | ◘ |
| $a'$ | ♥ | – |
| $a''$ | – | ◘ |

There is no *metaphysical* problem about $a$ or $a'$ or $a''$. They are all equally good worms. If the predicates we use apply to $a$ in $w_1$ then we are talking about an individual which exists also in $w_2$. If they apply only to $a'$ or to $a''$ we are talking about individuals which do not exist in both worlds. What looked like a metaphysical debate turns therefore into a debate about which of the infinitely many available worms we take our predicates to be applying to. If we are unsure what to say about whether or not a worm exists in this or that other world or time, that may, on the Quinean account, betray no more than the indeterminacy of reference – that our speech habits are not precise enough to isolate just which worm we intend to be talking about. One might describe this as 'naturalised metaphysics' in the manner of Quine's 'naturalized epistemology' (Quine 1969). What it means is that there is no *philosophical* obligation to advance any theory of just which worms are well-behaved relative to any particular predicate, and we advance no such theory.[7]

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[6]See for instance the discussion on p. 93 of Kaplan 1978, or p. 27 of Mackie 2006.

[7]Montague 1960 (p. 152, of Montague 1974) makes a similar point by saying that the metaphysical project of specifying the ontological status of entities of a certain sort must not be confused with the project of *defining* those entities.

We can at last consider the purported way in which the difference between times and worlds bears on individuation. Lewis prefers stageism to wormism for both times and worlds.[8] But he seems more sympathetic to temporal worms than to modal worms, on the ground that there are fewer natural counterpart relations for temporal continuity than for modal continuity. This is because temporal counterparts are related to what does happen, whereas modal counterparts are only constrained by what logically *could* happen. Change over time, at least in our world, seems constrained in ways in which change from world to world is not. Both Lewis 1986a (pp. 214-220) and Sider 1996 argue that while temporal worms need not cause trouble (because there are natural counterpart relations which determine 'identity through time') there are too many modal counterpart relations to provide analogously robust restrictions on the available five-dimensional worms. On p. 217f of Lewis 1986a Lewis refers to Quine's article, but appears to treat Quine as a stageist, so that a predicate like ***human*** takes the *stage* to be a person. Lewis does not seem to consider the possibility that according to Quine it is only the whole worm which could be a person.

Nevertheless Quine, like Lewis, also mounts an argument against worlds on the basis of continuity. In his case it is part of an argument that the parallel between times and worlds makes clear why worlds will not do for continuants what time will do. On p. 861 Quine points out that most ordinary temporal re-identification depends on individuals being 'bodies', and that the individuation of bodies turns on "continuity of displacement, distortion, and chemical change." He then goes on

> These considerations cannot be extended across worlds, because you can change anything to anything by easy stages through some connecting series of possible worlds. The devastating difference is that the series of momentary cross-sections of our real world is uniquely imposed on us, for better or for worse, whereas all manner of paths of continuous gradation from one possible world to another are free for the thinking up.

The passage is curious because earlier in the paper Quine has made it clear that, as far as ontology is concerned, temporal 'objects' are also 'free for the thinking

[8]Our concern in this chapter is not to adjudicate between these two approaches, but merely describe how Quine's worm approach bears on the world-time parallel. Nor have we any comments to make on how the two approaches affect questions about the relation between individuals (like statues) and the material of which they are made. (On such issues see works like Baker 1997, Rea 1997, and others.)

up'. The thrust of Quine's paper is not the ontological difference between world-bound and trans-world worms – for all that his ulterior motive may be to exclude worms whose manifestations are not all taken from the actual world. The issue concerns which of these worms count as bodies, because their manifestations are linked in a well-behaved way – by "continuity of displacement, distortion and chemical change." And it is here that Quine sees a difference between times and worlds. His reason, given in the passage from p. 861 quoted above, is that "you can change anything to anything by easy stages through some connecting series of possible worlds."[9] Certainly you can define such a worm, but 'easy stages' has to assume more. The problem is not that you can make up a worm in any arbitrary way which connects the manifestation of an object in one world with a manifestation of the same object in another world. Quine has already admitted the possibility of arbitrary discontinuous worms in the case of times, and has no problem with an ontology which includes such peculiar things. He is correct in his observation on p, 862 that it is 'discouragingly easy' to define a predicate which applies to any worm of manifestations. What rules out most worms in the present context is that they do not satisfy the predicate ***body***. Quine's reference to 'easy stages' in the quoted passage has to mean that the move from one world to another for this worm satisfies the constraints of good behaviour imposed by ***body***. But that implies that Quine must accept that there *are* such constraints. And such constraints seem plausible. Learning to apply a predicate seems to involve learning whether that predicate *would* apply to thus and such an individual whether or not it ever has or ever will. We know what it would be like to walk out of the room in the next five minutes whether or not we in fact do so. What is missing in Quine's paper is an argument that *predicates* cannot constrain modal 'change' in the way in which they constrain temporal change. If the things which satisfy the predicate only do so in a well-behaved way in a well-behaved world there is no guarantee that any 'individual' – however free it may be for the thinking up – will be sufficiently well-behaved to satisfy the predicate in question.

We might tell you that we left the key in the letterbox this morning, and you might reply that you took it out this afternoon. We can symbolise this by

(1) $\exists x(\boldsymbol{key}\ x \wedge \boldsymbol{in}\ x \wedge \boldsymbol{out}\ x)$

[9]Quine presumably has in mind examples like Chisholm's Adam/Noah case (see p. 168 above), though Chisholm is concerned with the re-identification of a *person* from world to world, and the manifestation, in any given world, of a worm which is a person need not, for a wormist, be itself a person in the world in which it is a manifestation.

where ***in*** *x* means that we put *x* in the letterbox, and ***out*** *x* means that you took *x* out of the letterbox. Suppose that the quantifier ∃ ranges over all sums of primitive simples. For (1) to be true there must be a sum which satisfies the predicates ***key***, ***in*** and ***out***, and Quine's point is simply that the predicates that we use in language have to apply to sums which are well behaved in terms of our interaction with the world. Quine of course is inclined to deny the literal truth or falsity of statements like

(2) That key could have been twice as big as it is.

If you accept sentences like (2) as able to be literally true then it seems that you are stuck with a problem anyway, whether or not you talk in terms of possible worlds. It is a problem about how much an object which is in fact a key in or out of a particular letterbox can change without going out of existence. There seems no problem with

(3) Although you took the key out of the letterbox you might have left it there.

But what about

(4) This key might not have been a key?

If you think that (4) can have a literal truth value then, in possible worlds terms, you are taking a stand about whether the predicate ***key*** could apply in this world to a mereological sum which could exist in worlds in which it did not satisfy that same predicate ***key***. Quine is right that the problem is about the application of predicates like ***key***, but his dismissal of worlds assumes that the predicates of ordinary language do not have the modal power to give guidance on such questions as the difference between (4) and (3).[10]

We have not in this chapter made any claims about whether a worm metaphysics is the *correct* way of accounting for these matters. All we have tried to show is that Quine's argument in 'Worlds Away', far from showing that worlds are worse off than times because of the problems of continuing identity, might actually indicate that their status is exactly similar.

[10]Chihara 1998 pp. 39-58 looks at this argument of Quine's in some detail and examines responses by Forbes 1985 and others. Much of this discussion is couched in terms of the 'necessity of origin' view in Kripke 1972, and is restricted to the identity of organisms.

# 16

# *Abstract and concrete*

Many actualists claim that possible worlds are *abstract*, and so not as real as our actual 'concrete' world. And here we appear to have a mismatch, because the presentism/eternalism debate seems to pay little attention to the ontological status of times, and does not seem to be a debate in which the presentist holds that times are abstract and the eternalist holds that they are concrete.[1] The fact that 8.15pm, NZDT, on Friday 31 December 2004 might be an abstract, necessarily existent entity in no way lessens the physical reality of what is then happening. Nor would the fact that times are abstract entities suggest that any one time is a privileged 'present time'. So it is by no means obvious that the debate between possibilists and actualists should be seen as a debate about whether worlds are concrete or abstract.

Still, actualists typically do seem to feel an obligation to give an 'abstract' analysis of worlds, and we shall examine some of the analyses that have been given, in terms of the temporal parallel. Stalnaker (2003, p. 28) speaks of a world as a 'way things might be', where the actual world is the way things are. On p. 27 he contrasts this with Lewis's 'I and all my surroundings'. (Lewis 1973a, p. 86.) Stalnaker describes a way things might be as a property or state of the world, not as a world itself. The way he speaks on p. 28 is that the world is a property — the property of something's (I and all my surroundings?) being a certain way. If that property is instantiated then that is the way the world is. Stalnaker does not tell us here what he takes a property to be, and that is unfortunate, since he later (p. 129) takes a property to be a 'function taking a possible world into a subset of the domain of that world'; and that requires that we must already have worlds in order to say what properties are. In fact Stalnaker (pp. 33-38) explicitly advocates treating propositions as sets of worlds rather than making worlds out of (maximal consistent) sets of propositions. When Stalnaker makes out that under certain assumptions his theory is the same as Adams's theory of maximal consistent sets of propositions (see p. 53 above) it would seem that the actual

[1]Rea 2003, p. 249 in defending four-dimensionalism takes times to be 'concrete sums of events'.

world is also abstract. In fact there seems good reason to interpret Stalnaker as claiming that *all* worlds are abstract, including the actual world. Otherwise, if being actual is a property that a world can have — as when we say that another world *might have been actual* — then it seems that if that (abstract) world had been actual it would have been concrete. And this suggests, at least if abstract and concrete are contraries, that being abstract is not essential to it.

Suppose that all possible worlds, including the actual world, are abstract. Then, even if the actual world is privileged, that privilege would not confer a different *ontological* status on it. All the possible worlds would *exist* in just the same way, but only one of them, as Stalnaker puts it on p. 28, would be *instantiated*. The distinction between existence and instantiation is in fact the clue to Plantinga's 'actualist' construction of possible worlds out of states of affairs (Plantinga 1976), as described on p. 55 above. We can make this precise by relating Plantinga's actualism to the modal and tense languages of first-order logic introduced in Chapter 8. These languages assume quantification over worlds and times, together with a temporal 'now' predicate applying to times, and a modal 'actual' predicate, $A$, applying to worlds. Plantinga has to allow quantification over worlds, since worlds are states of affairs, and his definition of a world on p. 258 involves quantification over states of affairs. As we showed in Chapter 4 there is no bar to extending his actualism to the temporal case. Assume **s** is a variable for states of affairs. Assume also a predicate $\boldsymbol{O}$ for 'obtains'. Because our language includes both tense and modality, we need Plantinga's notion (Plantinga 1985, p. 90, and p. 56 above) of a *temporally invariant* state of affairs, together also with a *modally invariant* state of affairs. A state of affairs $s$ is *temporally invariant* if it is not possible for $s$ *ever* to obtain unless $s$ *always* obtains : $\Box(\otimes\boldsymbol{O}\mathbf{s} \supset \oplus\boldsymbol{O}\mathbf{s})$. A *world* is a maximal temporally invariant state of affairs:

$$W\mathbf{s} =_{\text{df}} \Box(\otimes\boldsymbol{O}\mathbf{s} \supset \oplus\boldsymbol{O}\mathbf{s}) \wedge \forall\mathbf{s}'((\Box(\otimes\boldsymbol{O}\mathbf{s}' \supset \oplus\boldsymbol{O}\mathbf{s}') \wedge \Diamond(\boldsymbol{O}\mathbf{s}' \wedge \boldsymbol{O}\mathbf{s})) \supset \Box(\boldsymbol{O}\mathbf{s} \supset \boldsymbol{O}\mathbf{s}'))$$

A state of affairs is modally invariant iff $\oplus(\Diamond\boldsymbol{O}\mathbf{s} \supset \Box\boldsymbol{O}\mathbf{s})$. A time is a maximal modally invariant state of affairs

$$T\mathbf{s} =_{\text{df}} \oplus(\Diamond\boldsymbol{O}\mathbf{s} \supset \Box\boldsymbol{O}\mathbf{s}) \wedge \forall\mathbf{s}'((\oplus(\Diamond\boldsymbol{O}\mathbf{s}' \supset \Box\boldsymbol{O}\mathbf{s}') \wedge \otimes(\boldsymbol{O}\mathbf{s}' \wedge \boldsymbol{O}\mathbf{s})) \supset \oplus(\boldsymbol{O}\mathbf{s} \supset \boldsymbol{O}\mathbf{s}'))$$

If we use **w** and **t** as world and time variables we can define $A\mathbf{w}$ as $W\mathbf{w} \wedge \boldsymbol{O}\mathbf{w}$, and $N\mathbf{t}$ as $T\mathbf{t} \wedge \boldsymbol{O}\mathbf{t}$. Our languages are in fact weaker than Plantinga's since the

only 'states of affairs' they quantify over are worlds and times. In appendix 3 we make some remarks about a formal modelling for Plantinga's metaphysics.

Plantinga's theory is the kind that Lewis 1986a calls 'magical ersatzism'. In his treatment of all the versions of ersatzism that he mentions, Lewis is scrupulous in discussing "the versions" rather than "authors" (p. 144) but, given our use of Plantinga as a metaphysician whose views on worlds carry over exactly to times, we can look at how Lewis's comments might apply when they do so. Lewis characterises ersatzism as the view that worlds are abstract entities which (p. 137) *represent* concrete worlds, and which (p. 138) exist as (abstract) parts of the actual (concrete) world. One of these abstract worlds is 'selected' by the concrete world. Lewis poses a challenge for the magical ersatzist. (p. 176) The selection relation must be either internal or external. To be internal would be to be like truth in a world — predictable from the nature of worlds and propositions themselves. So Plantinga must clearly suppose it to be external. Lewis takes selection to be a *relation* between an abstract world and the concrete actual world, but of course for Plantinga, whether a state of affairs obtains or not is simply a fact. There is no thing which *does* the selecting. Lewis's complaints about this external relation (p. 180) seem to suggest that he thinks that, because obtaining is an external relation, an explanation is owed of why a donkey talks iff the state of affairs of there being a talking donkey obtains. The redundancy theory of obtaining goes like this: What is it for the state of affairs of a donkey's talking to obtain? It's for a donkey to be talking.[2] As we stressed in Chapter 4 actual truth for Plantinga, as for Adams, is a primitive notion unrelativised to a world, and certainly for Prior, and possibly for Adams and Plantinga, this primitive truth is present truth.

Imagine a debate between two theorists of time about whether times are abstract or concrete. Could this debate take place between two eternalists? The answer we believe is yes. We see no reason why an eternalist should not believe in the entities of mathematics. Quine certainly does. (See, for instance, Quine 1953a, p. 122, or 1960, p. 267.) On the other hand there are eternalists (see for instance Sider 2001, p. 110f) who are suspicious of times as abstract entities.[3] On the presentist side, Prior (1967, p. 189) certainly objected to instants on the

[2]There is a sense in which Lewis's own notion of truth is absolute but not world relative. That is because he regards truths in a world as (absolute) truths about world-bound things. But, unlike Plantinga's, Lewis's notion of absolute truth is not a notion of absolute *contingent* truth, and depends on his idiosyncratic attitude to individuals.

[3]Dyke 2002b argues for a version of eternalism (she calls it a version of 'B-theory') in which truth applies to tokens, so as to avoid speaking of truth at a time. (She is principally concerned to argue against a date-based semantics.)

ground of their abstractness. The most that could be said to link these two debates would be that a presentist could not allow times to be concrete if that were interpreted to mean that other times *are now* concrete. And an actualist could not allow worlds to be concrete if that were interpreted to mean that other worlds *are actually* concrete.

So much for the question of what worlds and times *are*. But perhaps the status of worlds and times themselves is not the important question. Stalnaker (2003, p. 121) is bothered about the status of non-actual individuals, and Prior was worried about individuals which once existed or will exist but do not exist at present. There is in fact something odd about the claim that past or possible *individuals* might be abstract. Is it plausible to say that Socrates is now abstract? And what of Sherlock Holmes who never existed? How could a sentence like

(1) Holmes is a flesh and blood person

even be *possible* of an abstract object? Yet (1) is true in the stories. For this reason we cannot suppose that Holmes is an object which is essentially abstract, since in every world in which he exists he is concrete. If then we are to apply the doctrine of abstract objects to non-existent objects like Holmes it would seem that we must allow that an object which is abstract in one world is concrete in another — that Holmes is abstract in our world, but concrete in worlds in which he exists. In fact a line like this is taken in Linsky and Zalta 1994. They argue that objects like Holmes *exist* in all worlds, but are only concrete in some of them. This account easily carries over to the temporal case, and may therefore be of assistance to presentists. What it means is that Socrates still *exists* but he is no longer concrete. Prior 1957, p. 30, says that for a thing to exist is for there to be facts about it. But he is then, on p. 34, in difficulty about whether it is possible to speak of Bucephalus. On the Linsky/Zalta account we are able to speak of Bucephalus since he is an abstract object. When we ordinarily say that he no longer exists what we really mean, on the Linsky/Zalta approach, is that he is no longer concrete. Similarly of course in the modal case — and it is indeed the modal case that interests Linsky and Zalta. One might feel that all this is no more than terminology. A possibilist/eternalist says that we may quantify over things which don't exist, and treats existence as a robust predicate. Linsky and Zalta want to keep *exists* to mark the domain of quantification, but then they need another word to mark the robust sense of existence used by the possibilist/eternalist. So they use the abstract/concrete distinction. Terminology is not what is important here. What is important is that the domain of quantification is restricted by a predicate. Some call this *exists*, others use

*concrete*.

Is there any reason why a possibilist/eternalist cannot accept the abstractionist construction of 'non-existent entities'? Linsky and Zalta explicitly acknowledge that their view is non-committal on the actualism/presentism vs possibilism/eternalism debate. Let's take Socrates. On the possibilist/eternalist way of thinking he is always available for quantification, but only *exists* at certain times in certain worlds. So what of a possibilist/eternalist who is perfectly happy to put the very same situation by saying that, in the sense of 'exists' which indicates no more than availability for quantification, everything, including Socrates, necessarily always exists, but only in the sense in which abstract objects are necessary existents. Unlike the more 'obvious' abstract entities, like the number 17, or perhaps a proposition, some, like Socrates, are only contingently abstract and become concrete at certain times and in certain worlds. We colloquially call this 'existence' and so we say that

(2) Socrates exists

is true at *t* in *w* iff the abstract object Socrates is concrete at *t* in *w*. Nothing in this procedure pre-judges the question of whether there is a privileged actual world or present time, or is incompatible with the indexical theory of actuality. In fact, this way of looking at things fits well with Plantinga's metaphysics. When we look at Plantinga's paper we see that Plantinga's real worry is that he believes that

> there neither are nor could have been things that do not exist. (p. 257).

What does he mean? Well, you might suppose that it is contradictory to say that there *are* things which do not exist, since 'there are' might be held to imply actual present existence. But Plantinga appears to think that *there could not have been* things which do not exist. There is of course an innocuous sense in which that is true if you read 'there are' to imply existence. For all it says is

(3) $\sim M\exists x(\sim x \textbf{\textit{ exists}} \wedge x \textbf{\textit{ exists}})$

And if the quantifier $\exists$ is read as existence entailing we also have

(4) $\sim\exists x(\sim x \textbf{\textit{ exists}} \wedge M\, x \textbf{\textit{ exists}})$.

But in fact when Plantinga talks about actualism it seems that he thinks that

*genuine* existential quantification can only *ever* apply to things in the actual world, where actuality is a metaphysical absolute, and not to be understood indexically. So that even

(5) $M\exists x$ ***purple cow*** $x$

would seem to be false ((5) is Plantinga's sentence (1) on p. 269, except that we have used $M$ in place of his 'possibly' and ***purple cow*** $x$ in place of his '$x$ is a purple cow.') Plantinga is aware that (5) has to be given a true interpretation, as does

(6) Possibly there is an object distinct from each object that exists in @.

(6) is Plantinga's (4) on p. 256 (but with @ for Plantinga's α). In the case of worlds Plantinga introduced maximal states of affairs where states of affairs are taken as primitive. On p. 259 he also takes properties as primitive, eschewing a set-theoretical reduction for two reasons. One is to avoid the problem of the identity of necessarily equivalent properties. That problem raises issues of a complex kind which are distinct from the topic of the present book. (See p. 33 above.) The other reason is to do with the *existence* of properties. Given Plantinga's view that existence is restricted to actual existence, and given the view that no set-theoretical construction made up from things which do not exist can itself exist, it would follow that most of the properties we need will not exist in set theory, since they will have extensions in other worlds which contain things which do not exist in the actual world. So properties are primitive abstract entities which exist necessarily. (p. 262.) But although a property necessarily exists nothing follows about whether it is instantiated or *exemplified*. Among properties are those Plantinga calls *essences*. (p. 262.) The property of being Plato is an essence, and therefore necessarily exists. But it is only exemplified in some worlds and at some times. Plato himself may be said to exist at those times and in those worlds in which his essence is exemplified. In fact our ordinary word 'exists' is a predicate in which $x$ ***exists*** is true in a world $w$ at a time $t$ iff the essence which is the value of the variable $x$ is exemplified at $t$ in $w$. How then are we to treat the kind of sentence discussed in Chapter 6:

(7) $\exists x($***famous*** $x \wedge P($***philosopher*** $x \wedge$ ***drink hemlock*** $x))$?

From some remarks on p. 269 it seems that Plantinga interprets $\exists x\alpha$ as requiring

that $x$ currently exist. That may be sensible if $x$ is an individual, but Plantinga's quantifiers range over essences, and on p. 268 he suggests that in that case different domains for different worlds "lose much of their significance." We shall describe a modification of Plantinga's view which keeps his metaphysics and keeps the spirit of his actualism while removing a minor anomaly which is a hangover from the 'standard' supposition that quantifiers range over concrete things rather than essences. The anomaly concerns Plantinga's discussion of what is in effect the Barcan Formula. On p. 269 Plantinga reads (5) as being true iff there is a world in which an essence $x$ is co-exemplified with the property of being a purple cow. But he reads

(8) $\exists x\, M\, \boldsymbol{purple\ cow}\ x$

as meaning that some essence $x$ is "co-exemplified with the property of possibly being a purple cow." Read that way (8) is indeed false if nothing which actually exists (i.e., no essence which is actually exemplified) could be a purple cow. And it could still be false even if it were possible for some essence which is not in fact exemplified to be co-exemplified with the property of being a purple cow, so that (8) could be false even if (5) is true, and so (8) does not follow from (5). But this is not the only way of reading (8), and, we suggest, it is not the way Plantinga should read it. To see why look at the embedded wff:

(9) $\boldsymbol{purple\ cow}\ x$

Now $x$ (strictly the entity assigned to the variable $x$) is an essence and (9) does not say that the essence is a purple cow — (9) says that the property of being a purple cow is co-exemplified with $x$. So in (8) the co-exemplification is within the scope of $M$, and (8) should be read:

(10) there is an essence $x$ such that it is possible that $x$ and the property of being a purple cow are co-exemplified.

And (10) appears to say the same thing as (5), since in Plantinga's metaphysics the *existence* of an essence and its exemplification are quite different things. (8) should not be confused with

(11) $\exists x(\boldsymbol{exists}\ x \wedge M\, \boldsymbol{purple\ cow}\ x)$

where $\boldsymbol{exists}\ x$ is taken to mean that $x$ is exemplified. (11) is certainly false, but

that is because ***exists*** is outside the scope of $M$, and therefore $x$ is required to be actually exemplified. In (8), by contrast, exemplification (as opposed to existence) only occurs inside the scope of $M$. What then of (7)? (7) appears to claim that there is an essence $x$ which is now co-exemplified with the property of being famous, and since ***famous*** has to be outside the scope of $P$ it would seem that $x$ must now exist. But here we see the beauty of Plantinga's theory. For, although it may look as if the co-exemplification of a property and an essence demands the exemplification of that essence, there is no reason why this must be so. On Plantinga's semantics one cannot analyse '$x$ and the property of being $F$ are co-exemplified' as

(12) $\exists y$($y$ exemplifies $x$ and $Fy$).

Here is why.[4] It is essential to Plantinga's project that an essence which is not exemplified in the actual world might be exemplified in another possible world. If (12) were to analyse such a possibility then (12) would have to be true in a non-actual world. But in (12) the $\exists$ cannot range over essences. It must express genuine *individual* quantification. But then it cannot apply to things which exist only in other possible worlds — for Plantinga there are no such things. It is the avoidance of this sort of quantification which provides the whole rationale for Plantinga's enterprise. And when we see that co-exemplification should not be analysed in terms of individual quantification we can leave it up to the meaning of the predicate to provide the criteria. It is part of the *meaning* of ***famous*** to determine what is to count as co-exemplification with it. For most affirmative predicates the co-exemplification of $x$ with $F$ at $t$ in $w$ requires that $x$ itself is exemplified at $t$ in $w$. This is why there cannot be a non-existent purple cow. So ***exists*** in

(13) $\exists x$(***exists*** $x \wedge$ ***purple cow*** $x$)

is redundant, and (13) is equivalent to

(14) $\exists x$ ***purple cow*** $x$.

(14) claims that there is an essence $x$ which is co-exemplified with the property

[4]This seems the thrust of the comments on p. 337 of Plantinga 1985, where he claims that the fact that a haecceity (essence) is the thisness of a particular object cannot be essential to that object. This seems to be his way of saying that facts about essences should not be analysed in terms of facts about the things whose essences they are.

of being a purple cow, and it is a part of the nature of being a purple cow that an essence can only be co-exemplified with it at *t* in *w* if *x* is itself exemplified at *t* in *w*. While the co-exemplification of *x* at *t* in *w* with the property of being famous at *t* in *w* probably requires that *x* be exemplified in *w* at some *t′* earlier than *t* it need not require that *x* be exemplified in *w at t*. Co-exemplification may be a bit of a misnomer here, since the co-exemplification of a property and an essence suggests a presupposition of a thing which does the exemplifying. Chapter 5 of Fine 2005 (especially pp. 179-186) contains a strong criticism of Plantinga's metaphysics based on the claim that Socrates' essence presupposes Socrates.[5] This is the doctrine that Plantinga (1985 pp. 93 and 324) calls 'existentialism', and if it is accepted then indeed Plantinga's metaphysics will not solve the problem of possibilia without already admitting them. Plantinga rejects such existentialism, and the best way to take his metaphysics is to refuse to admit such entities as a physical Socrates into your domain of quantification. Where we ordinarily speak of physical things we should more properly speak of the exemplification of essences and properties. Exemplification and co-exemplification would be primitive second-order features of essences and properties. If asked what it is for Socrates' essence to be exemplified we point to Socrates and say 'that's what it is'. But what we are pointing to is not a *thing* but the exemplification of an essence. The metaphysical question then turns on whether exemplification involves a primitive notion of contingent truth; and whether it involves a primitive notion of tensed truth. Plantinga certainly takes it that exemplification is contingent, and, although he is mostly silent on the question of tense, nothing he says prevents exemplification from being tensed, or impugns the world-time parallel.

It might be instructive to compare two attitudes to ontology. Recall the opening sentence of Lewis 1986a: "The world we live in is a very inclusive thing." Contrast this with a rather different attitude — one which considers that metaphysical insight is best gained by looking at *truth* rather than things. The second sentence of Wittgenstein 1921 reads: "The world is the totality of facts not of things." What Plantinga's metaphysics does is ensure that the domain of

[5]On p. 181f Fine uses the analogy of a 'Platonic idealist' who proposes the elimination of material things by deriving them from their singletons. Obviously if like Fine (and perhaps set theory itself) you take a singleton to presuppose the entity of which it is the singleton then indeed you are stuck in a vicious circularity. On p. 216 Fine points out that such things as essences could not possibly be people, so that the essence of a non-existent person never could *be* a person. Plantinga of course has to say that it is not so that the essence could ever *be* a person, merely that it could be *co-exemplified* with the property of being a person.

quantification consists entirely of abstract objects.[6] Where we used to speak of a concrete object as if it were in a domain, we now speak of the exemplification of an essence. From Plantinga's point of view this is all to the good. Domains are part of semantics and maybe it is ludicrous to suppose them populated by concrete individuals — particularly if you want to talk about the non-existent. By populating them instead with necessarily existing abstract entities we arrive at exactly the same semantic structures used in earlier chapters in quantifying over erstwhilia and possibilia. On Plantinga's metaphysics you don't need an additional non-abstract *thing* to give you genuine existence. All you need is the *fact* — which may be contingent and tensed — that an essence is exemplified. Admittedly, Plantinga, on this construal, might have to take being an essence as primitive, and others have echoed Fine's objections.[7] but one might argue that every theory has to have its primitives, and with essences as primitive Plantinga can avoid taking ordinary objects as primitive. Even if essences cannot be defined they do satisfy certain principles that mark them off from the range of all properties. One such principle is that co-exemplified essences are identical:

(15) For any essences $E_1$ and $E_2$, if $E_1$ and $E_2$ are co-exemplified then $E_1 = E_2$.

A consequence of (15) is

(16) For any essences $E_1$ and $E_2$, if $E_1$ and $E_2$ are co-exemplified then where $P$ is any property it is logically necessary that if $E_1$ and $P$ are co-exemplified then $E_2$ and $P$ are co-exemplified.

It is of course no part of our book to take sides on whether Plantinga has produced a viable metaphysics. It is enough that it is a version of actualism sufficiently precisely formulated to allow a comparison with the temporal situation; and we have seen that Plantinga's treatment of non-existent entities in terms of the exemplification os properties and essences can apply in the temporal

[6]Curiously Quine 1976c, p. 503, also suggests that the ontology of the physical universe might consist solely of abstract objects, though not for Plantinga's reasons.

[7]The issue is briefly canvassed on pp. 117-120 of Chihara 1998. Menzel 1991 considers a theological motivation for a Plantinga-like metaphysics. If you follow Aquinas in thinking of essences as 'ideas in the mind of God', you can permit God to decide to actualize an essence without the antecedent presupposition of the existence of a possible entity waiting to be created. Creation *ex nihilo* is therefore compatible with foreknowledge of things which do not currently exist.

case exactly as it applies in the modal case.

# 17

# *Supervenience*

One way of objecting to possible worlds is that we should only believe in things which are part of the causal order. Such objections are brought by Stalnaker 1984, p.49, and McGinn 1981, pp.153-158. But what is causation? Suppose we take the analysis in Lewis 1973b. For Lewis, causation depends on a prior notion of *counterfactual dependence*, where a proposition (set of worlds) *b* counterfactually depends on a proposition *a* in a world *w* iff *a* and *b* are true in *w*, and there is a world in which *a* and *b* are both false, which is closer to *w* than any world in which *a* is false and *b* is true.[1] So if *that* notion is involved in causation, the very description of a world in causal terms brings in reference to other worlds. On the other hand, if these other worlds are not required for ontology, then causation also is not required for ontology, and the claim that other worlds and their contents are not real because they are not involved in causation collapses. One form of the causation objection to possible worlds seems to assume that causation is a relation between *things*, and physical things at that — so that if you claim that worlds are abstract, you may want to deny their involvement in causation on the ground that abstract entities cannot enter into causal relations.[2] But it is not at all clear that this (if true) marks a serious

[1]Where *a* is the proposition expressed by $\alpha$, and *b* is the proposition expressed by $\beta$, this amounts to the claim that $\alpha$ and $\beta$ and $\sim\alpha \mathrel{\Box\!\!\rightarrow} \sim\beta$ are all true in *w*.

[2]That abstract entities are not involved in causation appears a widely held view. See for instance Lewis 1986a, pp. 109 and 179, Divers 2002, p. 133, Davies 1983, p. 132. See also the issues raised by Benacerraf's work in the philosophy of mathematics in, among others, Benacerraf 1973, pp. 671 and 673, Maddy 1996, p. 62, Wagner 1996, p. 74 and Stalnaker 1996, p. 104. There have been those who have argued that numbers are not necessary for science. See Field 1980. If Field is right then at least times and worlds are no worse off. And in any case such arguments do not bear on their comparative reality. Mellor (1995, p. 162) says that causation primarily relates what he calls *facta* and derivatively events. He uses this term because although *facta* have many features in common with facts they have important differences. In lots of ways one can think of them as propositions, but we do not wish to impose on Mellor a construal which he might reject. Mellor's account of causation is that the claim that *a* causes *b* is the claim that the truth of *a* (or as Mellor puts it the fact that *a*) raises the probability of *b*. The claim that *b* is probable is a modal claim, in the sense that what is probably so may in fact not be actually so. Mellor has in fact argued

ontological difference between worlds and times.

The argument that worlds and times have a different status claims that while a past event can cause a present event, a merely possible event cannot cause an actual event. At first sight this may seem obvious, but a closer examination shows that matter may not be quite so clear. Consider a temporal example:

(1) The soil is damp today because it rained yesterday

If (1) is true then its two component sentences

(2) The soil is damp today

and

(3) It rained yesterday

must both be true today, and in the case of (3) its truth today is equivalent to the truth yesterday of

(4) It is raining.

So much is straightforward, but if that is what is happening in the temporal case it is not hard to present a modal parallel

(5) The earthquake caused it to be possible to build a cricket ground instead of a shipping basin.

The truth of (5) requires the truth of

(6) The earthquake occurred

and

(7) It was possible to build a cricket ground instead of a shipping basin

and while indeed a cricket ground was built, the truth of (5) does not entail that

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elsewhere (Mellor 1987) that causation is an intentional notion. Whether or not these accounts are along the right lines it is at least not obvious that worlds are not involved in the causal order.

a cricket ground was in fact built.[3] To be sure this possibility is more than a mere *logical* possibility, and it is indeed true that statements of *logical* possibility and logical necessity are non-contingent, and so not the kind of propositions which can enter into informative causal statements. But that is only to be expected, and does not alter the fact that the modal propositions involved in other senses of possibility and necessity may well be contingent and, as in (5), may be components in meaningful causal statements.

But, even if this is so, someone may object that it is some underlying non-modal fact which provides the genuine cause. Indeed it may be said that ultimately all metaphysically genuine facts are non-modal. Take for instance:

(8) Tallulah would have been in College Station on Thursday 8 September 2005 if her car had not broken down on Wednesday 7 September 2005.

(8) has the form

(9) $\alpha \mathrel{\Box\!\!\rightarrow} \beta$

where $\alpha$ and $\beta$ are respectively

(10) Tallulah's car does not break down on Wednesday 7 September 2005

and

(11) Tallulah is in College Station on Thursday 8 September 2005.

Recall the discussion of $\Box\!\!\rightarrow$ on p. 65. Where only eternal sentences like (10) and (11) are involved the semantics of $\Box\!\!\rightarrow$ may be expressed as:

(12) $\alpha \mathrel{\Box\!\!\rightarrow} \beta$ is true at a world $w_1$ iff there is a world $w_2$ at which $\alpha$ and $\beta$ are both true, which is closer to $w_1$ than any world $w_3$ at which $\alpha$ is true but $\beta$ is false.

But if we are given all the non-modal facts about what happens in $w_1$ we are

[3]That (7) states an *effect* is immaterial. Consider: 'The water ran down the channel because it could do so.'

given, it is said, all we could possibly need for the truth of (8). It would not be possible to have a world in which everything happened just as in $w_1$, but in which (8) is false.

What should be said about this reductionist argument — that modal facts are not required because they are determined by non-modal facts? It is instructive to contrast it with the argument we have discussed in previous chapters, that tensed facts are not required, because they are determined by untensed facts. How this can be in a tenseless world is part of the explanatory point of the indexical semantics. We can summarise the debate between presentism and eternalism, and between actualism and possibilism, in the following way: A presentist takes as basic a primitive and unanalysed notion of truth, according to which truth, plain and simple, and present truth are identified. By contrast, an eternalist claims that the basic primitive is truth at a time. In the modal case an actualist holds that simple truth is identical with actual truth. A possibilist will say that the basic notion is truth (at a time) in a world, and that the reason we don't need an operator for 'it is actually the case that' is that 'it is actually the case that $\alpha$' is true (at $t$) in $w$ iff $\alpha$ itself is true (at $t$) in $w$.[4] Where an eternalist can treat a tensed sentence as one which can change its truth value from time to time, a possibilist can admit a sentence $\alpha$ which may be true (at $t$) in $w_1$, but false (at $t$) in $w_2$, i.e., which can change its truth value from world to world. Such a sentence may be called contingent, even though its truth or falsity in $w_1$ or $w_2$ is non-contingent, in exactly the same way as its truth or falsity at $t_1$ or $t_2$ is tenseless.

To put the matter briefly, eternalism reduces the presentist's primitive notion of (tensed) present truth to a tenseless notion of truth at a time, while possibilism reduces the actualist's primitive notion of (contingent) actual truth to a non-contingent notion of truth in a world. So the modal parallel of a reduction of the tensed to the tenseless would be a reduction of the contingent to the non-contingent, of the kind mentioned on pp. 41 and 50. The reason we have repeated these points here is because, while questions about the reduction of the tensed to the tenseless have been discussed in the manner we have just described, the parallel modal case has been supposed to be, not a reduction of the contingent to the non-contingent, but rather a reduction of the modal to the non-modal, or, as Melia 2003, p. 1 puts it, to the 'categorical'. Whatever one may say about the standard attempts to reduce the modal to the non-modal, one thing they have in common is that, except for Lewis's modal realism, they do not attempt to reduce

[4]Bear in mind the more complex uses of 'actually' which motivated the semantics introduced in chapters 7 and 8. These uses are irrelevant to the present topic, as are the analogous uses of 'now'.

the contingent to the non-contingent. Typically, they attempt to 'reduce' modal properties — like say fragility — to non-modal properties — like say having a certain physical structure.[5] And whether something has a certain physical structure is a contingent fact which is supposed to provide the analysis of some other contingent fact. It would be as if we were using one tensed fact to provide the analysis of another tensed fact.

Still the question of whether, in this sense, the modal *is* reducible to the non-modal might still be held to have implications for the world-time parallel. For there is an argument which claims that, unlike the situation in the modal case, in which a reduction of the modal to the non-modal may seem plausible, many temporal facts require facts about other times. That is, it might be held that

(13) The present could be exactly as it is even if the past or future were different

but

(14) The actual could not be exactly as it is if what might have been were different.

Look at a temporal example:

(15) It rained yesterday in College Station.

(15) is true at a time $t_1$ because there was rain in College Station on the day $t_2$ preceding $t_1$. If there had not been (15) would be false at $t_1$. Further, this depends essentially on what happened at $t_2$. However full a description is given of what happened at $t_1$, it will not determine, it is claimed, the truth of (15). To account for the truth of (15) we need other times. This appears to contrast with the modal case, where it seemed that we could not be given a full description of what happens in $w_1$ without it thereby being determined whether (8) is true or false there.

The principal problem in addressing the purported difference between (13) and (14) is to articulate what they mean. For if it is true that yesterday was raining, it is a present truth — a truth today — that it rained yesterday. And lest

[5]The argument that the modal must be based on the actual is at the heart of Mondadori and Morton 1976. In addition to Mondadori and Morton see also Sider, 2001, p. 41, and 2003. Tooley 1997, p. 266, expresses the common demand that modal truth demands a 'categorical base'. These issues are discussed further in Rini and Cresswell 2009.

this should be thought a presentist way of putting it, an eternalist could put it that it is a truth about Tuesday that there is rain on Monday — or perhaps that there is rain on the day preceding the day of utterance. To sustain a meaningful (13) one has to give a clear sense to 'the present' which does not include all present truths. That is to say, among truths we must be able to form a subclass of which we can say that they together constitute what is now taking place; and it is by no means obvious that this is possible. There have certainly been philosophers who have maintained that the past exists only in so far as it has left present traces,[6] and even ordinary facts about the present are not obviously ones which are independent of other times. In order to see the importance of this we shall set out a mini-dialogue between two presentists:

**P1** We both agree that only what is present is real, and we both agree that there are (present) truths about the past, and (perhaps) about the future. But I maintain that there is a sub-class of truths which are *purely* about the present, and are made so by facts which exist *now*. Other present truths, by contrast, are present truths about the past — they are made so because certain facts once existed, but do so no longer.

**P2** I'm glad you agree that only the present exists, but I'm puzzled why you should suppose that a truth about the past is made so by something which no longer exists. If something was once the case, we presentists should agree that there must be a present fact which makes it so, and any distinction between facts solely about the present, and facts about the past or future, is unsustainable.

**P1** Perhaps I was hasty in talking about facts, since they are entities about which there is metaphysical disagreement. So let me give you a simple example of a statement about the present:

(16) There is a sideboard to the left of the dining room door at 57 Stanley Ave, Palmerston North, New Zealand.

If I say this at 12.20pm NZST on 4 August 2006, then there is a sense, loved by eternalists, in which I could have equally said,

[6]See for instance Prior 1967, p. 28, quoting p. 38 f of Łukasiewicz 1922. Bigelow 1996 puts the matter by saying that the presently existing world has a certain property, like say having once contained a now famous philosopher who drank hemlock. See also Smith 1993, pp. 160-163, and Hinchliff 1996, pp. 123 f.

(17) There is a sideboard to the left of the dining room door at 57 Stanley Ave, Palmerston North, New Zealand at 12.20pm NZST on 4 August 2006

but whether we are talking of (16) or (17), given the present tense of 'is', they are solely about a single time.

**P2** I'm not going to quibble with you about the difference between (16) and (17). We can ignore the dispute between us and the eternalists here, and I'm even going to overlook your use of truth at a time, since we presentists know how to analyse that. But I have worries about (17). What if the 'sideboard' only came into existence a few seconds before 12.20, and disappeared a few seconds after 12.20? Although a world in which this happened might coincide with ours at 12.20, (17) could, with reason, be held to be false there, on the ground that this would not count as a truth about a *sideboard.* A sideboard, it could be said, is the kind of thing which must have a continuing existence; and so the truth, even of (17), involves other times. More generally, causal facts cannot be about a single time. Causation can relate an event which happens at one time with another event which happens at another time. When we look at a far off galaxy though a telescope we do not see it as it is *now*, but as it was many years ago.

**P1** You may be right about statements like (17), but that is because such statements are logical constructions out of more basic primitive statements which genuinely *are* solely about a single time.

**P2** Now I'm even more puzzled. (17) was introduced to avoid presupposing any particular metaphysics; it was supposed to be an everyday example of things ordinary people might wish to say. You now seem to be telling me that you have a metaphysical theory which protects you from objections based on ordinary non-philosophical sentences, about whose truth or falsity we can agree.

We shall not continue this dialogue, since it is not our aim to take sides on the question of whether facts about the past or future can be reduced to facts about the present. Our purpose is to point out that this is a debate *within* presentism, and tells us *nothing* about the dispute between presentism and eternalism, i.e., nothing about whether reality is tensed or tenseless. Indeed the whole debate

could be re-written as a debate between two eternalists, E1 and E2, in which E1 maintains that all facts depend on basic metaphysical facts which group themselves into distinct equivalence classes in such a way that each class represents everything that is solely about a particular time, and is independent of what occurs at any other time; while E2 maintains that no such distinction is possible.

Whatever may be said about the dialogue between P1 and P2, and about its modal counterpart, the issues they raise really do seem to depend on some kind of metaphysics in which there is a collection of simple propositions, which determine how things are — a metaphysics in the style of Russell and Wittgenstein, in which what is the case is a maximal set of mutually independent atomic facts.[7] We may take the atomic facts to be tensed or tenseless, and as contingent or non-contingent. The idea of a metaphysics of non-contingent facts may have seemed strange to these 'logical atomists', yet it is the view whereby the metaphysical basis of truth simpliciter is truth in a world, which is non-contingent in the same way as the tenseless view of time bases present truth on tenseless truth at a time. It is fairly clear that Wittgenstein supposed that atomic facts are contingent. Russell too had supposed it to be a contingent matter what facts there are.[8] But Russell certainly,[9] and Wittgenstein probably, seem to have supposed that atomic facts are tenseless. If the atomic facts are contingent but tenseless then, given primitive modality, you can define a possible world as a maximal collection of atomic facts, but you cannot define a time in an analogous way. Of course, if times are constituents of the atomic facts — as it might be that

[7]A simple model along these lines is the Tractatus-like model assumed in Cresswell 1972 and 1973. Wittgenstein's purpose in the *Tractatus Logico-Philosophicus* (Wittgenstein 1921) was primarily metaphysical. As such the use of his work to present a possible world as a collection of atomic facts may be historically inaccurate — though perhaps no more inaccurate than any view about what worlds really are.

[8]It is plausible to take facts as true propositions, but that cannot be the view of the Russell of Russell 1918 (see especially p.70). Russell is clear that while facts must appear in a catalogue of what there is propositions do not. Wittgenstein certainly speaks of a world's being constituted by the existence of facts (Wittgenstein 1921, §2 and 2.04). But he is clear that properties and relations hold between 'objects', and there is some reason to suppose that these 'objects' are necessarily existing simples. (See Wittgenstein 1921, 2.024-2.0271.) Part of Wittgenstein's reason seems to be to avoid the need for what Russell (1918, p. 37f) called a 'general fact' which says that the atomic facts listed are *all* the facts. Each simple object marks a position in logical space, and it is not a contingent matter how many of them there are. (We are not of course claiming to be giving an authoritative exegesis of these notoriously difficult authors.)

[9]See Russell 1915. Time gets little mention in the lectures on logical atomism (Russell 1918) and in fact most of Russell's examples are tensed. But his remarks on p. 88f of Russell 1918 make it pretty clear that his views on time are still those of the 1915 paper.

a particular point of logical space is filled in a certain way at a certain time; or perhaps that the points themselves are spacetime points, which are (tenselessly) filled in a certain way — then indeed, times and worlds might be said to have a different status. However, all that that shows is that any metaphysical difference between times and worlds arises in this form of atomism if it is built into its formulation. We have seen in previous chapters that there are no semantic or logical arguments which force you either way on the metaphysics of times and worlds, so that of course there is nothing to prevent a metaphysics whose atomic facts are contingent but tenseless; but nothing either to prevent a metaphysics based on tensed atomic facts. We would however make the following observation. In divorcing metaphysics from semantics in this way there is a danger of isolating it from other theoretical constraints, and thereby preserving its independence at the cost of its relevance. This is particularly so given that the quantification involved in ordinary existential sentences like (17) will almost certainly not appear at the level of atomic facts. In Chapter 9 we argued that temporal and modal discourse both have quantificational structure. They are thus on a par with (17), and if it is true that worlds do not enter as constituents of atomic facts neither do things like sideboards, whose existence most of us acknowledge to be unproblematic.

So it seems that the debate about (13) and (14) has turned into the kind of debate that took place in the early years of the 20th century, provoked by the metaphysics of logical atomism, about whether reality is constituted by atomic facts. We have stressed that that debate is independent of the debate between actualists and possibilists; and in fact it does not even seem to be a debate with any significant consequences for the foundations of modality. Imagine the following dialogue between two possibilists:

**M1** We both agree that the basic notion of truth is truth in a world, and we accept the indexical theory of actuality, according to which anyone who claims in a world $w$ that a sentence $\alpha$ is (actually) true simpliciter is making a claim which is to count as true iff $\alpha$ is true in $w$, and we agree that this is to be understood as a reduction of contingent truth to non-contingent truth-in-a-world, in just the same way as an eternalist reduces tensed truth to tenseless truth-at-a-time. However we differ between ourselves. I follow David Lewis (Lewis, 1986b, p.ix) and hold that a world is constituted by "local matters of particular fact". What that means, for me, is that all modal truths in a world are determined by (or supervene on if you prefer that language) non-modal facts.

**M2** I'm surprised that you call yourself a follower of David Lewis. I can see how the logical modalities might be so determined, though it is in a somewhat trivial sense. For if something is logically possible, or logically necessary, then that fact, that it is so possible or necessary, does not itself change from world to world.[10] But there are many modal statements whose truth values do change from world to world. On Lewis's account the sentence (8) above has the form (9), and so, by (12), is true at a world $w_1$ iff there is a world $w_2$ at which $\alpha$ and $\beta$ are both true, which is closer to $w_1$ than any world $w_3$ at which $\alpha$ is true but $\beta$ is false. Doesn't the truth of (9) depend on a fact (about the nearness of worlds) which is additional to the particular facts you are so fond of?

**M1** Not at all. Nearness of worlds is a logical absolute in the sense that whether or not it is a fact that $w_2$ is closer to $w_1$ than $w_3$ is, does not itself change from world to world. The reason why (8) is contingent is that its value at different worlds depends on how near various other worlds are to the world in question. It is not as if the 'fact' that '$w_2$ is closer to $w_1$ than $w_3$ is' might be included in some worlds but not in others.

**M2** What you say may be correct, but you have given me no help in expressing sentences like (8) above. (8) appears to make a modal claim. I suppose that, on the Tractarian picture that you also seem fond of, (8) might be represented in a 'canonical' language which provides a metaphysically accurate description of the world, as an infinite disjunction of infinite conjunctions of atomic propositions or their negations; and none of these seems to have the modal structure of (8). But I would not even begin to know how to undertake a translation of (8) into such a language.

**M1** I agree that there is no hope of the kind of translation that you have in mind. But that is not necessary. Here's how it goes. We must distinguish two claims here, a *semantic* claim and a *metaphysical* claim. By not insisting that the nearness relation among worlds be *defined* in terms of

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[10]M2 is a little careless here. Although what is said is correct, it is only half the story. As noted in Chapter 3, there are sentences, like (2) on p, 39, which can change their truth value with the passage of time; but when they are true are necessarily true, and when they are false are necessarily false. So M2 the situation is a little more complex than assumed in Meyer 2006 p.28, who claims that while sentences of the form $\Box\varphi$ have an absolute truth value, sentences like always-$\varphi$ and sometimes-$\varphi$ do not. The fact is that $\Box\varphi$ can vary from time to time, but not from world to world; while always-$\varphi$ and sometimes-$\varphi$ can vary from world to world, but not from time to time.

the particular facts of the worlds in question, sentences like (8) can be accommodated without a reductive analysis *even when worlds are no more than* collections of matters of basic particular fact.[11] Given (12), (8) will be true in $w_1$ provided that (10) and (11) are true in a world $w_2$ closer to $w_1$ than any world $w_3$ in which (10) is true but (11) is not. This depends on two things. Firstly it depends on whether (10) and (11) are true or false in $w_2$ and $w_3$, and we are assuming that this depends only on the particular facts in those worlds. Secondly it depends on the comparative nearness of $w_2$ and $w_3$ from $w_1$. This fact may seem an additional fact to the matters of particular fact that constitute each world, and you are right to wonder whether or not, and if so how, it can be reduced to the particular facts. Curiously, it does not matter whether facts about nearness are so reducible, for whether or not, they cannot be extra facts about any particular world. What would it be for the truth of (8) at $w_1$ to depend on *more* than the matters of particular fact which hold at $w_1$? Well, there would have to be a $w_1$ and a $w_1$* which coincide on all matters of particular fact, but where (say) $w_2$ is closer to $w_1$ than $w_3$ is, but $w_2$ is not closer to $w_1$* than $w_3$ is. But if $w_1$ and $w_1$* coincide on matters of particular fact then, in the metaphysics I favour, they are identical, and so what is claimed cannot happen.

**M2** All the same, you still haven't told me what kind of non-modal facts your facts like (8) might depend on.

**M1** Well let me ask you something? What is *your* analysis of worlds?

**M2** I don't have one. I take them as primitive.

**M1** But then aren't you taking nearness of worlds as a modal primitive?

**M2** No more than you. Let me quote your own argument back at you. Nearness of worlds (for me as well as for you) is a logical absolute. As you said yourself, it does not matter whether or not you can give a reductive analysis of it. Your worlds are determined by matters of particular fact, because that is how you have defined them. I take worlds as primitive. They are what they are. Of course it is because each world

[11] Mondadori and Morton 1976 (p. 250) stress that modal facts may depend on non-modal facts without the necessity for a 'non-circular paraphrase'. The point is also made in Dyke 2007.

is what it is that makes it similar to this or that other world — but I see no reason for postulating any collection of atomic facts to *make* it the world it is. It just *is* the world it is. Analysis has to stop somewhere. But whichever of us is right about where to stop, the argument that Lewis's account of counterfactuals does not require a reductive analysis of nearness is the same for both of us.

**M1** I guess the authors of this book will have some ideas about where to stop. They will remind us that they have not intended to write a treatise on metaphysics, and it is perhaps appropriate that we should stop our discussion at this point. While there may be all kinds of interesting things to say about what kind of primitive facts might go to make up a world or a time, there is nothing in this question which gives any help in the debate between actualism and possibilism, or between presentism and eternalism.

**M2** In fact the only connection our dispute has with the world-time parallel is that it is analogous to the dispute between P1 and P2, or between E1 and E2, which presumably supports the view that such a parallel exists.

**M1** I agree. As far as our authors are concerned, all they have tried to do is examine arguments in the metaphysics of time and modality, to see whether there are any which reveal a serious ontological difference between times and worlds. They feel that their examination of the arguments has strongly suggested that if there *is* a difference in ontological status then it is difficult to see how logical argument could reveal it. If logical argument is unavailable then metaphysics is only position stating. And at least one of the authors is sympathetic to a view that would presumably have been held by the logical positivists, that if formal versions of apparently conflicting claims are intertranslatable, then there is a very real question whether these metaphysical claims have robust content — indeed whether perhaps they are even literally meaningless. But that of course goes beyond anything they argue for in this book.

**M2** You are right that this does seem a good place to end the book. I hope that it will be widely read.

**M1** I couldn't agree more.

# Appendices

# Appendix 1

## *The equivalence of $\mathfrak{L}_{multi}$, $\mathfrak{L}_{xtw}$ and $\mathfrak{L}_{i}$*

In this appendix we state and prove theorems which entail a semantic equivalence between the languages $\mathfrak{L}_{\text{multi}}$, $\mathfrak{L}_{x\mathbf{tw}}$ and $\mathfrak{L}_{i}$, as defined in Chapter 8 on pp. 98 and 100. In order to do this we require a formal definition of truth for these languages. It is safest to give this in the style of Chapter 5. Despite the difference between $\mathfrak{L}_{\text{multi}}$, $\mathfrak{L}_{x\mathbf{tw}}$ and $\mathfrak{L}_{i}$, a model for all these languages has the form $\langle W,R,T,<,K,C,D,V\rangle$ as defined on p. 67. The difference comes in how the truth values of wff are obtained. In $\mathfrak{L}_{\text{multi}}$ an assignment $\mu$ gives values (from D) only to the individual variables (since these are the only kind of variables in $\mathfrak{L}_{\text{multi}}$) but, in conjunction with V, determines a truth value $V^{\text{multi}}{}_{\mu}(\alpha)$ to a wff $\alpha$ of $\mathfrak{L}_{\text{multi}}$ which gives truth values relative to a pair $\langle u,s\rangle$ as described on pp. 90 and 92, where $u$ is a sequence of worlds (members of W) and $s$ a sequence of times (members of T). In $\mathfrak{L}_{x\mathbf{tw}}$ an assignment $\mu$ gives values from D to the individual variables, from W to the world variables, and from T to the temporal variables, but then $V_{\mu}$ just gives truth values relative to worlds and times in the ordinary way. Specifically it works like this. For $\mathfrak{L}_{\text{multi}}$, where $u$ is a sequence of worlds, $s$ a sequence of times, and $\mu$ an assignment, $V^{\text{multi}}{}_{\mu}(\alpha,u,s)$ denotes the value of $\alpha$ determined by $\langle W,R,T,<,K,C,D,V\rangle$ with respect to $u$, $s$ and $\mu$. $V^{\text{multi}}{}_{\mu}(\alpha,u,s)$ is defined recursively as

| | |
|---|---|
| $[V^{\text{multi}}\varphi]$ | $V^{\text{multi}}{}_{\mu}(\varphi x_1 \ldots x_n,u,s) = 1$ if $\langle \mu(x_1), \ldots, \mu(x_n), u_0, s_0\rangle \in V(\varphi)$ and 0 otherwise. |
| $[V^{\text{multi}}{}_{\perp}]$ | $V^{\text{multi}}{}_{\mu}(\perp,u,s) = 0$ |
| $[V^{\text{multi}}\textbf{\textit{then}}]$ | $V^{\text{multi}}{}_{\mu}(\textbf{\textit{then}}_i,u,s) = 1$ if $s_0 = s_i$, and 0 otherwise. |
| $[V^{\text{multi}}\textbf{\textit{actual}}]$ | $V^{\text{multi}}{}_{\mu}(\textbf{\textit{actual}}_i,u,s) = 1$ if $u_0 = u_i$, and 0 otherwise. |
| $[V^{\text{multi}}{}_{\supset}]$ | For any wff $\alpha$ and $\beta$, $V^{\text{multi}}{}_{\mu}(\alpha \supset \beta,u,s) = 1$ if either $V^{\text{multi}}{}_{\mu}(\alpha,u,s) = 0$ or $V^{\text{multi}}{}_{\mu}(\beta,u,s) = 1$; otherwise $V^{\text{multi}}{}_{\mu}(\alpha \supset \beta,u,s) = 0$. |
| $[V^{\text{multi}}G]$ | For any wff $\alpha$ and for any $t \in T$, $V^{\text{multi}}{}_{\mu}(G\alpha,u,s) = 1$ if for every $s'$ just like $s$ except that $s_0 < s'_0$, $V^{\text{multi}}{}_{\mu}(\alpha,u,s') = 1$; otherwise $V^{\text{multi}}{}_{\mu}(G\alpha,u,s) = 0$. |

[$V^{multi}H$] For any wff $\alpha$ and for any $t \in T$, $V^{multi}_{\mu}(H\alpha,u,s) = 1$ if for every $s'$ just like $s$ except that $s'_0 < s_0$, $V^{multi}_{\mu}(\alpha,u,s') = 1$; otherwise $V^{multi}_{\mu}(H\alpha,u,s) = 0$.

[$V^{multi}L$] For any wff $\alpha$, $V^{multi}_{\mu}(L\alpha,u,s) = 1$ if for every $u'$ just like $u$ except that $u_0 R u'_0$, $V^{multi}_{\mu}(\alpha,u',s) = 1$; otherwise $V^{multi}_{\mu}(L\alpha,u,s) = 0$.

[$V^{multi}K$] For any wff $\alpha$, $V^{multi}_{\mu}(K\alpha,u,s) = 1$ if for every $u'$ just like $u$ and $s'$ just like $s$ except that $K(u_0,s_0,u'_0,s'_0)$, $V^{multi}_{\mu}(\alpha,u',s') = 1$; otherwise $V^{multi}_{\mu}(L\alpha,u,s) = 0$.

[$V^{multi}\forall$] $V^{multi}_{\mu}(\forall x\alpha,u,s) = 1$ if $V^{multi}_{\rho}(\alpha,u,s) = 1$ for every $x$-alternative $\rho$ of $\mu$, and 0 otherwise.

[$V^{multi}\Box\!\rightarrow$] $V_{\mu}(\alpha \Box\!\rightarrow \beta,u,s) = 1$ if there is some $u'$ such that $u$ and $u'$ coincide except at 0, and (i) $V_{\mu}(\alpha,u',s) = 1$ and $V_{\mu}(\beta,u',s) = 1$, and (ii) if there is any $u''$ coinciding with $u'$ except at 0, which is such that $V_{\mu}(\alpha,u,s'') = 1$ and $V_{\mu}(\beta,u,s'') = 0$, then $u'_0 <_{\langle u_0,s_0\rangle} u''_0$. Otherwise $V_{\mu}(\alpha \Box\!\rightarrow \beta,u,s) = 0$.

[$V^{multi}Ref$] $V^{multi}_{\mu}(Ref_i\alpha,u,s) = 1$ if $V^{multi}_{\mu}(\alpha,u,s[0/i]) = 1$, where $s[0/i]$ is $s$ with $s_0$ in place of $s_i$; and 0 otherwise.

[$V^{multi}Ref^{\mathbf{m}}$] $V^{multi}_{\mu}(Ref^{\mathbf{m}}_i\alpha,u,s) = 1$ if $V^{multi}_{\mu}(\alpha,u[0/i],s) = 1$, where $u[0/i]$ is $u$ with $u_0$ in place of $u_i$; and 0 otherwise.

For $\mathfrak{L}_{x\mathbf{tw}}$ the case is simpler and can follow the definition of $V_{\mu}$ on p. 67 with a very few additions, which all depend on the fact that $\mathfrak{L}_{x\mathbf{tw}}$ contains temporal and modal variables, and that $\mu$ now assigns values to the temporal and modal variables, so that $\mu(\mathbf{t}) \in T$ and $\mu(\mathbf{w}) \in W$. For that reason the rules stated on p. 67 need to be understood in such a way that $\mu$ is an extended assignment. The only new atomic wff have the form $N\mathbf{t}$ and $A\mathbf{w}$. The additional truth rules are therefore

[V$N$] $V_{\mu}(N\mathbf{t},w,t) = 1$ if $\mu(\mathbf{t}) = t$, and $V_{\mu}(N\mathbf{t},w,t) = 0$ otherwise.

[V$A$] $V_{\mu}(A\mathbf{w},w,t) = 1$ if $\mu(\mathbf{w}) = w$, and $V_{\mu}(A\mathbf{w},w,t) = 0$ otherwise.

[V$\forall^{\mathbf{t}}$] $V_{\mu}(\forall\mathbf{t}\alpha,w,t) = 1$ if $V_{\rho}(\alpha,w,t) = 1$ for every $\mathbf{t}$-alternative $\rho$ of $\mu$, and 0 otherwise.

[V$\forall^{\mathbf{w}}$] $V_{\mu}(\forall\mathbf{w}\alpha,w,t) = 1$ if $V_{\rho}(\alpha,w,t) = 1$ for every $\mathbf{w}$-alternative $\rho$ of $\mu$, and 0 otherwise.

The principal theorem to be proved is not unexpected. It makes sense, following Tarski 1936, to understand the truth of a formula with respect to an assignment to the variables in terms of the satisfaction of a wff by a sequence. The idea is that an assignment $\mu$ in $\mathfrak{L}_{x\mathbf{tw}}$ corresponds to a sequence $s$ of times when $\mu(\mathbf{t}_i) =$

$s_i$, and $\mu(\mathbf{w}_i) = u_i$. We can express this formally as follows. Where $\mu$ is an assignment to the individuals of D let $\mu^*$ be an extension of $\mu$ which coincides with $\mu$ on the individual variables, but in addition gives values from T to the temporal variables and values from W to the world variables. So $\mu$ is an assignment to $\mathfrak{L}_{multi}$ and $\mu^*$ is an assignment to $\mathfrak{L}_{x\mathbf{tw}}$. Then

THEOREM 8.1 For any wff $\alpha$ of $\mathfrak{L}_{x\mathbf{tw}}$, any sequence $u$ of worlds and $s$ of times, and any assignment $\mu$, $V_{\mu^*}(\alpha,u_0,s_0) = V^{multi}_{\mu}(\tau(\alpha),u,s)$, provided that for $1 \leq i$, $\mu^*(\mathbf{t}_i) = s_i$ and $\mu^*(\mathbf{w}_i) = u_i$.

Theorem 8.1 requires some lemmas. Assume that $\alpha$ is a wff of $\mathfrak{L}_{multi}$.

LEMMA 8.2 $V^{multi}_{\mu}(Ref_j \oplus Ref_i \textbf{\textit{then}}_j \alpha,u,s) = 1$ iff $V^{multi}_{\mu}(\alpha,u,s') = 1$ for every $s'$ which is just like $s$ except that $s'_i = t$ for some $t \in$ T, provided that $\textbf{\textit{then}}_j$ does not occur in $\alpha$.

LEMMA 8.3 $V^{multi}_{\mu}(Ref^{\mathbf{m}}_j \Box Ref^{\mathbf{m}}_i \textbf{\textit{actually}}_j \alpha = 1$ iff $V^{multi}_{\mu}(\alpha,u',s) = 1$ for every $u'$ which is just like $u$ except that $u'_i = w$ for some $w \in$ W, provided that $\textbf{\textit{actually}}_j$ does not occur in $\alpha$.

To prove lemmas 8.2 and 8.3 we need two preliminary lemmas. In the proofs which follow we use the notation *ts* to denote the sequence obtained from *s* by replacing $s_0$ with *t*, and *wu* to denote the sequence obtained from *u* by replacing $u_0$ with *w*.

LEMMA 8.4 If, for $1 \leq j$, $\textbf{\textit{then}}_j$ does not occur in $\alpha$, and *s* and *s'* coincide on all positions except possibly the j-th, then $V^{multi}_{\mu}(\alpha,u,s) = V^{multi}_{\mu}(\alpha,u,s')$.

*Proof*: By induction on $\alpha$. $V^{multi}_{\mu}(\varphi x_1 ... x_n,u,s) = 1$ iff $\langle \mu(x_1),...,\mu(x_n),u_0,s_0 \rangle \in V(\varphi)$, iff $\langle \mu(x_1),...,\mu(x_n),u_0,s'_0 \rangle \in V(\varphi)$, (since *s* and *s'* coincide except at j, and so coincide at 0) iff $V^{multi}_{\mu}(\varphi x_1 ... x_n,u,s') = 1$. $V^{multi}_{\mu}(\perp,u,s) = 0 = V^{multi}_{\mu}(\perp,u,s')$. $V^{multi}_{\mu}(\textbf{\textit{then}}_i,u,s) = 1$ iff $s_0 = s_i$. But i is not j and *s* and *s'* coincide everywhere except at j, and so $s_0 = s_i$ iff $s'_0 = s'_i$, and so $V^{multi}_{\mu}(\textbf{\textit{then}}_i,u,s) = V^{multi}_{\mu}(\textbf{\textit{then}}_i,u,s')$. $V^{multi}_{\mu}(\textbf{\textit{actually}}_i,u,s) = 1$ iff $u_0 = u_i$ iff $V^{multi}_{\mu}(\textbf{\textit{actually}}_i,u,s')$. If $\textbf{\textit{then}}_j$ does not occur in $\alpha \supset \beta$ then $\textbf{\textit{then}}_j$ does not occur in $\alpha$ or in $\beta$, and so $V^{multi}_{\mu}(\alpha \supset \beta,u,s) = 1$ iff $V^{multi}_{\mu}(\alpha,u,s) = 0$ or $V^{multi}_{\mu}(\beta,u,s) = 1$, iff, by the induction hypothesis, $V^{multi}_{\mu}(\alpha,u,s') = 0$ or $V^{multi}_{\mu}(\beta,u,s') = 1$, iff, $V^{multi}_{\mu}(\alpha \supset \beta,u,s') = 1$. Suppose $V^{multi}_{\mu}(\forall x \alpha,u,s) = 0$. Then there is some *x*-alternative $\rho$ of $\mu$ such that

$V^{multi}_{\rho}(\alpha,u,s) = 0$. Now ***then***$_j$ does not occur in $\forall x\alpha$ and so does not occur in $\alpha$, so $V^{multi}_{\rho}(\alpha,u,s') = 0$, and so $V^{multi}_{\mu}(\forall x\alpha,u,s') = 0$. Similarly, if $V^{multi}_{\mu}(\forall x\alpha,u,s') = 0$ then $V^{multi}_{\mu}(\forall x\alpha,u,s) = 0$. Take $G\alpha$. If there is no ***then***$_j$ in $G\alpha$ then there is none in $\alpha$. Further, if $s$ and $s'$ coincide on all positions except possibly the j-th, then so do $ts$ and $ts'$ for any $t \in$ T. Suppose $V^{multi}_{\mu}(G\alpha,u,s) = 0$. Then for some $t \in$ T such that $s_0 < t$, $V^{multi}_{\mu}(\alpha,u,ts) = 0$, and so, by the induction hypothesis, $V^{multi}_{\mu}(\alpha,u,ts') = 0$. But then $V^{multi}_{\mu}(G\alpha,u,s') = 0$. Similarly if $V^{multi}_{\mu}(G\alpha,u,s') = 0$. The proofs for $H$, $L$, $K$, $\Box$ and $\Box\!\!\rightarrow$ are analogous. (In fact what is doing the trick about these 'ordinary' operators is that they depend only on changes to $s_0$ or $u_0$. So that, provided ***then***$_j$ does not occur in wff governed by them it will not occur in their arguments, and so $ts$ and $ts'$ will coincide for any $t \in$ T, provided $s$ and $s'$ do, and therefore satisfy the induction hypothesis.[1]) $V^{multi}_{\mu}(Ref_i\alpha,u,s) = 1$ iff $V^{multi}_{\mu}(\alpha,u,s[0/i]) = V^{multi}_{\mu}(\alpha,u,s'[0/i])$ iff $V^{multi}_{\mu}(Ref_i\alpha,u,s') = 1$. ■

LEMMA 8.5 If, for $1 \leq j$, ***actually***$_j$ does not occur in $\alpha$, and $u$ and $u'$ coincide on all positions except possibly the j-th, then $V^{multi}_{\mu}(\alpha,u,s) = V^{multi}_{\mu}(\alpha,u',s)$.

(The proof of lemma 8.5 is just like 8.4, *mutatis mutandis*.)

*Proof of lemma 8.2*: $V^{multi}_{\mu}(Ref_j \oplus Ref_i$***then***$_j\alpha,u,s) = 1$ iff $V^{multi}_{\mu}(\oplus Ref_i$***then***$_j\alpha,u,s[0/j]) = 1$ iff, for all $t \in$ T:

(i) $V^{multi}_{\mu}(Ref_i$***then***$_j\alpha,u,t(s[0/j])) = 1$

iff

(ii) $V^{multi}_{\mu}($***then***$_j\alpha,u,(t(s[0/j]))[0/i]) = 1$

iff

(iii) $V^{multi}_{\mu}(\alpha,u,((t(s[0/j]))[0/i][j/0]) = 1$.

Now consider the relation between $(t(s[0/j]))[0/i][j/0]$ and $s'$, where $s'$ is as stated in the theorem. First, for any k except 0, j or i, $((t(s[0/j])[0/i][j/0])_k = s'_k$. Second, $((t(s[0/j])[0/i])_j = s_0 = s'_0$, and so $((t(s[0/j])[0/i][j/0])_0 = s'_0$. Third, $((t(s[0/j])[0/i][j/0])_i = t = s'_i$. So $(t(s[0/j]))[0/i][j/0]$ and $s'$ coincide except possibly at j. But ***then***$_j$ does not occur in $\alpha$, and so, by lemma 8.4, (iii) holds iff

(iv) $V^{multi}_{\mu}(\alpha,u,s') = 1$

[1] In fact if we were to use the neighbourhood semantics described in footnote 6 on p. 67 we could do the inductions for all the ordinary operators at one blow.

for every $s'$ like $s$ except at i. ■

Lemma 8.3 is proved analogously, relying on lemma 8.5. We can now prove theorem 8.1.

*Proof of theorem 8.1*: The proof is by induction on wff of $\mathcal{L}_{x\mathbf{tw}}$.

$V_{\mu*}(\varphi x_1...x_n,u_0,s_0) = 1$
iff
$\langle \mu(x_1),...,\mu(x_n),u_0,s_0\rangle \in V(\varphi)$
iff
$V^{\text{multi}}{}_{\mu}(\varphi x_1...x_n,u,s) = 1$
iff
$V^{\text{multi}}{}_{\mu}(\tau(\varphi x_1...x_n),u,s) = 1$

$V^{\text{multi}}{}_{\mu}(\bot,u,s) = 0 = V_{\mu*}(\bot,u_0,s_0) = V_{\mu*}(\tau(\bot),u_0,s_0)$

$V_{\mu*}(N\mathbf{t}_i,u_0,s_0) = 1$ iff $\mu(\mathbf{t}_i) = s_0$. Now $\mu(\mathbf{t}_i) = s_i$, and so $V_{\mu*}(N\mathbf{t}_i,u_0,s_0) = 1$ iff $s_i = s_0$, iff $V^{\text{multi}}{}_{\mu}(\textbf{\textit{then}}_i,u,s) = 1$. $V_{\mu*}(A\mathbf{w}_i,u_0,s_0) = 1$ iff $\mu(\mathbf{w}_i) = u_0$. Now $\mu(\mathbf{w}_i) = u_i$, and so $V_{\mu*}(A\mathbf{w}_i,u_0,s_0) = 1$ iff $u_i = u_0$, iff $V^{\text{multi}}{}_{\mu}(\textbf{\textit{actual}}_i,u,s) = 1$.

$V_{\mu*}(\alpha \supset \beta,u_0,s_0) = 1$
iff
$V_{\mu*}(\alpha,u_0,s_0) = 0$ or $V_{\mu*}(\beta,u_0,s_0) = 1$
iff
$V^{\text{multi}}{}_{\mu}(\tau(\alpha),u,s) = 0$ or $V^{\text{multi}}{}_{\mu}(\tau(\beta),u,s) = 1$
iff
$V^{\text{multi}}{}_{\mu}(\tau(\alpha \supset \beta),u_0,s_0) = 1$.

For the operators $G$, $H$, $L$, $\Box$, $K$ and $\Box\!\!\rightarrow$, all we need to remember is that their semantics only works on $u_0$ and $s_0$, and that (a) in $\mathcal{L}_{\text{multi}}$ all the other terms of $u$ and $s$ are simply carried along as parameters, and that (b) in $\mathcal{L}_{x\mathbf{tw}}$ the assignments to the world and time variables are also carried along as parameters. We can illustrate this in the case of $G$. Assume $V_{\mu*}(G\alpha,u_0,s_0) = 0$, then for some $t \in T$ such that $u_0 < t$, $V_{\mu*}(\alpha,u_0,t) = 0$, i.e. $V_{\mu*}(\alpha,u_0,(ts)_0) = 0$, and so, by the induction hypothesis $V^{\text{multi}}{}_{\mu}(\alpha,u,ts) = 0$, and so $V^{\text{multi}}{}_{\mu}(G\alpha,u,s) = 0$. Similarly if $V^{\text{multi}}{}_{\mu}(G\alpha,u,s) = 0$, and analogously for $H$, $L$, $K$, $\Box$ and $\Box\!\!\rightarrow$.

Take $\forall x\alpha$. Suppose $V_{\mu*}(\forall x\alpha,u_0,s_0) = 0$. Then for some $x$-alternative $\rho*$ of $\mu*$, $V_{\rho*}(\alpha,u_0,s_0) = 0$. Since $x$ is not $\mathbf{w}_i$ or $\mathbf{t}_i$ then $\rho*(\mathbf{w}_i) = \mu*(\mathbf{w}_i)$ and $\rho*(\mathbf{t}_i) =$

$\mu^*(\mathbf{t}_i)$, and since $u_i = \mu^*(\mathbf{w}_i)$ and $s_i = \mu^*(\mathbf{t}_i)$ then $u_i = \rho^*(\mathbf{w}_i)$ and $s_i = \rho^*(\mathbf{t}_i)$, and so, by the induction hypothesis, $V^{multi}_{\rho}(\tau(\alpha),u,s) = 0$, and so $V^{multi}_{\mu}(\forall x\tau(\alpha),u,s) = 0$, and so $V^{multi}_{\mu}(\tau(\forall x\alpha),u,s) = 0$. If $V^{multi}_{\mu}(\tau(\forall x\alpha),u,s) = 0$ then $V^{multi}_{\mu}(\forall x\tau(\alpha),u,s) = 0$ and so for some $x$-alternative $\rho$ of $\mu$, $V^{multi}_{\rho}(\tau(\alpha),u,s) = 0$. Since $x$ is not $\mathbf{w}_i$ or $\mathbf{t}_i$ then $\rho^*(\mathbf{w}_i) = \mu^*(\mathbf{w}_i)$ and $\rho^*(\mathbf{t}_i) = \mu^*(\mathbf{t}_i)$, and since $u_i = \mu^*(\mathbf{w}_i)$ and $s_i = \mu^*(\mathbf{t}_i)$ then $u_i = \rho^*(\mathbf{w}_i)$ and $s_i = \rho^*(\mathbf{t}_i)$, and so by the induction hypothesis, $V_{\rho^*}(\alpha,u_0,s_0) = 0$. So $V_{\mu^*}(\forall x\alpha,u_0,s_0) = 0$.

Assume that $u_k = \mu^*(\mathbf{w}_k)$ and $s_k = \mu^*(\mathbf{t}_k)$ for $1 \leq k$. $V_{\mu^*}(\forall \mathbf{t}_i\alpha,u_0,s_0) = 1$ iff, for every $\mathbf{t}_i$-alternative $\rho^*$ of $\mu^*$, $V_{\rho^*}(\alpha,u_0,s_0) = 1$. By the induction hypothesis, $V_{\rho^*}(\alpha,u_0,s_0) = 1$ iff $V^{multi}_{\rho}(\alpha,u,s') = 1$ where $s'$ differs from $s$ only in that $s'_i = \rho^*(\mathbf{t}_i)$. And this will happen for every $\mathbf{t}_i$-alternative $\rho^*$ of $\mu^*$ iff for every $t \in$ T such that where $s'$ is just like $s$ except that $s'_i = t$, $V^{multi}_{\mu}(\alpha,u,s') = 1$. By lemma 8.2 this latter will hold iff $V^{multi}_{\mu}(Ref_j \oplus Ref_i \textbf{\textit{then}}_j\alpha,u,s) = 1$, provided j is so chosen that $\textbf{\textit{then}}_j$ does not occur in $\alpha$; i.e., iff $V^{multi}_{\mu}(\tau(\forall \mathbf{t}_i\alpha),u,s) = 1$.

Assume that $u_k = \mu^*(\mathbf{w}_k)$ and $s_k = \mu^*(\mathbf{t}_k)$ for $1 \leq k$. $V_{\mu^*}(\forall \mathbf{w}_i\alpha,u_0,s_0) = 1$ iff, for every $\mathbf{w}_i$-alternative $\rho^*$ of $\mu^*$, $V_{\rho^*}(\alpha,u_0,s_0) = 1$. By the induction hypothesis, $V_{\rho^*}(\alpha,u_0,s_0) = 1$ iff $V^{multi}_{\rho}(\alpha,u',s) = 1$ where $u'$ differs from $u$ only in that $u'_i = \rho^*(\mathbf{w}_i)$. And this will happen for every $\mathbf{w}_i$-alternative $\rho^*$ of $\mu^*$ iff for every $t \in$ T such that where $u'$ is just like $u$ except that $u'_i = w$, $V^{multi}_{\mu}(\alpha,u',s) = 1$. By lemma 8.3 this latter will hold iff $V^{multi}_{\mu}(Ref_j \Box Ref_i \textbf{\textit{actually}}_j\alpha,u,s) = 1$, provided j is so chosen that $\textbf{\textit{actual}}_j$ does not occur in $\alpha$; i.e., iff $V^{multi}_{\mu}(\tau(\forall \mathbf{w}_i\alpha),u,s) = 1$. ■

For formulae with no free variables theorem 8.1 becomes simply

THEOREM 8.6 For any sequences $s$ and $u$, $\alpha$ is true at $u_0$ and $s_0$ iff $\tau(\alpha)$ is true at $u$ and $s$.

We could have made use of two definitions in the statement of the last few results:

[Def$\oplus_i$] $\oplus_i\alpha =_{df} Ref_j \oplus Ref_i \textbf{\textit{then}}_j\alpha$ (where j is so chosen as not to occur in $\alpha$).

[Def$\Box_i$] $\Box_i\alpha =_{df} Ref_j \Box Ref_i \textbf{\textit{actually}}_j\alpha$ (where j is so chosen as not to occur in $\alpha$).

Then lemmas 8.2 and 8.3 can be stated as:

COROLLARY 8.7 (a) $V^{multi}_{\mu}(\oplus_i\alpha,u,s) = 1$ iff, for every $s'$ which coincides with $s$ except at the i-th position, $V^{multi}_{\mu}(\alpha,u,s') = 1$

(b) $V^{multi}_{\mu}(\Box_i\alpha,u,s) = 1$ iff, for every $u'$ which coincides with $u$ except at the i-th position, $V^{multi}_{\mu}(\alpha,u',s) = 1$

$\Box_i$ is used on p.486 of Peacocke 1978, who was one of the earliest to see the need for indexed operators of this kind. Although Peacocke does not set out a recursive model-theoretic semantics for $\Box_i$ it is clear (as Forbes 1983 notes on p. 291) that its semantics has to be as described here. $\Box_0$ is simply $\Box$.[2] If $\Box_i$ is taken as primitive in $\mathfrak{L}_{multi}$ in place of $Ref^{\mathbf{m}}_i$, $Ref^{\mathbf{m}}_i\alpha$ can be defined as $\Box_i(\boldsymbol{actual}_i \supset \alpha)$.

THEOREM 8.8 $V^{multi}_{\mu}(\Box_i(\boldsymbol{actual}_i \supset \alpha),u,s) = V^{multi}_{\mu}(Ref_i\alpha,u,s)$

*Proof*: $V^{multi}_{\mu}(\Box_i(\boldsymbol{actual}_i \supset \alpha),u,s) = 1$ iff, for every $u'$ which coincides with $u$ except at the i-th position, $V^{multi}_{\mu}(\boldsymbol{actual}_i \supset \alpha,u',s) = 1$; i.e. iff, for every $u'$ which coincides with $u$ except at the i-th position,

(i) if $u'_0 = s'_i$ then $V^{multi}_{\mu}(\alpha,u',s) = 1$.

But $u_0 = u'_0$ (since $u$ and $u'$ coincide except at i) and so the only $u'$ which coincides with $u$ except at the i-th position and is such that $u'_0 = u_i$ is $u[0/i]$, and so (i) holds iff

(ii) $V^{multi}_{\mu}(\alpha,u[0/i],s) = 1$

iff $V^{multi}_{\mu}(Ref^{\mathbf{m}}_i\alpha,u,s) = 1$. ■

(Obviously the analogous principles hold in the temporal case, where $Ref_i\alpha$ can be defined as $\oplus_i(\boldsymbol{then}_i \supset \alpha)$.)

We now show the equivalence of $\mathfrak{L}_{xtw}$ and the purely extensional

[2]Peacocke also has an operator $A_i$, which corresponds to $\boldsymbol{actually}_i$. See also Forbes 1985, p. 91, footnote 28. In Forbes 1983 the indices are sometimes 'possibilities', or 'incomplete' worlds, but Forbes points out that the semantics is as stated for $\Box_i$ above when the indices are worlds. On p. 87f Forbes 1989 speaks of a hierarchy of languages and introduces indexed operators, using Peacocke's notation. Each language extends the hierarchy by adding higher numerical indices, and contains languages lower in the hierarchy. Goranko 1996 discusses a similar hierarchy, and proves that the hierarchy is of strictly increasing expressive power.

indexical language $\mathfrak{L}_i$. $\mathfrak{L}_i$ can also be interpreted by $\langle W,R,T,<,K,C,D,V\rangle$. Models for $\mathfrak{L}_{x\mathbf{tw}}$ and $\mathfrak{L}_i$ differ only in that in a model for $\mathfrak{L}_{x\mathbf{tw}}$, R, <, K and C are directly used in evaluating formulae, while in models for $\mathfrak{L}_i$, V must be constrained to assign the R, <, K and C in the model to the predicates R, <, K, and C in the language. Truth in $\mathfrak{L}_i$ is defined in the standard way for an ordinary (extensional) many-sorted logic.[3] It is worth commenting on the atomic case. Where $\varphi$ is an n-place predicate of $\mathfrak{L}_{x\mathbf{tw}}$ we have that

[V$\varphi^i$] $\quad V_{\mu^*}(\varphi x_1 ... x_n \mathbf{wt}) = 1$ iff $\langle \mu^*(x_1),...,\mu^*(x_n),\mu^*(\mathbf{w}),\mu^*(\mathbf{t})\rangle \in V(\varphi)$.

We can illustrate the special predicates by R:

[VR] $\quad V_{\mu^*}(\mathbf{w}R\mathbf{w}') = 1$ iff $\langle \mu^*(\mathbf{w}),\mu^*(\mathbf{w}')\rangle \in V(R)$

But $V(R) = R$, and so $V_{\mu^*}(\mathbf{w}R\mathbf{w}') = 1$ iff $\mu^*(\mathbf{w})R\mu^*(\mathbf{w}')$ — distinguishing of course between R as a predicate of $\mathfrak{L}_i$ and R as a relation in the model which interprets that predicate. <, K and C can be treated analogously.

We shall show how to translate every wff in a language $\mathfrak{L}_i$ into a wff of a language $\mathfrak{L}_{x\mathbf{tw}}$, noting that such languages are exactly equivalent to the multiply-indexed languages $\mathfrak{L}_{\text{multi}}$ from Chapter 7. It is convenient to amalgamate [**w**] and [**t**] as defined on pp. 97 and 96:

(1) $\quad [\mathbf{w},\mathbf{t}]\alpha =_{\text{df}} \Box\oplus((A\mathbf{w} \wedge N\mathbf{t}) \supset \alpha)$

From (1) we have, for any model $\langle W,R,T,<,K,C,D,V\rangle$, and any $w \in W$ and $t \in T$, and assignment $\mu$,

(2) $\quad V_\mu([\mathbf{w},\mathbf{t}]\alpha,w,t) = V_\mu(\alpha,\mu(\mathbf{w}),\mu(\mathbf{t}))$.

We shall use the letter $\tau$ for the translation of wff of $\mathfrak{L}_i$ into wff of $\mathfrak{L}_{x\mathbf{tw}}$. This $\tau$ must of course be distinguished from the $\tau$ defined in chapter 8 on p. 98. The present $\tau$ is defined as follows:

[3] We use $V_\mu$ ambiguously. In $\mathfrak{L}_{x\mathbf{tw}}$, $V_\mu(\alpha,w,t)$ is a world and time-relative assignment for $\alpha$ in $\mathfrak{L}_{x\mathbf{tw}}$; while in $\mathfrak{L}_i$, for $\alpha$ in $\mathfrak{L}_i$, $V_\mu(\alpha)$ is not relative to $w$ and $t$. But each is uniquely determined by the same $\langle W,R,T,<,K,C,D,V\rangle$, when $V(R) = R$, $V(<) = <$, $V(K) = K$ and $V(C) = C$, and by the same assignment $\mu$. Note that although we have not required $\mathfrak{L}_i$ to contain a predicate of identity of individuals nothing prevents a model in which there is a predicate $\varphi_=$ for which $V(\varphi_=) = \{\langle a,a,w,t\rangle$: $a \in D$, $w \in W$ and $t \in T\}$.

$\tau(\varphi x_1 \ldots x_n \mathbf{wt}) = [\mathbf{w},\mathbf{t}]\varphi x_1 \ldots x_n$
$\tau(\bot) = \bot$
$\tau(\mathbf{t} = \mathbf{t}') = \otimes(N\mathbf{t} \wedge N\mathbf{t}')$
$\tau(\mathbf{w} = \mathbf{w}') = \Diamond(A\mathbf{w} \wedge A\mathbf{w}')$
$\tau(\mathbf{t} < \mathbf{t}') = \otimes(N\mathbf{t} \wedge FN\mathbf{t}')$
$\tau(\mathbf{w}R\mathbf{w}') = \Diamond(A\mathbf{w} \wedge MA\mathbf{w}')$
$\tau(K(\mathbf{w},\mathbf{t},\mathbf{w}',\mathbf{t}') = \Diamond\otimes(N\mathbf{t} \wedge A\mathbf{w} \wedge K_{\Diamond}(N\mathbf{t}' \wedge A\mathbf{w}'))$
$\tau(C(\mathbf{t},\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3) = \Box\oplus((N\mathbf{t} \wedge A\mathbf{w}_1) \supset ((A\mathbf{w}_2 \vee A\mathbf{w}_3) \Box\!\!\rightarrow A\mathbf{w}_2))$
$\tau(\alpha \supset \beta) = \tau(\alpha) \supset \tau(\beta)$
$\tau(\forall x\alpha) = \forall x\tau(\alpha)$
$\tau(\forall \mathbf{w}\alpha) = \forall \mathbf{w}\tau(\alpha)$
$\tau(\forall \mathbf{t}\alpha) = \forall \mathbf{t}\tau(\alpha)$

One of the things we notice immediately about this translation is that for any wff $\alpha$ of $\mathfrak{L}_i$, $\tau(\alpha)$ is a tenseless non-contingent sentence, whose truth value does not change from world to world or from time to time. This of course is to be expected, since $\mathfrak{L}_i$ is the tenseless, and modal-free, language in which the semantics of a tense and modal language can be expressed. In the case of the atomic wff of $\mathfrak{L}_i$ the following facts will be helpful:

(3) $V_\mu(\mathbf{w}R\mathbf{w}') = 1$ iff $\mu(\mathbf{w})R\mu(\mathbf{w}')$
$V_\mu(\mathbf{t} < \mathbf{t}') = 1$ iff $\mu(\mathbf{t}) < \mu(\mathbf{t}')$
$V_\mu(K(\mathbf{w},\mathbf{t},\mathbf{w}',\mathbf{t}')) = 1$ iff $K(\mu(\mathbf{w}),\mu(\mathbf{t}),\mu(\mathbf{w}'),\mu(\mathbf{t}'))$
$V_\mu(C(\mathbf{t},\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3)) = 1$ iff $\mu(\mathbf{w}_2) <_{\langle\mu(\mathbf{w}_1,\mu(\mathbf{t})\rangle} \mu(\mathbf{w}_3)$

(In the last clause of (3) $\mathbf{w}_1$, $\mathbf{w}_2$ and $\mathbf{w}_3$ of course refer to any three world variables, irrespective of their place in the standard ordering of world variables.)

THEOREM 8.9 In any model $\langle W,R,T,K,C,D,V\rangle$, for any wff $\alpha$ of $\mathfrak{L}_i$ and any $w^* \in W$, $t^* \in T$ and assignment $\mu$, $V_\mu(\alpha) = V_\mu(\tau(\alpha),w^*,t^*)$

*Proof*: The theorem is proved by induction on the construction of wff of $\mathfrak{L}_i$. We note that where $\varphi$ is an 'ordinary' n-place predicate of $\mathfrak{L}_{x\mathbf{tw}}$ it is an n+2-place predicate of $\mathfrak{L}_i$. Despite this, the value provided by $V(\varphi)$ is in both cases a set of n+2-tuples of which the first n terms are taken from D and the last two from W and T respectively. In this theorem it is the atomic cases which are the complex ones, because the translation of atomic wff involving the 'designated' predicates of $\mathfrak{L}_i$, <, R, K and C is not completely straightforward. For an ordinary n+2-place predicate $\varphi$ of $\mathfrak{L}_i$ we have the following:

$V_\mu(\varphi x_1 ... x_n \mathbf{w}\mathbf{t}) = 1$

iff

$\langle \mu(x_1),...,\mu(x_n),\mu(\mathbf{w}),\mu(\mathbf{t})\rangle \in V(\varphi)$

iff

$V_\mu(\varphi x_1 ... x_n,\mu(\mathbf{w}),\mu(\mathbf{t})) = 1$

iff

$V_\mu([\mathbf{w},\mathbf{t}]\varphi x_1 ... x_n,w^*,t^*) = 1$

iff

$V_\mu(\tau(\varphi x_1 ... x_n \mathbf{w}\mathbf{t}),w^*,t^*) = 1.$

$\tau(\mathbf{t} = \mathbf{t}')$ is $\otimes(N\mathbf{t} \wedge N\mathbf{t}')$, and $V_\mu(\otimes(N\mathbf{t} \wedge N\mathbf{t}'),w^*,t^*) = 1$ iff, for some $t \in T$, $V_\mu(N\mathbf{t} \wedge N\mathbf{t}',w^*,t) = 1$, iff, for some $t \in T$,

(i) $V_\mu(N\mathbf{t},w^*,t) = 1$

and

(ii) $V_\mu(N\mathbf{t}',w^*,t) = 1$

and by [V*N*] on p. 210, (i) holds iff $\mu(\mathbf{t}) = t$, and (ii) holds iff $\mu(\mathbf{t}') = t$. But this will be so for some $t \in T$ iff $\mu(\mathbf{t}) = \mu(\mathbf{t}')$, iff $V_\mu(\mathbf{t} = \mathbf{t}') = 1$.

$\tau(\mathbf{w} = \mathbf{w}')$ is $\otimes(A\mathbf{w} \wedge A\mathbf{w}')$, and $V_\mu(\otimes(A\mathbf{w} \wedge A\mathbf{w}'),w^*,t^*) = 1$ iff, for some $w \in W$, $V_\mu(A\mathbf{w} \wedge A\mathbf{w}',w^*,t) = 1$, iff

(i) $V_\mu(A\mathbf{w},w^*,t) = 1$

and

(ii) $V_\mu(A\mathbf{w}',w^*,t) = 1$

and by [V*A*] on p. 210, (i) holds iff $\mu(\mathbf{w}) = w$, and (ii) holds iff $\mu(\mathbf{w}') = w$. But this will be so for some $w \in W$ iff $\mu(\mathbf{w}) = \mu(\mathbf{w}')$, iff $V_\mu(\mathbf{w} = \mathbf{w}') = 1$.

$\tau(\mathbf{t} < \mathbf{t}')$ is $\otimes(N\mathbf{t} \wedge FN\mathbf{t}')$, and $V_\mu(\otimes(N\mathbf{t} \wedge FN\mathbf{t}'),w^*,t^*) = 1$ iff, for some $t \in T$, $V_\mu(N\mathbf{t} \wedge FN\mathbf{t}',w^*,t) = 1$, iff, for some $w \in W$,

(i) $V_\mu(N\mathbf{t},w,t^*) = 1$

and

(ii) $V_\mu(FN\mathbf{t}',w,t^*) = 1$

and (i) holds iff $\mu(\mathbf{t}) = t$, and (ii) holds iff for some $t'$ such that $t < t'$

(iii) $V_\mu(N\mathbf{t},w^*,t') = 1$

i.e., iff $\mu(\mathbf{t}') = t'$. So $V_\mu(\otimes(N\mathbf{t} \wedge FN\mathbf{t}'),w^*,t^*) = 1$ iff there are $t$ and $t'$ such that $t = \mu(\mathbf{t})$ and $t' = \mu(\mathbf{t}')$ and $t < t'$, iff $\mu(\mathbf{t}) < \mu(\mathbf{t}')$, iff, by (3), $V_\mu(\mathbf{t} < \mathbf{t}') = 1$.

$\tau(\mathbf{w}R\mathbf{w}')$ is $\Diamond(A\mathbf{w} \wedge MA\mathbf{w}')$, and $V_\mu(\Diamond(A\mathbf{w} \wedge MA\mathbf{w}'),w^*,t^*) = 1$ iff, for some $w \in W$, $V_\mu(A\mathbf{w} \wedge MA\mathbf{w}',w,t^*) = 1$, iff

(i) $V_\mu(A\mathbf{w},w,t^*) = 1$

and

(ii) $V_\mu(MA\mathbf{w}',w,t^*) = 1$

and (i) holds iff $\mu(\mathbf{w}) = w$, and (ii) holds iff for some $w'$ such that $wRw'$

(iii) $V_\mu(A\mathbf{w},w',t^*) = 1$

iff $\mu(\mathbf{w}') = w'$. So $V_\mu(\Diamond(A\mathbf{w} \wedge MA\mathbf{w}'),w^*,t^*) = 1$ iff there are $w$ and $w'$ such that $w = \mu(\mathbf{w})$ and $w' = \mu(\mathbf{w}')$ and $wRw'$, i.e., iff $\mu(\mathbf{w})R\mu(\mathbf{w}')$, i.e., iff $V_\mu(\mathbf{w}R\mathbf{w}') = 1$.

$\tau(K(\mathbf{w},\mathbf{t},\mathbf{w}',\mathbf{t}')$ is $\Diamond\otimes(N\mathbf{t} \wedge A\mathbf{w} \wedge K_\Diamond(N\mathbf{t}' \wedge A\mathbf{w}'))$, and $V_\mu(\Diamond\otimes(N\mathbf{t} \wedge A\mathbf{w} \wedge K_\Diamond(N\mathbf{t}' \wedge A\mathbf{w}')),w^*,t^*) = 1$ iff for some $w \in W$ and some $t \in T$, $V_\mu(N\mathbf{t} \wedge A\mathbf{w} \wedge K_\Diamond(N\mathbf{t}' \wedge A\mathbf{w}'),w,t) = 1$, iff

(i) $V_\mu(N\mathbf{t},w,t) = 1$ and $V_\mu(A\mathbf{w},w,t) = 1$

and

(ii) $V_\mu(K_\Diamond N\mathbf{t}',w,t) = 1$ and $V_\mu(K_\Diamond A\mathbf{w}',w,t) = 1$

and (i) holds iff $\mu(\mathbf{t}) = t$ and $\mu(\mathbf{w}) = w$, and (ii) holds iff for some $w' \in W$ and $t \in T$ such that $K(w,t,w',t')$

(iii) $V_\mu(A\mathbf{w},w',t') = 1$ and $V_\mu(N\mathbf{t},w',t') = 1$.

And (iii) holds iff $\mu(\mathbf{w}') = w'$ and $\mu(\mathbf{t}') = t'$. So $V_\mu(\Diamond\otimes(N\mathbf{t} \wedge A\mathbf{w} \wedge K_\Diamond(N\mathbf{t}' \wedge A\mathbf{w}')),w^*,t^*) = 1$ iff $\mu(\mathbf{w}) = w$ and $\mu(\mathbf{t}) = t$, and there are $w'$ and $t'$ such that $w' = \mu(\mathbf{w}')$ and $t' = \mu(\mathbf{t}')$ and $K(w,t,w',t')$, i.e., iff $K(\mu(\mathbf{w}),\mu(\mathbf{t}),\mu(\mathbf{w}'),\mu(\mathbf{t}'))$, i.e., iff $V_\mu(K(\mathbf{w},\mathbf{t},\mathbf{w}',\mathbf{t}')) = 1$.

$\tau(C(\mathbf{t},\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3))$ is $\Box\oplus((N\mathbf{t} \wedge A\mathbf{w}_1) \supset ((A\mathbf{w}_2 \vee A\mathbf{w}_3) \Box\!\!\rightarrow A\mathbf{w}_2)$. Suppose $V_\mu(\Box\oplus((N\mathbf{t} \wedge A\mathbf{w}_1) \supset ((A\mathbf{w}_2 \vee A\mathbf{w}_3) \Box\!\!\rightarrow A\mathbf{w}_2),w^*,t^*) = 1$. Then for every $t \in T$ and $w_1 \in W$, we have that if

(i) $V_\mu(N\mathbf{t} \wedge A\mathbf{w}_1,w_1,t) = 1$

then

(ii) $V_\mu((A\mathbf{w}_2 \vee A\mathbf{w}_3) \Box\!\!\rightarrow A\mathbf{w}_2, w_1, t) = 1.$

Suppose that $w_1 = \mu(\mathbf{w}_1)$ and $t = \mu(\mathbf{t})$. Then (i) holds then, and so (ii) holds. If (ii) holds we have that there is some $w_2$ such that

(iii) $V_\mu(A\mathbf{w}_2 \vee A\mathbf{w}_3, w_2, t) = 1$

and

(iv) $V_\mu(A\mathbf{w}_2, w_2, t) = 1.$

and, if there is a $w_3$ such that

(v) $V_\mu(A\mathbf{w}_2 \vee A\mathbf{w}_3, w_3, t) = 1$

but

(vi) $V_\mu(A\mathbf{w}_2, w_3, t) = 0$

then $w_2 <_{\langle w_1, t\rangle} w_3$. Now from (iv) $w_2 = \mu(\mathbf{w}_2)$, and from (v) either $\mu(\mathbf{w}_2) = w_3$ or $\mu(\mathbf{w}_3) = w_3$. But by (vi) $\mu(\mathbf{w}_2) \neq w_3$, and so if (v) and (vi) then $\mu(\mathbf{w}_3) = w_3$. So that for any $t$, $w_1$, $w_2$ and $w_3$, we have $w_2 <_{\langle w_1, t\rangle} w_3$, provided that $t = \mu(\mathbf{t})$, $w_1 = \mu(\mathbf{w}_1)$, $w_2 = \mu(\mathbf{w}_2)$ and $w_3 = \mu(\mathbf{w}_3)$, which is to say that $\mu(\mathbf{w}_2) <_{\langle \mu(\mathbf{w}_1), \mu(\mathbf{t})\rangle} \mu(\mathbf{w}_3)$, i.e., $V_\mu(C(\mathbf{t}, \mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3)) = 1$, as required.

Now suppose $V_\mu(C(\mathbf{t}, \mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3) = 1$. Then $\mu(\mathbf{w}_2) <_{\langle \mu(\mathbf{w}_1), \mu(\mathbf{t})\rangle} \mu(\mathbf{w}_3)$. Now

(vii) If $V_\mu(N\mathbf{t} \wedge A\mathbf{w}_1, w_1, t) = 1$ then $t = \mu(\mathbf{t})$ and $w_1 = \mu(\mathbf{w}_1)$

So we have to show that

(viii) $V_\mu((A\mathbf{w}_2 \vee A\mathbf{w}_3) \Box\!\!\rightarrow A\mathbf{w}_2, \mu(\mathbf{w}_1), \mu(\mathbf{t})) = 1.$

Now

(ix) $V_\mu(A\mathbf{w}_2 \vee A\mathbf{w}_3, \mu(\mathbf{w}_2), \mu(\mathbf{t})) = 1$ and $V_\mu(A\mathbf{w}_2, \mu(\mathbf{w}_2), \mu(\mathbf{t})) = 1$

and further

(x) For any $w_3$, if $V_\mu(A\mathbf{w}_2 \vee A\mathbf{w}_3, \mu(\mathbf{w}_3), \mu(\mathbf{t})) = 1$, and $V_\mu(A\mathbf{w}_2, w_3, \mu(\mathbf{t})) = 0$, then $w_3 = \mu(\mathbf{w}_3)$.

But we are given that $\mu(\mathbf{w}_2) <_{\langle \mu(\mathbf{w}_1), \mu(\mathbf{t})\rangle} \mu(\mathbf{w}_3)$. So we have (viii). And (vii) ensures that the only $w_1$ and $t$ which can satisfy $N\mathbf{t} \wedge A\mathbf{w}_1$ are $\mu(\mathbf{w}_1)$ and $\mu(\mathbf{t})$, and so $V_\mu(\tau(\alpha \Box\!\!\rightarrow \beta), w^*, t^*) = 1$. ■

The reverse translation $\tau^*$ from wff $\alpha$ of $\mathfrak{L}_{x\mathbf{tw}}$ to wff of $\mathfrak{L}_i$ (not to be confused

with the $\tau^*$ of $\mathcal{L}_{x\mathbf{tw}}$ into $\mathcal{L}_{multi}$ on p. 99) is a generalisation of a perfectly standard procedure in modal and tense logic.[4] Where $\alpha$ is any wff of $\mathcal{L}_{x\mathbf{tw}}$, and $\mathbf{w}^*$ and $\mathbf{t}^*$ are a world variable and a temporal variable (which represent the actual and the present) the translation $\tau^*(\alpha)(\mathbf{w}^*,\mathbf{t}^*)$ is defined as follows:

$\tau^*(\varphi x_1 \ldots x_n)(\mathbf{w}^*,\mathbf{t}^*) = (\varphi x_1 \ldots x_n \mathbf{w}^* \mathbf{t}^*)$
$\tau^*(N\mathbf{t})(\mathbf{w}^*,\mathbf{t}^*) = (\mathbf{t} = \mathbf{t}^*)$, where $\mathbf{t}$ is not $\mathbf{t}^*$
$\tau^*(A\mathbf{w})(\mathbf{w}^*,\mathbf{t}^*) = (\mathbf{w} = \mathbf{w}^*)$, where $\mathbf{w}$ is not $\mathbf{w}^*$
$\tau^*(\bot)(\mathbf{w}^*,\mathbf{t}^*) = \bot$
$\tau^*(\alpha \supset \beta)(\mathbf{w}^*,\mathbf{t}^*) = (\tau^*(\alpha)(\mathbf{w}^*,\mathbf{t}^*) \supset \tau^*(\beta)(\mathbf{w}^*,\mathbf{t}^*))$
$\tau^*(\forall x\alpha)(\mathbf{w}^*,\mathbf{t}^*) = \forall x(\tau^*(\alpha)(\mathbf{w}^*,\mathbf{t}^*))$
$\tau^*(\forall \mathbf{w}\alpha)(\mathbf{w}^*,\mathbf{t}^*) = \forall \mathbf{w}(\tau^*(\alpha)(\mathbf{w}^*,\mathbf{t}^*))$ [provided $\mathbf{w}$ is not $\mathbf{w}^*$[5]]
$\tau^*(\forall \mathbf{t}\alpha)(\mathbf{w}^*,\mathbf{t}^*) = \forall \mathbf{t}(\tau^*(\alpha)(\mathbf{w}^*,\mathbf{t}^*))$ [provided $\mathbf{t}$ is not $\mathbf{t}^*$]
$\tau^*(G\alpha)(\mathbf{w}^*,\mathbf{t}^*) = \forall \mathbf{t}(\mathbf{t}^* < \mathbf{t} \supset \tau^*(\alpha)(\mathbf{w}^*,\mathbf{t}))$ [where $\tau^*(\alpha)(\mathbf{w}^*,\mathbf{t})$ indicates $\tau^*(\alpha)$ with free $\mathbf{t}$ replacing $\mathbf{t}^*$]
$\tau^*(H\alpha)(\mathbf{w}^*,\mathbf{t}^*) = \forall \mathbf{t}(\mathbf{t} < \mathbf{t}^* \supset \tau^*(\alpha)(\mathbf{w}^*,\mathbf{t}))$
$\tau^*(L\alpha)(\mathbf{w}^*,\mathbf{t}^*) = \forall \mathbf{w}(\mathbf{w}^* R \mathbf{w} \supset \tau^*(\alpha)(\mathbf{w},\mathbf{t}^*))$ [where $\tau^*(\alpha)(\mathbf{w},\mathbf{t}^*)$ indicates $\tau^*(\alpha)$ with free $\mathbf{w}$ replacing $\mathbf{w}^*$]
$\tau^*(\Box\alpha)(\mathbf{w}^*,\mathbf{t}^*) = \forall \mathbf{w}(\tau^*(\alpha)(\mathbf{w},\mathbf{t}^*))$
$\tau^*(K\alpha)(\mathbf{w}^*,\mathbf{t}^*) = \forall \mathbf{w}(K(\mathbf{t}^*,\mathbf{w}^*,\mathbf{w},\mathbf{t}) \supset \tau^*(\alpha)(\mathbf{w},\mathbf{t}))$
$\tau^*(\alpha \,\Box\!\!\rightarrow \beta)(\mathbf{w}^*,\mathbf{t}^*) = \exists \mathbf{w}(\tau^*(\alpha)(\mathbf{w},\mathbf{t}^*) \wedge \tau^*(\beta)(\mathbf{w},\mathbf{t}^*) \wedge \forall \mathbf{w}'((\tau^*(\alpha)(\mathbf{w},\mathbf{t}^*) \wedge \sim\tau^*(\beta)(\mathbf{w},\mathbf{t}^*) \supset C(\mathbf{t}^*,\mathbf{w}^*,\mathbf{w},\mathbf{w}'))$

Assume that $\langle W,R,T,<,K,C,D,V\rangle$ is a model, and that $\mu$ and $\mu^*$ are assignments to the variables of $\mathcal{L}_{x\mathbf{tw}}$ and $\mathcal{L}_i$ respectively with the property that for every individual variable $x$, $\mu(x) = \mu^*(x)$.

THEOREM 8.10 For any wff $\alpha$ of $\mathcal{L}_{x\mathbf{tw}}$, $V_\mu(\alpha,w,t) = 1$ iff $V_{\mu^*}(\tau^*(\alpha)(\mathbf{w}^*,\mathbf{t}^*)) = 1$ provided $\mu(\mathbf{w}^*) = w$ and $\mu(\mathbf{t}^*) = t$.

Theorem 8.10 is proved by induction on the construction of wff of $\mathcal{L}_{x\mathbf{tw}}$. We note that where $\varphi$ is an n-place predicate of $\mathcal{L}_{x\mathbf{tw}}$ it is an n+2-place predicate of $\mathcal{L}_i$. On

[4]See for instance, Hughes and Cresswell 1996, p. 183.

[5]If $\mathbf{w}$ *is* $\mathbf{w}^*$ take a bound alphabetic variant of $\forall \mathbf{w}\alpha$ in which $\mathbf{w}$ is not $\mathbf{w}^*$.

the assumption that $\mu(\mathbf{w}^*) = w$ and $\mu(\mathbf{t}^*) = t$ we have that

$$V_{\mu^*}(\varphi x_1 \ldots x_n \mathbf{w}^* \mathbf{t}^*) = 1$$

iff

$$\langle \mu(x_1), \ldots, \mu(x_n), \mu(\mathbf{w}^*), \mu(\mathbf{t}^*) \rangle \in V(\varphi)$$

iff

$$\langle \mu(x_1), \ldots, \mu(x_n), w, t \rangle \in V(\varphi)$$

iff

$$V_{\mu^*}(\varphi x_1 \ldots x_n, w, t) = 1$$

The case of $\perp$ is trivial, as are the inductions for $\supset$, and $\forall$ for variables of all kinds. The modal and tense operators require a little more effort.

Take the case of $G$, and suppose that $V_\mu(G\alpha, w, t) = 0$. Now $\tau^*(G\alpha)(\mathbf{w}^*, \mathbf{t}^*)$ is $\forall \mathbf{t}(\mathbf{t}^* < \mathbf{t} \supset \tau^*(\alpha)(\mathbf{w}^*, \mathbf{t}))$, and so we have to show that where $\mu^*(\mathbf{w}^*) = w$, and $\mu^*(\mathbf{t}^*) = t$, then $V_{\mu^*}(\tau^*(G\alpha)(\mathbf{w}^*, \mathbf{t}^*) = 0$. Now if $V_\mu(G\alpha, w, t) = 0$ there will be some $t'$ such that $t < t'$ and $V_\mu(\alpha, w, t') = 0$. Let $\rho^*$ be the $\mathbf{t}$-alternative of $\mu^*$ such that $\rho^*(\mathbf{t}) = t'$. Then, by the induction hypothesis $V_{\rho^*}(\tau^*(\alpha)(\mathbf{w}^*, \mathbf{t})) = 0$. But $t < t'$ and $\rho^*(\mathbf{t}^*) = \mu(\mathbf{t}^*) = t$, and so $\rho^*(\mathbf{t}^*) < \rho^*(\mathbf{t})$. So, by (3), $V_{\rho^*}(\mathbf{t}^* < \mathbf{t} \wedge \tau^*(\alpha)(\mathbf{w}, \mathbf{t}^*)) = 0$, and since $\rho^*$ is a $\mathbf{t}$-alternative of $\mu$, $V_{\mu^*}(\forall \mathbf{t}(\mathbf{t}^* < \mathbf{t} \supset \tau^*(\alpha)(\mathbf{w}, \mathbf{t}^*)) = 0$. Now suppose that $V_{\mu^*}(\forall \mathbf{w}(\mathbf{w}^* R \mathbf{w} \supset \tau^*(\alpha)(\mathbf{w}, \mathbf{t}^*)) = 0$. Then for some $\mathbf{t}$-alternative $\rho^*$ of $\mu^*$, $V_{\rho^*}(\mathbf{t}^* < \mathbf{t} \wedge \tau^*(\alpha)(\mathbf{w}, \mathbf{t}^*)) = 0$, and so

(i) $V_{\rho^*}(\mathbf{t}^* < \mathbf{t}) = 1$

and

(ii) $V_{\rho^*}(\tau^*(\alpha)(\mathbf{w}^*, \mathbf{t})) = 0.$

From (i) by (3) $\rho^*(\mathbf{t}^*) < \rho^*(\mathbf{t})$. Now $\rho^*$ is a $\mathbf{t}$-alternative of $\mu^*$, and so $\rho$ and $\mu$ coincide, and further $\rho^*(\mathbf{t}^*) = \mu^*(\mathbf{t}^*) = t$, and so, from (ii) by the induction hypothesis, $V_\mu(\alpha, w, \rho(\mathbf{t})) = 0$, where $t < \rho(\mathbf{t})$, and so $V_\mu(G\alpha, w, t) = 0$, as required.

The case for $H$ is exactly analogous, with $t' < t$ in place of $t < t'$, and for $L$ but with $wRw'$ in place of $t < t'$. For $K$ the only difference is that if $V_\mu(K\alpha, w, t) = 0$ there will be some $w' \in W$ and $t' \in T$ such that $K(w, t, w', t')$ and $V_\mu(\alpha, w', t') = 0$, and we choose $\rho^*$ to be the $\mathbf{w}$-$\mathbf{t}$-alternative of $\mu^*$ in which $\rho(\mathbf{w}) = w'$, and $\rho(\mathbf{t}) = t'$. Similarly if $V_\mu(\forall \mathbf{w}(K(\mathbf{w}^*, \mathbf{t}^*, \mathbf{w}, \mathbf{t}) \supset \tau^*(\alpha)(\mathbf{w}, \mathbf{t})), w, t) = 1$, then for some $\mathbf{w}$,$\mathbf{t}$-alternative $\rho^*$ of $\mu^*$ we will have $K(\rho(\mathbf{w}^*), \rho(\mathbf{t}^*), \rho(\mathbf{w}), \mu(\mathbf{t}))$ and $V_\mu(\tau^*(\alpha)(\mathbf{w}, \mathbf{t})) = 0$. So by the induction hypothesis, since $\rho^*(\mathbf{w}^*) = w$, $\rho^*(\mathbf{t}^*) = t$, $\rho^*(\mathbf{w}) = w'$ and $\rho^*(\mathbf{t}) = t'$, $K(w, t, w', t')$ and $V_\rho(\alpha, w', t') = 0$, and so $V\rho(K\alpha, w, t) = 0$.

Suppose $V_\mu(\alpha \,\Box\!\!\rightarrow\, \beta, w_1, t) = 1$. Then there is some $w_2$ such that

(i) $V_\mu(\alpha, w_2, t) = 1$ and $V_\mu(\beta, w_2, t) = 1$

and

(ii) For every $w_3 \in W$ if $V_\mu(\alpha, w_3, t) = 1$ but $V_\mu(\beta, w_3, t) = 0$ then $w_2 <_{\langle w_1, t\rangle} w_3$.

Now $\tau^*(\alpha \,\Box\!\!\rightarrow\, \beta)(\mathbf{w}^*, \mathbf{t}^*)$ is $\exists \mathbf{w}(\tau^*(\alpha)(\mathbf{w}, \mathbf{t}^*) \wedge \tau^*(\beta)(\mathbf{w}, \mathbf{t}^*) \wedge \forall \mathbf{w}'((\tau^*(\alpha)(\mathbf{w}', \mathbf{t}^*) \wedge \sim\tau^*(\beta)(\mathbf{w}', \mathbf{t}^*)) \supset C(\mathbf{t}^*, \mathbf{w}^*, \mathbf{w}, \mathbf{w}'))$. Let $\rho^*$ be the $\mathbf{w}$-alternative of $\mu^*$ such that $\rho^*(\mathbf{w}) = w_2$. Then from (i), by the induction hypothesis we have

(iii) $V_{\rho^*}(\tau^*(\alpha)(\mathbf{w}, \mathbf{t}^*)) = 1$ and $V_{\rho^*}(\tau^*(\beta)(\mathbf{w}, \mathbf{t}^*)) = 1$

and from (ii) and the induction hypothesis we have, for every $\mathbf{w}'$-alternative $\sigma^*$ of $\rho^*$, that if

(iv) $V_{\sigma^*}(\tau^*(\alpha)(\mathbf{w}', \mathbf{t}^*)) = 1$ and $V_{\sigma^*}(\tau^*(\beta)(\mathbf{w}', \mathbf{t}^*)) = 0$

then

(v) $\sigma^*(\mathbf{w}) <_{\langle \sigma^*(\mathbf{w}^*), \sigma^*(\mathbf{t})\rangle} \sigma^*(\mathbf{w}')$

From (v) by (3) we have

(vi) $V_{\sigma^*}(C(\mathbf{t}^*, \mathbf{w}^*, \mathbf{w}, \mathbf{w}')) = 1$

and so (vi) is true if (iv) is, i.e., we have

(vii) $V_{\sigma^*}((\tau^*(\alpha)(\mathbf{w}', \mathbf{t}^*) \wedge \sim\tau^*(\beta)(\mathbf{w}', \mathbf{t}^*) \supset C(\mathbf{t}^*, \mathbf{w}^*, \mathbf{w}, \mathbf{w}')) = 1$

for every $\mathbf{w}'$-alternative $\sigma^*$ of $\rho^*$, and so

(vii) $V_{\rho^*}(\forall \mathbf{w}'((\tau^*(\alpha)(\mathbf{w}', \mathbf{t}^*) \wedge \sim\tau^*(\beta)(\mathbf{w}', \mathbf{t}^*) \supset C(\mathbf{t}^*, \mathbf{w}^*, \mathbf{w}, \mathbf{w}')) = 1$.

But $\rho^*$ is a $\mathbf{w}$-alternative of $\mu^*$ and so from (ii) and (iii) we have

(ix) $V_{\mu^*}(\tau^*(\alpha \,\Box\!\!\rightarrow\, \beta)(\mathbf{w}^*, \mathbf{t}^*)) = 1$, as required. ■

# Appendix 2

## *Language and metalanguage*

The results of Appendix 1 establish that straightforward sentences of natural language require a framework which is equivalent in power to that provided by explicit quantification over times and worlds. Given that power, the possibility emerges of showing that the tense and modal language has itself the power to express the truth conditions of a modal predicate language. It might be tempting to express this by saying that such a language has the power to express its own semantics. That however would not be correct, because it is known that such languages lead to paradox, and our book is not about the semantic paradoxes. All we will say is that if you have a tense and modal object language with the resources described in Chapters 6 and 8 you can express its indexical semantics in a similar metalanguage which includes but extends the object language with a mechanism for referring to the sentences of the object language and saying things about them.[1]

The indexical semantics assumed so far uses the panoply of set theory. But in fact much less will do the trick, and a three-sorted indexical language in the style of $\mathfrak{L}_i$ is sufficient to provide the framework for the indexical semantics

[1]The aim of explaining tense in a tensed metalanguage appears in Prior 1957, p. 9. Explaining modality in a modal metalanguage became popular in the 70s among philosophers who wished to address modal language in the semantical framework advocated in Davidson 1967. See in particular Davies 1978, Gupta 1978 and Peacocke 1978, and also Davies 1981 pp.187-242. More recent work along similar lines is found in Chihara 1998. These authors were to a large extent concerned with using a metalanguage whose expressive resources did not outrun the resources of the object language, and they did not normally consider object languages which could express quantification over worlds. (This is not entirely so. Gupta on p. 467 speaks of analogues of worlds in a 'Bressanian' logic; and Peacocke 1978, pp. 485-489 and Davies 1981, pp. 220-238 consider some of the phenomena which led us in chapters 6 and 8 to the need for world quantification.) Authors like those just mentioned did not attempt to handle tense in the analogous way, and as we have noted in footnote 6 on p. 145 above, Davies follows Evans 1985 in claiming that the world/time parallel should be resisted.

for the tense and modal languages of Chapter 8.[2] Given the results of Appendix 1 it will follow therefore that one cannot argue that the semantics of tense and modality demand an untensed non-modal metalanguage — and even if one could, the mechanisms for tense and modality are exactly analogous. We consider what happens when an indexical language $\mathfrak{L}_i^+$ is used to provide the semantics for an object language $\mathfrak{L}_{x\mathbf{tw}}$, and then proceed to look at how the semantics expressed in $\mathfrak{L}_i^+$ can also be expressed in a tensed and modal metalanguage $\mathfrak{L}_{x\mathbf{tw}}^+$. Where $\alpha$ is a wff of $\mathfrak{L}_{x\mathbf{tw}}$ we use $\lceil\alpha\rceil$ for a term whose semantic value in the metalanguage $\mathfrak{L}_i^+$ or $\mathfrak{L}_{x\mathbf{tw}}^+$ is the expression $\alpha$ in the object language. The domain $D^+$ of these metalanguages must contain D and, in addition, contain all the symbols and wff of $\mathfrak{L}_{x\mathbf{tw}}$ and all the value assignments to the variables of $\mathfrak{L}_{x\mathbf{tw}}$.[3] Assume that the metalanguage has predicates ***true***, and ***sat*** to refer to truth and satisfaction in the object language. ***true*** is a four-place relation in $\mathfrak{L}_i^+$, whose first argument is a world, whose second is a time, whose third argument is a value assignment to the variables of $\mathfrak{L}_{x\mathbf{tw}}$, and whose fourth argument is a formula of $\mathfrak{L}_{x\mathbf{tw}}$. By making ***true*** involve reference to a value assignment it can apply to formulae with free variables. ***true*** is a two-place relation in $\mathfrak{L}_{x\mathbf{tw}}^+$, whose first argument is a value assignment to the variables of $\mathfrak{L}_{x\mathbf{tw}}$, and whose second argument is a formula of $\mathfrak{L}_{x\mathbf{tw}}$. Where $\alpha$ is a closed wff of $\mathfrak{L}_{x\mathbf{tw}}$ ***true*** becomes a one-place predicate in $\mathfrak{L}_{x\mathbf{tw}}^+$, reflecting the fact that in $\mathfrak{L}_{x\mathbf{tw}}$ there is a sense in which truth is 'absolute'. For both languages ***true*** is based on a 'satisfaction'

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[2]The question what kind of metalanguage to use forms the topic of an interesting exchange between Graham Priest and Hugh Mellor (Priest 1986, 1987; Mellor 1986) and has links with the results of the present appendix, and with points made in Cresswell 2006, though with important differences such as the extension to the modal case. A comment is perhaps in order here about A.N.Prior's attitude to metalogic. It is known that Prior was not sympathetic to metalogic. This at least is how you might read the reservations expressed in Prior 1967, pp. 41-45, and the remarks about expensive teacups in Prior 2003, p. 211. He may have reasoned somewhat as follows: The purpose of formal languages in philosophy is to enable the articulation of positions which are obscurely expressed in ordinary language. When you do metalogic you must use a language in which to do it. If you do semantics with, say, set theory you are simply using a formal language one level up. If you can use and understand *that* language without further semantic ascent it would seem that you can use and understand the bottom level without semantic ascent. But if you *cannot* understand a language without semantic ascent it would seem that you cannot understand a language anyway. For someone like Prior the tensed metalanguage would express the 'real' truth about the indexical theory of time, and he would claim to understand and use *that* metalanguage in just the way the rest of us understand and use the set-theoretical metalanguage. The interpretation to the two metalanguages can be given in standard set-theoretical terms because the results obtained could be generalised in such a way as to take you from any language to the next one up.

[3]Note that value assignments in $\mathfrak{L}_{x\mathbf{tw}}$ assign values only from D, not from the whole of $D^+$. It is important to stress that $D^+$, though containing D, is not contained in D. There is no suggestion that any of the languages we consider should be capable of expressing its own truth predicate, which we know to be impossible. In particular ***sat*** and ***true*** are not in $\mathfrak{L}_{x\mathbf{tw}}$ or $\mathfrak{L}_i$. (Further, a meta-metalanguage referring to $\mathfrak{L}_i^+$ would have to have a $\boldsymbol{sat}^+$ and a $\boldsymbol{true}^+$ distinct from ***sat*** and ***true***.)

relation, which, for an n-place predicate φ of $\mathfrak{L}_{x\mathbf{tw}}$, relates n members of the domain to ⌜φ⌝ at each world-time pair. Assume a three-place predicate ≈, so that ≈*abc* means that *c* is an individual variable of $\mathfrak{L}_{x\mathbf{tw}}$, and *a* and *b* are value assignments to the variables of $\mathfrak{L}_{x\mathbf{tw}}$ which coincide except possibly on *c*. The satisfaction relation determines the truth of all sentences relative to an assignment of values to the variables. In what follows one can regard the metalanguages, whether tense/modal or purely indexical as actually being *used* to express metalinguistic facts, just as when we say things like '$V_\mu(\alpha,w,t) = 1$' in ordinary semantics. Any model ⟨W,R,T,<,K,C,D,V⟩ will be a model for $\mathfrak{L}_i^+$ provided D is extended to $D^+$, and V is extended to $V^+$ by providing values for the predicates which describe features of the object language $\mathfrak{L}_{x\mathbf{tw}}$. Any such $V^+$ corresponds with an interpretation $V^{x\mathbf{wt}}$ to $\mathfrak{L}_{x\mathbf{tw}}$ in the sense that for any n-place predicate φ, any $a_1,...,a_n \in D$, $w \in W$ and $t \in T$,

(1) $\langle a_1,...,a_n,w,t\rangle \in V^{x\mathbf{wt}}(\varphi)$ iff $\langle w,t,a_1,...,a_n,\ulcorner\varphi\urcorner\rangle \in V^+(\boldsymbol{sat})$.

What the group of equivalences which follow do is constrain how the predicate ***true*** applied to complex wff of $\mathfrak{L}_i^+$ must behave if it is to reflect the intended meaning in a model in which $V^+(\boldsymbol{sat})$ satisfies (1) for some particular interpretation to the predicates of $\mathfrak{L}_{x\mathbf{tw}}$.

(2) $\forall\mathbf{w}\forall\mathbf{t}(\boldsymbol{true}(\mathbf{w},\mathbf{t},\mu,\ulcorner\varphi x_1...x_n\urcorner) \equiv \boldsymbol{sat}(\mathbf{w},\mathbf{t},\mu(x_1),...,\mu(x_n),\ulcorner\varphi\urcorner))$

(3) $\forall\mathbf{w}\forall\mathbf{t}(\boldsymbol{true}(\mathbf{w},\mathbf{t},\mu,\ulcorner N\mathbf{t}'\urcorner) \equiv \mathbf{t} = \mu(\mathbf{t}'))$

(4) $\forall\mathbf{w}\forall\mathbf{t}(\boldsymbol{true}(\mathbf{w},\mathbf{t},\mu,\ulcorner A\mathbf{w}'\urcorner) \equiv \mathbf{w} = \mu(\mathbf{w}'))$

(5) $\forall\mathbf{w}\forall\mathbf{t}\sim\boldsymbol{true}(\mathbf{w},\mathbf{t},\mu,\ulcorner\bot\urcorner))$

(6) $\forall\mathbf{w}\forall\mathbf{t}(\boldsymbol{true}(\mathbf{w},\mathbf{t},\mu,\ulcorner\alpha\supset\beta\urcorner)$
$\equiv (\boldsymbol{true}(\mathbf{w},\mathbf{t},\mu,\ulcorner\alpha\urcorner) \supset \boldsymbol{true}(\mathbf{w},\mathbf{t},\mu,\ulcorner\beta\urcorner)))$

(7) $\forall\mathbf{w}\forall\mathbf{t}(\boldsymbol{true}(\mathbf{w},\mathbf{t},\mu,\ulcorner\forall x\alpha\urcorner) \equiv \forall z(\approx\mu z\ulcorner x\urcorner \supset \boldsymbol{true}(\mathbf{w},\mathbf{t},z,\ulcorner\alpha\urcorner)))$

(8) $\forall\mathbf{w}\forall\mathbf{t}(\boldsymbol{true}(\mathbf{w},\mathbf{t},\mu,\ulcorner\forall\mathbf{w}\alpha\urcorner)$
$\equiv \forall z(\approx\mu z\ulcorner\mathbf{w}\urcorner \supset \boldsymbol{true}(\mathbf{w},\mathbf{t},z,\ulcorner\alpha\urcorner)))$

(9) $\forall\mathbf{w}\forall\mathbf{t}(\boldsymbol{true}(\mathbf{w},\mathbf{t},\mu,\ulcorner\forall\mathbf{t}\alpha\urcorner) \equiv \forall z(\approx\mu z\ulcorner\mathbf{t}\urcorner \supset \boldsymbol{true}(\mathbf{w},\mathbf{t},z,\ulcorner\alpha\urcorner)))$

(10) $\forall\mathbf{w}\forall\mathbf{t}(\boldsymbol{true}(\mathbf{w},\mathbf{t},\mu,\ulcorner G\alpha\urcorner)$
$\equiv \forall\mathbf{t}'(\mathbf{t} < \mathbf{t}' \supset \boldsymbol{true}(\mathbf{w},\mathbf{t}',\mu,\ulcorner\alpha\urcorner)))$

(11) $\forall\mathbf{w}\forall\mathbf{t}(\boldsymbol{true}(\mathbf{w},\mathbf{t},\mu,\ulcorner H\alpha\urcorner)$
$\equiv \forall\mathbf{t}'(\mathbf{t}' < \mathbf{t} \supset \boldsymbol{true}(\mathbf{w},\mathbf{t}',\mu,\ulcorner\alpha\urcorner)))$

(12) $\forall\mathbf{w}\forall\mathbf{t}(\boldsymbol{true}(\mathbf{w},\mathbf{t},\mu,\ulcorner L\alpha\urcorner)$
$\equiv \forall\mathbf{w}'(\mathbf{w}R\mathbf{w}' \supset \boldsymbol{true}(\mathbf{w}',\mathbf{t},\mu,\ulcorner\alpha\urcorner)))$

(13) $\forall\mathbf{w}\forall\mathbf{t}(\boldsymbol{true}(\mathbf{w},\mathbf{t},\mu,\ulcorner\Box\alpha\urcorner) \equiv \forall\mathbf{w}'\boldsymbol{true}(\mathbf{w}',\mathbf{t},\mu,\ulcorner\alpha\urcorner))$

(14) $\forall\mathbf{w}\forall\mathbf{t}(\boldsymbol{true}(\mathbf{w},\mathbf{t},\mu,\ulcorner K\alpha\urcorner)$

$$\equiv \forall \mathbf{w}'(\mathrm{K}(\mathbf{w},\mathbf{t},\mathbf{w}',\mathbf{t}') \supset \textbf{\textit{true}}(\mathbf{w}',\mathbf{t},\mu,\ulcorner\alpha\urcorner)))$$

(15) $$\forall \mathbf{w}\forall \mathbf{t}(\textbf{\textit{true}}(\mathbf{w},\mathbf{t},\mu,\ulcorner\alpha \mathrel{\Box\!\!\rightarrow} \beta\urcorner) \equiv \exists \mathbf{w}(\textbf{\textit{true}}(\mathbf{w}',\mathbf{t},\mu,\ulcorner\alpha\urcorner) \wedge \textbf{\textit{true}}(\mathbf{w}',\mathbf{t},\mu,\ulcorner\beta\urcorner) \wedge \forall \mathbf{w}'((\textbf{\textit{true}}(\mathbf{w}',\mathbf{t},\mu,\ulcorner\alpha\urcorner) \wedge \sim\textbf{\textit{true}}(\mathbf{w}',\mathbf{t},\mu,\ulcorner\alpha\urcorner)) \supset \mathrm{C}(\mathbf{t}^*,\mathbf{w}^*,\mathbf{w},\mathbf{w}')))$$

A word should be said here about the use of '$\mu(x)$' etc. as individual symbols in the metalanguage. For absolute precision one should introduce a three-place relation ***val***, interpreted so that $\langle b,c,a\rangle$ satisfies ***val*** iff $b$ is a value assignment to the variables of $\mathfrak{L}_{x\mathbf{tw}}$ (and is therefore in $\mathrm{D}^+$), $c$ is a variable of $\mathfrak{L}_{x\mathbf{tw}}$, and $a$ is the value of $c$ under assignment $b$. One can then express (2) as

(16) $\forall \mathbf{w}\forall \mathbf{t}\forall z(\textbf{\textit{true}}(\mathbf{w},\mathbf{t},z,\ulcorner\varphi x_1...x_n\urcorner) \equiv \exists y_1...\exists y_n(\textbf{\textit{val}}(z,\ulcorner x_1\urcorner,y_1) \wedge ... \wedge \textbf{\textit{val}}(z,\ulcorner x_n\urcorner,y_n) \wedge \textbf{\textit{sat}}(\mathbf{w},\mathbf{t},y_1,...,y_n,\ulcorner\varphi\urcorner))$.

(3) and (4) become

(17) $\forall \mathbf{w}\forall \mathbf{t}\forall z(\textbf{\textit{true}}(\mathbf{w},\mathbf{t},z,\ulcorner N\mathbf{t}'\urcorner) \equiv \exists \mathbf{t}''(\textbf{\textit{val}}(z,\ulcorner\mathbf{t}'\urcorner,\mathbf{t}'') \wedge \mathbf{t} = \mathbf{t}''))$

and

(18) $\forall \mathbf{w}\forall \mathbf{t}\forall z(\textbf{\textit{true}}(\mathbf{w},\mathbf{t},z,\ulcorner A\mathbf{w}'\urcorner) \equiv \exists \mathbf{w}''(\textbf{\textit{val}}(z,\ulcorner\mathbf{w}'\urcorner,\mathbf{w}'') \wedge \mathbf{w} = \mathbf{w}''))$

The extension of (16) to **w** and **t** is important when we recall that wff containing world and time variables have to be translated into a language $\mathfrak{L}_{\mathrm{multi}}$ which does not contain such variables, and so any complex expressions like $\mu(\mathbf{w})$ or $\mu(\mathbf{t})$ which occur in (3) or (4) have to be paraphrased in terms which use some variable $\mathbf{t}_i$ or $\mathbf{w}_i$ — as happens in (17) and (18). The case of terms like $\ulcorner\alpha\urcorner$ or $\ulcorner\varphi\urcorner$ is a little different, since each one of them refers to a specific expression of the object language. In this appendix we take them as primitive. However, there are dangers in such a treatment. Consider (2). If (2) is unpacked in accordance with (16) we see that the only occurrences of the variables $x_1,...,x_n$ are in expressions flanked by $\ulcorner$ and $\urcorner$. If such expressions are just primitive names then we can express (16) by giving $\varphi x_1...x_n$ the name ***Rodney*** and each $x_i$ the name $\textbf{\textit{Janet}}_i$. Then (16) becomes

(19) $\forall \mathbf{w}\forall \mathbf{t}\forall z(\textbf{\textit{true}}(z,\textbf{\textit{Rodney}}) \equiv \exists y_1...\exists y_n(\textbf{\textit{val}}(z,\ulcorner\textbf{\textit{Janet}}_1\urcorner,y_1) \wedge ... \wedge \textbf{\textit{val}}(z,\ulcorner\textbf{\textit{Janet}}_n\urcorner,y_n) \wedge \textbf{\textit{sat}}(\mathbf{w}',\mathbf{t}',y_1,...,y_n,\ulcorner\varphi\urcorner))$.

Given that all we have done is provide alternative names for the linguistic expressions, (19) is just as correct as (16), but of course it does not generalise in the way in which (2) does, and must be assumed case by case for every atomic

expression. This certainly counts against one motivation for such a semantics, since one wants a more general style of principle. In fact what one wants to say is that when an atomic formula is made up of an n-place predicate followed by n individual variables, a world variable and a temporal variable, then that formula is true with respect to a value assignment iff the n+2-tuple of entities assigned to the variables by that assignment satisfy the predicate in question. Such a principle can be formalised as a single sentence in a more elaborate metalanguage than the one we provide, and if we were in the business of assessing the adequacy of a formal metalanguage that would be a desideratum, indeed perhaps an essential requirement, both for (2) and for all the other clauses. Luckily it is sufficient for our purposes to give the truth clauses in a schematic form, standing in for an infinite number of wff. What is important is that whatever works in $\mathcal{L}_{x\mathbf{tw}}^{+}$ can be made to work in $\mathcal{L}_{i}^{+}$ and *vice versa*.[4] And even more important of course is that there is clearly no difference here in the way we approach time and the way we approach modality.

We next use $\tau$ as defined on p. 216 in Appendix 1, to translate (2)-(15) into wff of $\mathcal{L}_{x\mathbf{tw}}^{+}$. They become

(20) $\forall\mathbf{w}\forall\mathbf{t}([\mathbf{w},\mathbf{t}](\boldsymbol{true}(\mu,\ulcorner\varphi x_1 \ldots x_n\urcorner) \equiv [\mathbf{w},\mathbf{t}]\boldsymbol{sat}(\mu(x_1),\ldots,\mu(x_n),\ulcorner\varphi\urcorner))$

(21) $\forall\mathbf{w}\forall\mathbf{t}([\mathbf{w},\mathbf{t}](\boldsymbol{true}(\mu,\ulcorner N\mathbf{t}'\urcorner) \equiv \otimes(N\mathbf{t} \wedge N\mu(\mathbf{t}'))$

(22) $\forall\mathbf{w}\forall\mathbf{t}([\mathbf{w},\mathbf{t}](\boldsymbol{true}(\mu,\ulcorner A\mathbf{w}'\urcorner) \equiv \Diamond(A\mathbf{w} \wedge A\mu(\mathbf{w}'))$

(23) $\forall\mathbf{w}\forall\mathbf{t}{\sim}[\mathbf{w},\mathbf{t}](\boldsymbol{true}(\mu,\ulcorner\perp\urcorner))$

(24) $\forall\mathbf{w}\forall\mathbf{t}([\mathbf{w},\mathbf{t}](\boldsymbol{true}(\mu,\ulcorner\alpha \supset \beta\urcorner) \equiv$
$([\mathbf{w},\mathbf{t}](\boldsymbol{true}(\mu,\ulcorner\alpha\urcorner) \supset [\mathbf{w},\mathbf{t}](\boldsymbol{true}(\mu,\ulcorner\beta\urcorner)))$

(25) $\forall\mathbf{w}\forall\mathbf{t}([\mathbf{w},\mathbf{t}](\boldsymbol{true}(\mu,\ulcorner\forall x\alpha\urcorner) \equiv \forall z(\approx\mu z\ulcorner x\urcorner \supset [\mathbf{w},\mathbf{t}](\boldsymbol{true}(z,\ulcorner\alpha\urcorner)))$

(26) $\forall\mathbf{w}\forall\mathbf{t}([\mathbf{w},\mathbf{t}](\boldsymbol{true}(\mu,\ulcorner\forall\mathbf{w}'\alpha\urcorner) \equiv \forall z(\approx\mu z\ulcorner\mathbf{w}'\urcorner \supset [\mathbf{w},\mathbf{t}](\boldsymbol{true}(z,\ulcorner\alpha\urcorner)))$

(27) $\forall\mathbf{w}\forall\mathbf{t}([\mathbf{w},\mathbf{t}](\boldsymbol{true}(\mu,\ulcorner\forall\mathbf{t}'\alpha\urcorner) \equiv \forall z(\approx\mu z\ulcorner\mathbf{t}'\urcorner \supset [\mathbf{w},\mathbf{t}](\boldsymbol{true}(z,\ulcorner\alpha\urcorner)))$

(28) $\forall\mathbf{w}\forall\mathbf{t}([\mathbf{w},\mathbf{t}](\boldsymbol{true}(\mu,\ulcorner G\alpha\urcorner) \equiv$
$\mathbf{t}'(\otimes(N\mathbf{t} \wedge FN\mathbf{t}') \supset [\mathbf{w},\mathbf{t}'](\boldsymbol{true}(\mu,\ulcorner\alpha\urcorner)))$

(29) $\forall\mathbf{w}\forall\mathbf{t}([\mathbf{w},\mathbf{t}](\boldsymbol{true}(\mu,\ulcorner H\alpha\urcorner) \equiv$
$\forall\mathbf{t}'(\otimes(N\mathbf{t}' \wedge FN\mathbf{t}) \supset [\mathbf{w},\mathbf{t}'](\boldsymbol{true}(\mu,\ulcorner\alpha\urcorner)))$

(30) $\forall\mathbf{w}\forall\mathbf{t}([\mathbf{w},\mathbf{t}](\boldsymbol{true}(\mu,\ulcorner L\alpha\urcorner) \equiv$
$\forall\mathbf{w}'(\Diamond(A\mathbf{w} \wedge MA\mathbf{w}') \supset [\mathbf{w}',\mathbf{t}]\boldsymbol{true}(\mu,\ulcorner\alpha\urcorner)))$

[4]In fact, given that our languages do not contain primitive terms, terms like $\ulcorner\varphi\urcorner$ will actually need to be represented by *predicates* $\ulcorner\varphi\urcorner x$, and the expressions containing them expressed in primitive notation by Russell's method. The notation $\ulcorner\varphi\urcorner$ and $\ulcorner\alpha\urcorner$ is meant to suggest the device of quasi-quotation introduced in Quine 1940 (second edition pp. 33-37). However Quine was not engaged in the business of formalising a metalanguage, so our use here has at best a somewhat tenuous connection with Quine's notation.

(31) $\forall\mathbf{w}\forall\mathbf{t}([\mathbf{w},\mathbf{t}](\textit{true}(\mu,\ulcorner\Box\alpha\urcorner) \equiv [\mathbf{w},\mathbf{t}']\forall\mathbf{w}'\textit{true}(\mu,\ulcorner\alpha\urcorner))$

(32) $\forall\mathbf{w}\forall\mathbf{t}([\mathbf{w},\mathbf{t}](\textit{true}(\mu,\ulcorner K\alpha\urcorner) \equiv$
$\forall\mathbf{w}'(\Diamond\otimes(N\mathbf{t}\wedge A\mathbf{w}\wedge K_{\Diamond}(N\mathbf{t}'\wedge A\mathbf{w}')\supset[\mathbf{w}',\mathbf{t}]\textit{true}(\mu,\ulcorner\alpha\urcorner)))$

(33) $\forall\mathbf{w}\forall\mathbf{t}([\mathbf{w},\mathbf{t}](\textit{true}(\mu,\ulcorner\alpha \Box\!\!\rightarrow \beta\urcorner) \equiv \exists\mathbf{w}([\mathbf{w}',\mathbf{t}]\textit{true}(\mu,\ulcorner\alpha\urcorner) \wedge$
$[\mathbf{w}',\mathbf{t}]\textit{true}(\mu,\ulcorner\beta\urcorner)\wedge\forall\mathbf{w}'([\mathbf{w}',\mathbf{t}]\textit{true}(\mu,\ulcorner\alpha\urcorner)\wedge\sim[\mathbf{w}',\mathbf{t}]\textit{true}(\mu,\ulcorner\alpha\urcorner))$
$\supset\Box\oplus((N\mathbf{t}\wedge A\mathbf{w}_1)\supset((A\mathbf{w}_2\vee A\mathbf{w}_3)\Box\!\!\rightarrow A\mathbf{w}_2)))$

The equivalences stated in (20)-(33) are all guaranteed by theorem 8.9, but in fact what we really want is the ability to express the truth conditions of sentences of $\mathfrak{L}_{x\mathbf{tw}}$ in a more direct manner than is obtained using the translation $\tau$ defined on p. 217 above. On p. 9 of Prior 1957, Prior is clear that there is a close connection between the *present* truth of a past tense sentence, and the *past* truth of the corresponding present tense sentence:

> But although the statement 'It will be the case that Professor Carnap is flying to the moon', that is 'Professor Carnap will be flying to the moon', is not exactly a statement *about* the statement 'Professor Carnap is flying to the moon', we may say that the future tense statement *is* true if and only if the present-tense statement *will be* true. Similarly the past-tense statement 'It has been the case that Professor Carnap is flying to the moon', that is 'Professor Carnap has been flying to the moon' *is* true if and only if the present-tense statement 'Professor Carnap is flying to the moon' *has been* true;...'

What Prior is after can be stated for $\mathfrak{L}_{x\mathbf{tw}}$ by the equivalences:

(34) $\Box\oplus(\textit{true}(\mu,\ulcorner\varphi x_1...x_n\urcorner) \equiv \textit{sat}(\mu(x_1),...,\mu(x_n),\ulcorner\varphi\urcorner))$

(35) $\Box\oplus(\textit{true}(\mu,\ulcorner N\mathbf{t}\urcorner) \equiv N\mu(\mathbf{t}))$

(36) $\Box\oplus(\textit{true}(\mu,\ulcorner A\mathbf{w}\urcorner) \equiv A\mu(\mathbf{w}))$

(37) $\Box\oplus\sim\textit{true}(\mu,\ulcorner\perp\urcorner))$

(38) $\Box\oplus(\textit{true}(\mu,\ulcorner\alpha\supset\beta\urcorner) \equiv (\textit{true}(\mu,\ulcorner\alpha\urcorner)\supset\textit{true}(\mu,\ulcorner\beta\urcorner)))$

(39) $\Box\oplus(\textit{true}(\mu,\ulcorner\forall x\alpha\urcorner) \equiv \forall z(\approx\mu z\ulcorner x\urcorner\supset\textit{true}(z,\ulcorner\alpha\urcorner)))$

(40) $\Box\oplus(\textit{true}(\mu,\ulcorner\forall\mathbf{w}\alpha\urcorner) \equiv \forall z(\approx\mu z\ulcorner\mathbf{w}\urcorner\supset\textit{true}(z,\ulcorner\alpha\urcorner)))$

(41) $\Box\oplus(\textit{true}(\mu,\ulcorner\forall\mathbf{t}\alpha\urcorner) \equiv \forall z(\approx\mu z\ulcorner\mathbf{t}\urcorner\supset\textit{true}(z,\ulcorner\alpha\urcorner)))$

(42) $\Box\oplus(\textit{true}(\mu,\ulcorner G\alpha\urcorner) \equiv G\textit{true}(\mu,\ulcorner\alpha\urcorner))$

(43) $\Box\oplus(\textit{true}(\mu,\ulcorner H\alpha\urcorner) \equiv H\textit{true}(\mu,\ulcorner\alpha\urcorner))$

(44) $\Box\oplus(\textit{true}(\mu,\ulcorner L\alpha\urcorner) \equiv L\textit{true}(\mu,\ulcorner\alpha\urcorner))$

(45) $\Box\oplus(\textit{true}(\mu,\ulcorner K\alpha\urcorner) \equiv K\textit{true}(\mu,\ulcorner\alpha\urcorner))$

(46) $\Box\oplus(\textit{true}(\mu,\ulcorner\Box\alpha\urcorner) \equiv \Box\textit{true}(\mu,\ulcorner\alpha\urcorner))$

(47) $\Box\oplus(\textbf{\textit{true}}(\mu,\ulcorner\alpha \Box\!\!\rightarrow \beta\alpha\urcorner) \equiv (\textbf{\textit{true}}(\mu,\ulcorner\alpha\urcorner)) \Box\!\!\rightarrow \textbf{\textit{true}}(\mu,\ulcorner\alpha\urcorner)))$

Prior of course would offer (34)-(47) as they stand as an adequate specification of the semantics of $\mathfrak{L}_{x\mathbf{tw}}$, so it is worth establishing that they are exactly equivalent to (20)-(33), even though in (20)-(33) we already have the result that the indexical semantics for $\mathfrak{L}_{x\mathbf{tw}}$ can be expressed in $\mathfrak{L}_{x\mathbf{tw}}^{+}$. Notice that the truth clauses in (34)-(47) are in the scope of $\Box\oplus$. This is necessary because they have to apply recursively to sentences at any level of embedding. To get (34)-(47) from (20)-(33) we need to the establish equivalences between the right hand sides of each of these wff in $\mathfrak{L}_{x\mathbf{tw}}$. The proofs make use of the following lemmas. For any $w^* \in$ W and $t^* \in$ T,

LEMMA A2.1 $\forall\mathbf{w}\forall\mathbf{t}([\mathbf{w},\mathbf{t}]\alpha \equiv [\mathbf{w},\mathbf{t}]\beta) \equiv \Box\oplus(\alpha \equiv \beta),w^*,t^*) = 1$, provided neither **w** not **t** is free in $\alpha$ or $\beta$.

*Proof*: Suppose $V_\mu(\forall\mathbf{w}\forall\mathbf{t}([\mathbf{w},\mathbf{t}]\alpha \equiv [\mathbf{w},\mathbf{t}]\beta),w^*,t^*) = 1$. Take any $w \in$ W and $t \in$ T, and let $\rho$ be the **w**-**t**-alternative of $\mu$ in which $\rho(\mathbf{w}) = w$ and $\rho(\mathbf{t}) = t$. Then

(i) $V_\rho([\mathbf{w},\mathbf{t}]\alpha \equiv [\mathbf{w},\mathbf{t}]\beta,w^*,t^*) = 1.$

So

(ii) $V_\rho([\mathbf{w},\mathbf{t}]\alpha,w^*,t^*) = V_\rho([\mathbf{w},\mathbf{t}]\beta),w^*,t^*).$

Now

(iii) $V_\rho([\mathbf{w},\mathbf{t}]\alpha,w^*,t^*) = V_\rho(\alpha,\rho(\mathbf{w}),\rho(\mathbf{t}))$

and

(iv) $V_\rho([\mathbf{w},\mathbf{t}]\beta),w^*,t^*) = V_\rho(\beta,\rho(\mathbf{w}),\rho(\mathbf{t}))$

But $\rho(\mathbf{w}) = w$, and $\rho(\mathbf{t}) = t$, and so $V_\rho(\alpha,w,t) = V_\rho(\beta,w,t)$, and so

(v) $V_\rho(\alpha \equiv \beta,w,t) = 1.$

But $w$ and $t$ were chosen to be *any* $w \in$ W and $t \in$ T, and so

(vi) $V_\rho(\Box\oplus(\alpha \equiv \beta),w^*,t^*) = 1.$

So, provided **w** and **t** are not free in $\alpha$ or $\beta$,

(vii) $V_\mu(\Box\oplus(\alpha \equiv \beta),w^*,t^*) = 1.$

Now suppose $V_\mu(\forall\mathbf{w}\forall\mathbf{t}([\mathbf{w},\mathbf{t}]\alpha \equiv [\mathbf{w},\mathbf{t}]\beta),w^*,t^*) = 0$. Then there is a **w**-**t**-

alternative ρ of μ such that

(viii) $V_\rho([\mathbf{w},\mathbf{t}]\alpha \equiv [\mathbf{w},\mathbf{t}]\beta,w^*,t^*) = 0$

So

(ix) $V_\rho([\mathbf{w},\mathbf{t}]\alpha,w^*,t^*) \neq V_\rho([\mathbf{w},\mathbf{t}]\beta),w^*,t^*)$

Now

(xi) $V_\rho([\mathbf{w},\mathbf{t}]\alpha,w^*,t^*) = V_\rho(\alpha,\rho(\mathbf{w}),\rho(\mathbf{t}))$

and

(xii) $V_\rho([\mathbf{w},\mathbf{t}]\beta),w^*,t^*) = V_\rho(\beta,\rho(\mathbf{w}),\rho(\mathbf{t}))$

and so $V_\rho(\alpha,\rho(\mathbf{w}),\rho(\mathbf{t})) \neq V_\rho(\beta,\rho(\mathbf{w}),\rho(\mathbf{t}))$, and so

(xiii) $V_\rho(\alpha \equiv \beta,\rho(\mathbf{w}),\rho(\mathbf{t})) = 0.$

and so

(xiv) $V_\rho(\Box\oplus(\alpha \equiv \beta),w^*,t^*) = 0$

So, provided **w** and **t** are not free in α or β,

(xv) $V_\mu(\Box\oplus(\alpha \equiv \beta),w^*,t^*) = 0.$ ■

LEMMA A2.2

(a) $V_\mu([\mathbf{w},\mathbf{t}]\forall\mathbf{t}'(\Diamond(N\mathbf{t} \wedge FN\mathbf{t}')) \supset [\mathbf{w},\mathbf{t}']\alpha),w^*,t^*)$
$= V_\mu[\mathbf{w},\mathbf{t}]G\alpha,w^*,t^*)$

(b) $V_\mu([\mathbf{w},\mathbf{t}]\forall\mathbf{t}'(\Diamond(N\mathbf{t}' \wedge FN\mathbf{t})) \supset [\mathbf{w},\mathbf{t}']\alpha),w^*,t^*)$
$= V_\mu[\mathbf{w},\mathbf{t}]H\alpha,w^*,t^*)$

(c) $V_\mu([\mathbf{w},\mathbf{t}]\forall\mathbf{t}'(\Box(A\mathbf{w} \wedge MA\mathbf{w}')) \supset [\mathbf{w}',\mathbf{t}]\alpha),w^*,t^*)$
$= V_\mu[\mathbf{w},\mathbf{t}]L\alpha,w^*,t^*)$

(d) $V_\mu([\mathbf{w},\mathbf{t}]\forall\mathbf{t}'(\Box\Diamond(N\mathbf{t} \wedge A\mathbf{w} \wedge K_\Diamond(N\mathbf{t}' \wedge A\mathbf{w}'))) \supset [\mathbf{w}',\mathbf{t}]\alpha),w^*,t^*)$
$= V_\mu[\mathbf{w},\mathbf{t}]K\alpha,w^*,t^*)$

(e) $V_\mu([\mathbf{w},\mathbf{t}]\forall\mathbf{t}'(\Box\oplus((N\mathbf{t} \wedge A\mathbf{w}_1) \supset ((A\mathbf{w}_2 \vee A\mathbf{w}_3) \mathbin{\Box\!\!\rightarrow} A\mathbf{w}_2)),w^*,t^*) = V_\mu[\mathbf{w},\mathbf{t}](\alpha \mathbin{\Box\!\!\rightarrow} \beta),w^*,t^*)$

*Proof*: We shall illustrate the case (a) and then indicate how the others would go. Suppose $V_\mu([\mathbf{w},\mathbf{t}]\forall\mathbf{t}'(\oplus(N\mathbf{t} \wedge FN\mathbf{t}') \supset [\mathbf{w},\mathbf{t}']\alpha),w^*,t^*) = 0$. Then

(i) $V_\mu(\forall\mathbf{t}'(\oplus(N\mathbf{t} \wedge FN\mathbf{t}') \supset [\mathbf{w},\mathbf{t}']\alpha),\mu(\mathbf{w}),\mu(\mathbf{t})) = 0$

So there is a $\mathbf{t}'$-alternative ρ of μ such that

(ii) $V_\rho(\oplus(N\mathbf{t} \wedge FN\mathbf{t}'),\mu(\mathbf{w}),\mu(\mathbf{t})) = 1$

and

(iii) $V_\rho([\mathbf{w},\mathbf{t}']\alpha),\mu(\mathbf{w}),\mu(\mathbf{t})) = 0$

Since $\mathbf{t}'$ is not free in $\alpha$, from (iii)

(iv) $V_\mu(\alpha,\mu(\mathbf{w}),\rho(\mathbf{t})) = 0$

But we already know from theorem 8.9 that (ii) holds iff $\rho(\mathbf{t}) < \rho(\mathbf{t}')$, and since $\rho$ is a $\mathbf{t}'$-alternative of $\mu$ then $\rho(\mathbf{t}) = \mu(\mathbf{t})$, and so there is some $t'$ such that $\mu(\mathbf{t}) < t'$ and

(v) $V_\mu(\alpha,\mu(\mathbf{w}),t') = 0$

and so

(vi) $V_\mu(G\alpha,\mu(\mathbf{w}),\mu(\mathbf{t})) = 0.$

So

(viii) $V_\mu[\mathbf{w},\mathbf{t}]G\alpha,w^*,t^*).$

Suppose $V_\mu[\mathbf{w},\mathbf{t}]G\alpha,w^*,t^*) = 0$. Then $V_\mu(G\alpha,\mu(\mathbf{w}),\mu(\mathbf{t})) = 0$ and there is some $t'$ such that $\mu(\mathbf{t}) < t'$ and

(ix) $V_\mu(\alpha,\mu(\mathbf{w}),t') = 0$

Let $\rho$ be the $\mathbf{t}'$-alternative of $\mu$ for which $\rho(\mathbf{t}') = t'$. Then

(x) $V_\mu(\alpha,\rho(\mathbf{w}),\rho(\mathbf{t}')) = 0$

But neither $\mathbf{w}$ not $\mathbf{t}$ is free in $\alpha$, and so

(xi) $V_\rho(\alpha,\rho(\mathbf{w}),\rho(\mathbf{t}')) = 0$

and so

(xii) $V_\rho([\mathbf{w},\mathbf{t}']\alpha,\mu(\mathbf{w}),\mu(\mathbf{t}')) = 0$

But given that $\rho(\mathbf{t}) < \rho(\mathbf{t}')$, we have

(xi) $V_\rho(\oplus(N\mathbf{t} \wedge FN\mathbf{t}'),\mu(\mathbf{w}),\mu(\mathbf{t})) = 1$

and so

(xii) $V_\rho(\oplus(N\mathbf{t} \wedge FN\mathbf{t}') \supset [\mathbf{w},\mathbf{t}']\alpha,\mu(\mathbf{w}),\mu(\mathbf{t})) = 0$

But ρ is a $\mathbf{t}'$-alternative of μ and so

(xiii) $V_\mu(\forall\mathbf{t}'(\oplus(N\mathbf{t} \wedge FN\mathbf{t}')) \supset [\mathbf{w},\mathbf{t}']\alpha),\mu(\mathbf{w}),\mu(\mathbf{t})) = 0$

and so

(xiv) $V_\mu([\mathbf{w},\mathbf{t}]\forall\mathbf{t}'(\oplus(N\mathbf{t} \wedge FN\mathbf{t}')) \supset [\mathbf{w},\mathbf{t}']\alpha),w^*,t^*) = 0.$

(b)-(d) are proved analogously. (e) is trickier, but we know from theorem 8.9 that $V_\mu([\mathbf{w},\mathbf{t}]\forall\mathbf{t}'(\Box\oplus((N\mathbf{t} \wedge A\mathbf{w}_1) \supset ((A\mathbf{w}_2 \vee A\mathbf{w}_3) \mathrel{\Box\!\!\rightarrow} A\mathbf{w}_2)),w^*,t^*) = 1$ iff $\mu^*(\mathbf{w}) <_{\langle\sigma^*(\mathbf{w}^*),\sigma^*(\mathbf{t})\rangle} \mu^*(\mathbf{w}')$. In the light of this, if (e) is unpacked, what it says is that there is some $w'$ at which α and β are both true, and if any $w''$ is such that α is true at $w''$ but β is not, then $w' <_{\langle w,t\rangle} w''$; which of course are the conditions for the truth of $\alpha \mathrel{\Box\!\!\rightarrow} \beta$. ■

These lemmas straightforwardly give

LEMMA A2.3 (20)-(33) are equivalent in all contexts to (34)-(47).

COROLLARY A2.4 (34)-(47) are valid in all models in which (2)-(15) are valid.

In the light of the results of this appendix, here is what a defender of tense and modal primitives might say to a defender of the indexical semantics about what we have been doing:

> The issue is not whether we quantify over times and worlds. Remember that although the $\mathfrak{L}_{x\mathbf{tw}}$ languages involve explicit quantification over worlds and times they are exactly equivalent to the multiply-indexed languages $\mathfrak{L}_{\text{multi}}$ presented in Chapter 7. So when we prove results like those in this appendix and the last in an indexical metalanguage or meta-metalanguage, we are entitled to construe those metalanguages in a tensed and modal metalanguage with ***then*** and ***actually*** operators. So each of my metalanguages will be equivalent to a metalanguage in the style of $\mathfrak{L}_i^+$, and we are entitled to interpret this in a tensed and modal way, knowing that your meta-metalanguage will assure you that we mean the same as you do. By all means say that you understand the equivalence because you understand the set-theoretical meta-metalanguage. But, in saying that, you admit that you *do* understand a tensed and modal

metalanguage, and that you could understand a tensed meta-metalanguage, and so on.

Given that they are logically equivalent, we do not need to take sides on whether an indexical treatment or a tense/modal treatment reflects a better metaphysics for time and modality. But more importantly for the purposes of this book what is clear is that there is no *logical* reason to prevent what is said about the temporal case from being equally be said about the modal case.

# Appendix 3

## *Plantinga's metaphysics*

A Plantinga-style language $\mathfrak{L}_P$ will extend the languages of Chapter 5 by allowing variables **s**, **s**′, or $\mathbf{s}_1$, $\mathbf{s}_2$ etc. for states of affairs, together with the predicate ***O***. These variables only occur in atomic wff of the form ***O*****s**. $\mathfrak{L}_P$ will not contain the time and world variables or the primitive predicates *A* or *N*, though as noted on p. 186 we can use **w** and **t** as world and time variables, provided we remember that they are simply variables for states of affairs. We can then interpret *A***w** to mean that a temporally invariant maximally possible state of affairs obtains, and *N***t** to mean that a maximal modally invariant state of affairs obtains:

$W\mathbf{s} =_{df} \Box(\otimes \boldsymbol{O}\mathbf{s} \supset \oplus \boldsymbol{O}\mathbf{s}) \wedge \forall \mathbf{s}'((\Box(\otimes \boldsymbol{O}\mathbf{s}' \supset \oplus \boldsymbol{O}\mathbf{s}') \wedge \Diamond(\boldsymbol{O}\mathbf{s}' \wedge \boldsymbol{O}\mathbf{s})) \supset \Box(\boldsymbol{O}\mathbf{s} \supset \boldsymbol{O}\mathbf{s}'))$

$T\mathbf{s} =_{df} \oplus(\Diamond \boldsymbol{O}\mathbf{s} \supset \Box \boldsymbol{O}\mathbf{s}) \wedge \forall \mathbf{s}'((\oplus(\Diamond \boldsymbol{O}\mathbf{s}' \supset \Box \boldsymbol{O}\mathbf{s}') \wedge \otimes(\boldsymbol{O}\mathbf{s}' \wedge \boldsymbol{O}\mathbf{s})) \supset \oplus(\boldsymbol{O}\mathbf{s} \supset \boldsymbol{O}\mathbf{s}'))$

$A\mathbf{s} =_{df} W\mathbf{s} \wedge \boldsymbol{O}\mathbf{s}$

$N\mathbf{s} =_{df} T\mathbf{s} \wedge \boldsymbol{O}\mathbf{s}.$

Plantinga will need to accept principles which guarantee that times and worlds exist, i.e., that temporally and modally fixed states of affairs exist and that each is included in a maximal one — principles such as:

(1) $\exists \mathbf{s} \Box(\otimes \boldsymbol{O}\mathbf{s} \supset \oplus \boldsymbol{O}\mathbf{s})$
(2) $\exists \mathbf{s} \oplus(\Diamond \boldsymbol{O}\mathbf{s} \supset \Box \boldsymbol{O}\mathbf{s})$
(3) $\forall \mathbf{s}(\Box(\otimes \boldsymbol{O}\mathbf{s} \supset \oplus \boldsymbol{O}\mathbf{s}) \supset \exists \mathbf{s}' \Diamond(A\mathbf{s}' \wedge \boldsymbol{O}\mathbf{s} \wedge \boldsymbol{O}\mathbf{s}'))$
(4) $\forall \mathbf{s}(\oplus(\Diamond \boldsymbol{O}\mathbf{s} \supset \Box \boldsymbol{O}\mathbf{s}) \supset \exists \mathbf{s}' \otimes(N\mathbf{s}' \wedge \boldsymbol{O}\mathbf{s} \wedge \boldsymbol{O}\mathbf{s}'))$

They are versions of the completion principles referred to on pp. 35 and 109. The

need for such principles is discussed by Pollock 1985 pp. 121-126. Given the essentially second-order nature of this semantics the possibility of a complete axiomatisation is unclear. We can however give a proof of relative consistency for Plantinga's metaphysics, showing that Plantinga's states-of-affairs metaphysics incorporates a model for our $\mathfrak{L}_{x\mathbf{tw}}$ languages. We do this by taking states of affairs to be sets of world-time pairs, and allowing quantification over them. A state of affairs $s$ may then be said to *obtain* at a time $t$ in a world $w$ iff $\langle w,t\rangle \in s$. We must then require that the sate of affairs variables, $\mathbf{s}$, $\mathbf{s}'$, etc. are assigned states of affairs $s$, $s'$ etc. With that change it is not difficult to verify that $V_\mu(W\mathbf{s},w,t) = 1$ iff there is some $w^*$ such that for any $w' \in W$, and $t' \in T$, $\langle w',t'\rangle \in \mu(\mathbf{s})$ iff $w' = w^*$, and that there is some $t^*$ such that $\langle w',t'\rangle \in \mu(\mathbf{s})$ iff $t' = t^*$. We may refer to this $w^*$ and $t^*$ as $w_s$ and $t_s$. (1)-(4) guarantee that there always will be such a $w_s$ and $t_s$. $V_{\mu^*}(W\mathbf{s},w,t) = 1$ iff $w = w_\mu(\mathbf{s})$, and $V_{\mu^*}(T\mathbf{s},w,t) = 1$ iff $t = t_{\mu(\mathbf{s})}$. From this we have that $V_\mu(\Box(W\mathbf{s} \supset \alpha),w,t) = V_\mu(\alpha,w_{\mu(\mathbf{s})},t)$, and $V_\mu(\oplus(T\mathbf{s} \supset \alpha),w,t) = V_\mu(\alpha,w,t_{\mu(\mathbf{s})})$, and, and so, $V_\mu([\mathbf{s},\mathbf{s}']\alpha),w,t) = V_\mu(\alpha,w_{\mu(\mathbf{s})}, t_{\mu(\mathbf{s}')})$. If we define $[\mathbf{s},\mathbf{s}']\alpha$ as $\Diamond\otimes(A\mathbf{s} \wedge N\mathbf{s}' \wedge \alpha)$, we have that for every $w \in W$ and $t \in T$,

(5) $\quad V_\mu(\alpha \equiv \forall\mathbf{s}\forall\mathbf{s}'((A\mathbf{s} \wedge N\mathbf{s}') \supset [\mathbf{s},\mathbf{s}']\alpha),w,t) = 1.$

Although Plantinga's object *language* only uses devices available to Plantinga, our *interpretation* of that language is a standard indexical one. But we have shown in Appendix 2 that any standard indexical semantics can be expressed in a modal and tense language of a kind acceptable to Plantinga. Quine's being a distinguished philosopher (see p. 56) can be expressed as

(6) $\quad \{\langle w,t\rangle$: There is an assignment $\mu$ such that, for some variables $\mathbf{w}$ and $\mathbf{t}$, $\mu(\mathbf{w}) = w$ and $\mu(\mathbf{t}) = t$ and $V_\mu(A\mathbf{w} \wedge N\mathbf{t}) = 1$ and Quine is a distinguished philosopher at $t$ in $w\}$.

In the case of individuals we have to change the domain of interpretation in the following way: Where D is a domain of possibilia, and $a \in D$, the 'essence', $a^*$, of $a$ is $\{\langle a',w,t\rangle: a = a', w \in W, t \in T\}$. Let $D^{ess}$ be the domain of all essences based on D, and let quantification over possibilia be replaced by quantification over essences. Predication is then analysed in terms of co-exemplification of essences as described in Chapter 16. An interpretation V to the predicates is replaced by an interpretation $V^{ess}$ in such a way that where $\varphi$ is an n-place predicate $V^{ess}(\varphi)$ is a set of n+2-tuples of the form $\langle e_1,...,e_n,w,t\rangle$, where $e_1, ..., e_n$ are essences. $V^{ess}(\varphi)$ stipulates just which n-tuples of them are co-exemplified with $\varphi$. Since essences exist in all worlds, the quantifiers can range

over all essences. We then have a straightforward correspondence with a possibilistic interpretation by letting V and $V^{ess}$ be so related that

(7) $\langle a_1,...a_n,w,t\rangle \in V(\varphi)$ iff $\langle a_1^*,...a_n^*,w,t\rangle \in V^{ess}(\varphi)$

Finally we assume a primitive 'exemplification' predicate ***exemplified***, which corresponds with the possibilistic predicate ***exists*** in such a way that $\langle a,w,t\rangle \in V^{ess}(\textbf{\textit{exists}})$ iff $\langle a^*,w,t\rangle \in V(\textbf{\textit{exemplified}})$. Where a possibilist says that *a* exists in a world *w* at a time *t* Plantinga says that *a*'s essence, *a** is exemplified in *w* at *t*.

It is important to stress that this possibilist modelling of Plantinga's metaphysics is just that. Structures made in this way differ from Plantinga's metaphysics in two respects. First, they take logically equivalent propositions and properties to be identical, and second they make the essences out of possibilia. The issues raised by the first only arise in the semantics of propositional attitudes, and would have to be faced in any case in a world and time semantics for extensions of the languages introduced in Chapters 7 and 8. So far as the second goes, even though possibilia appear in the original domain they are not in the domain of quantification for the Plantinga languages. To get Plantinga's own metaphysics all we need to ensure is that the structures he has in mind can be embedded into the possibilistic structures we have just defined, and that is enough for a relative consistency proof, and to provide an answer for worries like that suggested on p. 164 of Hazen 1996, that aspects of Plantinga's metaphysics may lead to paradox, by showing that it need be no more paradoxical than the standard set-theoretical account.

# Appendix 4

# *Interval semantics*

Some phenomena from natural language might suggest that while temporal sentences are true or false at worlds they are not true or false at instants but only at time *intervals*.[1] One might claim that the present time (the 'specious present') is not a single moment but an interval. This means that overlapping that interval are other 'presents' extending that first one further into the past and further into the future. This process cannot stop until we have all of time, and *that* is why there can be no privileged moment in the way that there can be a privileged world. Put in the manner of Chapter 4 the argument would be that while we can, following Adams and Plantinga, assume an unanalysed notion of *actual* truth there is no analogous notion of unanalysed present truth. In Prior 1968b, Prior is acutely aware that utterances take time, and on p. 25 of 2003 says "It is, in short, not what is the case at an instant, but what most signally and irreducibly is not, that presents the hardest problem for the tense-logician." In Chapter 2 (p. 35) we stated that a semantical index *i* satisfies the principle that ~α is true at *i* iff α is not true at *i*. If we have temporal operators like [**t**], as introduced on p. 94, the principle can be stated as

(1) $[\mathbf{t}]\sim\alpha \equiv \sim[\mathbf{t}]\alpha$

where [**t**] stands for an arbitrary temporal operator. And in general, where **i** is a variable for *any* index, we have

(2) $[\mathbf{i}]\sim\alpha \equiv \sim[\mathbf{i}]\alpha$.

Now the discussion in Chapter 2 suggested that (1) does not hold for intervals, and therefore that an interval is not an index of truth. In terms of what we said

[1]Interval semantics is the principal theme of Cresswell 1985b, where sentences like (3) are considered at length. Other work up to that time is mentioned in the bibliographies of the articles collected there.

there about ***on Monday*** this was so, since for α to be true on Monday meant for it to be true at some instant on Monday. But in the case of a sentence like

(3) Bugsy polishes all the boots

one can imagine that (1) may well apply to it, and that if *t* is not an interval of Bugsy's polishing all the boots then

(4) Bugsy does not polish all the boots

is true at *t*. If Bugsy polishes all the boots between 2.00pm and 3.45pm on Monday it may well be true that he does *not* polish them all between 2.50 and 3.15. The point of truth at an interval is that it is not to be analysed as truth at every instant in that interval, and so nothing prevents (3) from satisfying (1). But then, what do we say about

(5) Bugsy polishes all the boots on Monday?

Surely (5) is true iff there is a sub-interval *t* of Monday at which (3) is true? To see that the situation is not quite so clear consider

(6) Tallulah sang exactly twice on Monday

For (6) to be true at *t* there would need to be two subintervals of Monday, $t_1$ and $t_2$, which do not overlap, each of which is an interval of Tallulah's singing; and there is no other sub-interval of Monday which is an interval of her singing. This means that the following semantics for [**Monday**] would be incorrect:

(7) [**Monday**]α is true at *t* (in world *w*) iff there is a sub-interval *t′* of Monday at which α is true

(7) is wrong because it would make (6) true iff there is a sub-interval of Monday at which Tallulah sings exactly twice; and this could be so however many *other* sub-intervals of Monday were also intervals at which she sang; but if she did then (6) would be *false*. In (6) the quantifier is in the 'exactly two' phrase, and this could perhaps have the semantics

(8) ***exactly twice*** α is true at *t* iff there are exactly two sub-intervals $t_1$ and $t_2$ of *t* such that α is true at $t_1$ and α is true at $t_2$.

This isn't quite right, since there may be a number of subintervals which in a sense all refer to the same occasion of α's truth as $t_1$ does, and others which refer to the same occasion as $t_2$ does. Still, it will do for now as an approximation for our purposes. We can then say that

(9) [**Monday**]α is true at $t$ iff α is true at $t_{\text{Monday}}$, where $t_{\text{Monday}}$ is the interval occupied by the Monday in question

(9) is in fact the semantics you would get by treating [**Monday**] as the equivalent in an interval-based semantics of [**t**] in a moment-based semantics. Then (6) can be formalised as

(10) [**Monday**] ***exactly twice sing Tallulah***.

What then of the simple

(11) Tallulah sings on Monday?

One can certainly define an operator ***once***, meaning at least once, whose semantics is that ***once*** α is true at $t$ iff α itself is true at some sub-interval $t'$ of $t$. We then note that there are many instances of 'default' quantifiers in natural language — as when we say that someone is a mother, meaning that she is someone's mother. If so we may take the interpretation of (11) to be:

(12) [**Monday**] ***once sing Tallulah***

where the ***once*** is a purely logical operator, which does not reach the surface. We can portray all these operators in terms of relations between indices. For [**Monday**] we can say that $t\text{R}_{\text{Monday}}t'$ iff $t = t_{\text{Monday}}$; for ***once*** that $t\text{R}_{\textit{once}}t'$ iff $y'$ is a subinterval of $t$; and so on. These are not the simple < used in the semantics of *P* and *F* on p. 15, and in that respect intervals behave more like modal operators, where each operator has its own 'accessibility' relation, which is not as a rule a linear one. The mere fact that the index is an *interval* does not stop it from being classical in the sense of satisfying (1). All it means is that the 'times' in the set T in the models of Chapter 5 are intervals, and relations between temporal indices are more complex than the straightforward < between instants. Intervals enable tense distinctions to be made which reflect what is called *aspect*. Thus for instance one can distinguish two past operators, which we shall call $P_{\textit{simple}}$ and $P_{\textit{perfect}}$.

(13) $P_{simple}\alpha$ is true at $t$ iff there is some $t'$ such that every moment in $t'$ precedes every moment in $t$ and $\alpha$ is true at $t'$.

(14) $P_{perfect}\alpha$ is true at $t$ iff there is some $t'$ such that $t$ is included in $t'$ and at least one moment in $t'$ precedes every moment in $t$, and no moment in $t'$ is after every moment in $t$.

Whether these operators reflect precisely the simple past and the perfect in English or any other language, they indicate the power of intervals. Another example is the English imperfective or 'progressive' operator. Thus in

(15) Bugsy is polishing all the boots

we are considering a moment of time which is part of an interval at which (3) is true. Suppose that Bugsy polishes all the boots over an interval $t^+$. That doesn't stop you being in the middle of this interval, and so you say that Bugsy is polishing all the boots. The relation associated with the imperfect in (15) might be just the subinterval relation, but as it happens the imperfect seems at least sometimes to involve other worlds, as in

(16) Bugsy was polishing all the boots, but collapsed and died before he had finished

where the interval at which (3) is true is not in the real world.[2]

What then of the argument which began this appendix, that you cannot privilege the present because it is an interval, but you *can* privilege the actual world because it is not? In assessing this observation we must distinguish the use of intervals as indices of truth, which therefore satisfy (1), and intervals as treated in Chapter 2 as families of indices, where the only genuine indices are instants. The 'specious present' only appears to be a problem if you simultaneously think of present truth as truth at an interval, but genuine truth as truth only at an instant. In interval semantics intervals themselves are genuine indices – and the equivalence results of Appendix 1 apply to all such indices.

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[2]Studies of this occur in such works as Dowty 1977. There are other interesting differences between the simple tenses and the progressive. Consider the difference between

(i) The pupils were standing when the teacher entered

and

(ii) The pupils stood when the teacher entered.

In (i) the standing was already in place before the entry, while (ii) at least suggests that the pupils were seated at the time of entry and subsequently stood.

It may be thought that the fact that one interval may be a part of another makes a difference. It is true that times seem to have a more precise structure than worlds; but if we think of worlds as histories we can speak of them as sharing a common part, and perhaps of one world being an extension of another. This is especially so when we recall that many 'modal' operators combine times and worlds. It is important to appreciate how the world-time parallel plays out in interval semantics. It is tempting to think that an interval corresponds with a *set* of worlds, where the individual worlds correspond to the moments which make up the interval. But that would be a mistake. In interval semantics it is the *interval itself* which corresponds to the world. The way in which an interval is made up out of moments is more like the way some philosophers make up a world out of states of affairs or propositions or some such. Just as the same moment can be a part of many intervals so the same state of affairs might be part of many worlds. And just as Aristotle is supposed to have claimed that moments, if they exist at all, are no more than limits of intervals, so some metaphysicians make worlds out of propositions or states of affairs or the like. So it seems that while interval semantics may make the phenomena of tense more complex it does not affect the claims made in this book.

# Appendix 5

## *Fatalism and the world-time parallel*

(with H. Kocurek[1])

This appendix discusses an argument against presentism in Rea 2006, that a presentist has to give up either bivalence or libertarianism. It is an interesting argument, since presentists like Prior seem to have supposed that it is eternalists who are committed to determinism. Admittedly Prior seems to want to give up bivalence, and in fact the search for a non-bivalent tense logic forms a large part of Chapters 9 and 10 of Prior 1957 (pp. 84-93 and 94-103), and Chapter 7 of Prior 1967 (pp. 113-136), though in Prior 1976, pp. 99-101, Prior suggests a solution that instead rejects the inevitability of past statements when they are ultimately about future events. Prior is in fact one of the most careful presentists, and in some ways anticipates Rea's argument. A particular advantage of looking at Prior's work is that he was insistent on presenting his views about time in systems of formal logic. We shall give an exposition of Rea's argument, and then compare it with an analogous argument against actualism, with the aim of showing that *if* Rea is successful against bivalent presentism his argument works equally against bivalent actualism.

Rea's 'main argument' (p. 518) works like this: Assume that all truth is present truth, and imagine that

(1) Sally is standing.

is now true and that, consequently

(2) 1000 years ago it was going to be the case that

[1]This appendix is based on work by Helga Kocurek, and presented to the 54th Annual Conference of the New Zealand Division of the Australasian Association of Philosophy at Auckland, New Zealand, in December 2007.

(3) Sally will stand in 1000 years.

All that is important about the 1000 years, is that, since Sally is assumed to be alive at the present time, she did not exist 1000 years ago.

Now Sally didn't exist 1000 years ago, and so, at least according to presentism as Rea understands it, no truth about her could then, as Rea puts it on p. 518 be

> even partly grounded in the occurrence of any event involving Sally or in any exercising of her agent-causal power.

So Sally had no choice 1000 years ago about whether (3) was true. But Rea assumes the principle

(M5) If the truth of a proposition $p$ at a past time $t'$ was not even partly grounded in the occurrence of any event involving S, or in any exercising of the agent-causal activity of S, then S has never had and never will have a choice about whether $p$ was true at $t'$. (Premise)[2]

What M5 does is claim that because Sally never had a choice 1000 years ago about whether (3) was true, she has no choice today about whether (2) is true, and therefore no choice about whether (1) is true. And the denial of choice about (1) is what contradicts her freedom.

Obviously M5 plays a crucial role in this argument, but it is not our intention to question it, and that for two reasons. There is no need for Rea to defend the premise, since all he requires is that a *presentist* be committed to it. Since we do not commit ourselves to any precise view about what presentism is we shall not address whether M5 is part of it. That is our first reason. Our second is that our aim is not to evaluate Rea's argument against presentism, but merely to present the modal parallel, so that we can claim that any version of actualism which accepts a modal equivalent of M5 will be subject to the same comments.

It will help to set the argument out formally, but to do that we need to extend the language of tense logic in a way which allows us to speak of 1000 years ago, and 1000 years hence, and to do that we have to make use of operators with an index which represents a temporal distance. What Prior always allowed (see Prior 1957 p. 11) was operators of the form *Pn* and *Fn* where, for any

[2]We have replaced Rea's $t_n$ by $t'$, since we will be following Prior in using $n$ as a variable for a temporal 'metric'.

formula α, *Pn*α means that α was true *n*-much ago, and *Fn*α means that α will be true *n*-much hence. To make the argument tighter we shall introduce two other symbols. Take $Gr_S$ and $C_S$ to be agent-relative sentential operators:

> $Gr_S\alpha$ = 'the truth of α is grounded in an event involving S or in an exercising of her agent-causal power'.
>
> $C_s\alpha$ = 'S has a choice about whether α is true'

Among the principles we shall need are

(MTL) (*PnFn*α ≡ α) is valid for any α and any *n* — so that *PnFn*α can be interchanged with α in any context

(*Pn*-distribution) *Pn*(α ⊃ β) ⊃ (*Pn*α ⊃ *Pn*β) is valid for any α, β and *n*.

In presenting Rea's argument in Priorian metric tense logic we shall assume that *n* is given the value of 1000 years and that Sally did not exist 1000 years ago. Then the claim that 1000 years ago it was not under Sally's control whether she would stand 1000 years later, i.e. the claim that (3) was not then under her control, may be represented as

(4) $Pn{\sim}Gr_{\text{Sally}}Fn(\textbf{\textit{Sally stands}})$.

Rea's M5 can then be represented as

(5) $Pn(\sim Gr_S\alpha \supset Fn{\sim}C_SPn\alpha)$.

(5) is a principle which holds for every *n* and every α. What (5) says is that, for any *n*, it was true *n*-much ago that if α was not grounded in S, then S would not be going to have a choice *n*-much later about whether α had been true *n*-much before.[3] In this case, since Sally did not ground any truths 1000 years ago, it would not be so 1000 years later (i.e. now) that she would now have any choice whether α had been true 1000 years before. We then proceed as follows:

(6) $PnFn{\sim}C_{\text{Sally}}PnFn(\textbf{\textit{Sally stands}})$ (From (4) and (5) with α as

[3]Here, and in what follows we use the phrase 'α is grounded in S' for Rea's phrase 'the truth of α is partly grounded in the occurrence of an event involving S or in the exercising of S's agent-causal power'.

*Fn*(***Sally stands***), by *Pn*-distribution)

(7) $PnFn\alpha \equiv \alpha$ (MTL)

(8) $\sim C_{\text{Sally}}$(***Sally stands***)

This argument is valid, so clearly any evaluation of it would have to concern itself with the status of the premises. In describing M5 Rea says on p. 519:

> M5 says, in short, that a person S has (or had or will have) a choice about a proposition *p* only if the truth of *p* is at least partly grounded in what S does.

The key phrase in this is the parenthetical remark 'or had or will have', because, if you look at (5) you will see that what it says is that *n*-much ago, if any $\alpha$ was not grounded *then* (in anything Sally did), this deprived Sally, not just of a choice *then* about $\alpha$, but also of a choice *n*-much later (when Sally *does* exist) about $Pn\alpha$. (5) is in fact the only premise which connects grounding at one time with choice at a later time, and we suspect that it may on its own be a statement of fatalism. (Note that (5) is weaker than Rea's M5 in one sense. M5 says that S *never* had or will have a choice about $Pn\alpha$. (5) says merely that S has no choice about $Pn\alpha$ *n*-much after the time when $\alpha$ is ungrounded. But the weaker version is enough for the argument.)

Be all that as it may, our interest is not so much in the plausibility or otherwise of the premises, but with what happens when you translate the argument into an argument about actualism. In this argument we can assume that the propositions involved are all temporally determinate propositions. In particular, suppose that 'Sally stands' is 'Sally stands at 3.20pm on Sunday 16 November 2009, GMT'.[4] In using metric tense logic it has been observed, at least since Prior 1957 p. 52, that there is no comparable ordering of worlds which would allow a natural modal metric. However, all that is required is that there be a pair of modal operators $M$ and $M^{-}$ which satisfy the following conditions:

| | |
|---|---|
| (MM) | ($MM^{-}\alpha \equiv \alpha$) is valid, so that $MM^{-}\alpha$ and $\alpha$ can be interchanged in any context |
| (*M*-distribution) | $M(\alpha \supset \beta) \supset (M\alpha \supset M\beta)$ and $M^{-}(\alpha \supset \beta) \supset (M^{-}\alpha \supset M^{-}\beta)$ |

[4]To avoid any appearance of begging questions against presentism we can assume that in the sentence 'stands' means 'either stood, or stands of will stand'.

are both valid.

$M$ and $M^{-}$ both look like 'possibility' operators, and one might question whether the distribution principle holds. But, like *Pn* and *Fn*, they are a very restricted kind of operator and satisfy principles of necessity operators as well as possibility operators. Just as *Pn* takes you back to a unique time, and *Fn* brings you back to the present, so, in possible worlds terms, $M$ takes you from one world to a unique other world, and $M^{-}$ brings you back. Now assume that

(9) $M\sim$(***Sally exists***)

Then we argue as follows:

(10) $MM^{-}$(***Sally stands***) (from (1) by MM)

Assuming that nothing is in the control of a non-existent person (9) and (10) give

(11) $M\sim Gr_{\text{Sally}}M^{-}$(***Sally stands***)

by $M$-distribution. The modal equivalent of M5 is

(12) $M(\sim Gr_{\text{S}}\alpha \supset M^{-}\sim C_{\text{S}}M\alpha)$

(13) $MM^{-}\sim C_{\text{Sally}}MM^{-}$(***Sally stands***) (From (11), (12) with $\alpha$ as $M^{-}$(***Sally stands***), by $M$-distribution)

(14) $\sim C_{\text{Sally}}$(***Sally stands***) (from (13) by MM)

Operators like $M$ and $M^{-}$ no doubt lack the naturalness of the corresponding temporal operators *Pn* and *Fn*, but they are certainly part of the arsenal of actualists like Adams, Plantinga and Stalnaker, who advocate the use of possible worlds made up out of things which exist in the actual world, using primitive truth and primitive modality. Adams for instance, as we saw of p. 53, understands a possible world to be a maximally possible set of propositions, which form a 'world-story'. For Adams, plain truth is equivalent to truth in the true world-story. This is not of course a definition of truth, since the actual truth of a world-story is determined by the truth simpliciter of the propositions in that world-story, but it does enable an actualist make sense of truth in a world. So let d be a 1-1 function which associates every world-story $w$ with another world-story $w'$; i.e.,

d is a function such that for $w$ and $w'$, $d(w) = d(w')$ iff $w = w'$. Ensure that the proposition that Sally exists is never a member of both $w$ and $d(w)$ for any $w$. Let $M$ and $M^{-}$ be operators whose meaning is specified by saying that $M\alpha$ is true according to a world-story $w$ iff the proposition expressed by $\alpha$ is a member of $d(w)$, and that $M^{-}\alpha$ is true according to a world-story $w$ iff the proposition expressed by $\alpha$ is a member of the unique $w'$ such that $w = d(w')$. Such a pair of operators would satisfy MM and *M*-distribution, whether or not they occur in any natural language.

At risk of repetition we shall exhibit the common logical structure of both arguments in a neutral way. Assume a 'point of origin', which we will simply call the *origin*. For the temporal argument the origin is the present time. For the modal argument it is the actual world. Now assume that this point has a *destination*. Although we use the word 'point', access to these 'points' must be available in a way which satisfies presentism in the one case, and actualism in the other case. In the temporal argument the destination is 1000 years ago, and is picked out by the metric operator $P_n$, when $n$ is 1000 years. In the modal version the origin's destination is picked out by $M$. $P_n$ is available to a presentist like Prior, and $M$ is available to an actualist like Adams, and there are converse operators $F_n$ and $M^{-}$, which have the property that $F_n\alpha$ and $M^{-}\alpha$ are each true at the destination iff $\alpha$ is true at the origin. The bivalence assumption is that both truth at the origin and truth at the destination make sense. Bivalent presentism must allow (present) truths about things which no longer exist. Rea (p. 515) acknowledges that presentists have to have a way of making sense of such truths. Similarly, bivalent actualism has to be able to analyse such truths as (33) on p. 78, or

(15) If Aristotle had not existed no one would have read his *Metaphysics*

which talks about what would be true of Aristotle (that no one has read his *Metaphysics*) in circumstances in which he does not exist. That such sentences have truth values is part of bivalent actualism, and is addressed by such actualist theories as Plantinga's theory of individual essences.[5] Assume that Sally stands

[5]An actualist might of course reject the modal argument by rejecting bivalence for worlds, yet accept Rea's argument by accepting bivalence for times. We have no comment to make about such a response, except to note that we can all agree that this is a possible response. However Adams 1974 at least is quite explicit (on. p. 190f of Loux 1979) that worlds are 'completely determinate', and the same would hold for Plantinga and Stalnaker. So bivalent actualism is not a straw-man position.

at the origin but that she doesn't even exist at the destination.

We shall set out both the temporal and the modal argument in two columns, one column representing what is true at the origin, and one representing what is true at the destination. In putting the matter this way we need to correct any misleading impression that we take truth at a point to be more basic than simple truth. For a presentist and an actualist truth is truth at the point of origin. That is where *Pn* and *M* become important. For the truth of α at the destination in the temporal case is the truth of *Pn*α at the point of origin, where truth at the origin is plain truth. In the modal case the truth of α at the destination is the truth of *M*α at the origin. The numbers in the following columns reflect the order in which the truth of each proposition at each point is established:

**I The temporal argument**

| Origin [the present time] | | Destination [1000 years ago] | |
|---|---|---|---|
| (a) | Sally stands | (b) | Sally doesn't exist |
| | | (c) | *Fn*(***Sally stands***) |
| (d) | *PnFn*(***Sally stands***) | (e) | *Fn*(***Sally stands***) is not grounded in Sally |
| | | (f) | *Fn*(Sally has no choice whether *PnFn*(***Sally stands***)) |
| (g) | *PnFn*(Sally has no choice whether *PnFn*(***Sally stands***)) | | |
| (h) | Sally has no choice whether Sally stands | | |

(a) and (b) are part of the assumptions of the example. Many of the other steps involve the equivalence of α at the destination with *Pn*α at the origin, and the equivalence of α at the origin with *Fn*α at the destination. (e) comes from (b) and (c) bearing in mind that 'true' in (e) means true at the destination — which can of course be expressed as plain truth (at the origin) using *Pn*.

We have already commented that *M* and $M^{-}$ are available to actualists like Adams, Plantinga and Stalnaker. In the temporal argument we assume that the same actual world is involved all the way through. In the modal argument you can either assume that there is a fixed moment of time and that the argument is couched in terms of what is happening or could have been happening at that time, or, as we have done above, you can restrict the argument to temporally fixed propositions which do not change their truth values in any given world with the

passage of time.

**II The modal argument**

| Origin [the actual world] | | Destination [the real world's *M*-twin] | |
|---|---|---|---|
| (a) | Sally stands | (b) | Sally doesn't exist |
| | | (c) | $M^{-}$(***Sally stands***) |
| (d) | $MM^{-}$(***Sally stands***) | (e) | $M^{-}$(***Sally stands***) is not grounded in Sally |
| | | (f) | $M^{-}$(Sally has no choice whether $MM^{-}$(***Sally stands***)) |
| (g) | $MM^{-}$(Sally has no choice whether $MM^{-}$(***Sally stands***)) | | |
| (h) | Sally has no choice whether Sally stands | | |

You can see, we hope, that the formally represented arguments really do exhibit an argument which can be stated without formal logic. But you will also see, we think, just how difficult it is to do so. Although we make no claims about the status of any of the premises of either argument we suspect that an actualist will probably take exception to the principle (12) on p. 247 above. Let's grant that at the destination no proposition α grounded in Sally is true. Why should it follow that in such a case it would not have been possible for it to be so that she *actually* has a choice about whether α is true? Sally in fact actually exists, and it seems at least imaginable to us that an actualist might well simply deny that any situation of the kind envisaged could limit her actual choices. Put more baldly, why should the mere fact that there could have been a world in which Sally does not exist limit her choices in *this* world? An actualist who makes this reply is questioning the move from (e) to (f) in argument II, and so is questioning (12). Here is how Rea's justification for a modal correlate of M5 might go.

> M5 says, in short, that a person S has (or could have had) a choice about a proposition *p* only if the truth of *p* is at least partly grounded in what S does.

This sounds innocuous until we look more carefully at the 'could have had'. We are imagining circumstances in which S doesn't exist, and so has no choices. But the 'could have had' refers to the possibility that S might have existed and makes

a claim about what would have been so then. Using our origin/destination terminology what this version of Rea's argument is claiming would be something like this, bearing in mind that an actualist can speak of what is true at the destination:

> Here is how things are at the destination: A proposition claiming what an agent S might have freely done could only be true if it was grounded in S. But suppose that S doesn't exist at the destination. Then, for an actualist, no proposition claiming what such an agent S (who does not exist) might freely have done had they existed would have been true.

Something like this last version is needed to make the argument work, and is the form it takes in (12) on p. 247 above. Rea's own statement of the (temporal) M5 is followed by discussing some possible objections. We have no quarrel with his replies to these objections because they seem objections to an innocuous way of taking his M5. But when the corresponding modal principle is unpacked as (12) it may well not be one which an actualist is likely to accept, and this raises the possibility of a presentist pointing out that Rea's M5 is not so obvious either. An actualist who questions (12) might go on to say that while they reject (12), they accept (5), and so, because of Rea's argument, they reject presentism. A presentist might reply:

> That's fine. Reject (12) and be an actualist. I have no problem with that. I don't even have a problem with your accepting (5). However, *I* happen to *reject* (5), because I can't see why the fact that it was once so that Sally didn't exist should limit her choices *now*, and so I don't accept that presentism leads to fatalism, any more than you accept that actualism leads to fatalism.

Whether these replies are adequate is not our concern, and we are of course taking no sides on the metaphysical positions — presentism, eternalism, bivalent actualism and possibilism — that we have mentioned. All we claim is that a presentist who wants to resist Rea's argument should look at how an actualist might resist the exactly parallel modal argument, because if it is plausible for an actualist to do so then it should be equally plausible for a presentist to resist the temporal argument.

# *Bibliography*


Adams, R.M., 1974, Theories of actuality, *Noûs*, Vol 8, pp 211-231, (Reprinted in Loux 1979, pp. 190-209)

— 1981, Actualism and thisness. *Synthese*, Vol 49, pp. 3-41

Åqvist, L. E. G. son, 1973, Modal logic with subjunctive conditionals and dispositional predicates. *Journal of Philosophical Logic*, Vol 2, pp. 1-26

Armstrong, D. M., 1989, *A Combinatorial Theory of Possibility*, Cambridge, Cambridge University Press

Bach, K., 1981, What's in a name. *Australasian Journal of Philosophy*, Vol 59, pp. 371-386

Baker, L.R., 2007, Temporal reality. *Time and Identity: Topics in Contemporary Philosophy, Vol 6*, ed. M. O'Rourke, Cambridge, Mass, MIT Press

Barcan, (Marcus) R.C., 1946, A functional calculus of first order based on strict implication. *The Journal of Symbolic Logic*, 11, 1-16

— 1961, Modalities and intensional languages. *Synthese*, Vol 13, pp. 303-322. [Reprinted as Chapter 1 of Marcus 1993, pp. 3-23.]

— 1962, Interpreting quantification. *Inquiry*, 5, 252-9

— 1993, *Modalities*, New York, Oxford University Press

Barwise, J., and J. Perry, 1983, *Situations and Attitudes*, Cambridge, Mass, MIT Press.

Benacerraf, P., 1973, Mathematical Truth, *The Journal of Philosophy*, Vol 70, pp. 661-679

Bigelow, J., 1991, Worlds enough for time. *Noûs*, Vol 25, pp. 1-19

— 1996, Presentism and Properties' *Noûs*, Vol 30, Supplement: *Philosophical Perspectives, 10, Metaphysics*, 35-52

Boolos, G., 1984, To be is to be the value of a bound variable (or to be some values of some variables). *The Journal of Philosophy*, Vol 81, pp. 449-450

Bourne, C., 2006. A Theory of Presentism. *Canadian Journal of Philosophy*, 1-24

— 2006, *A Future for Presentism*, Oxford, Clarendon Press

Bricker, P., 1989, Quantified modal logic and the plural *de re*. *Midwest Studies in Philosophy*, Vol 14: *Contemporary Perspectives in the Philosophy of Language* II, (ed. P. French *et al.)*, University of Notre Dame Press

— 2001, Island universes and the analysis of modality. in *Reality and Humean Supervenience: Essays on the Philosophy of David Lewis* , ed. G. Preyer and F. Siebelt, Lanham, Rowman and Littlefield

— 2008, Concrete possible worlds. In Sider, Hawthorne and Zimmerman, 2008, pp. 111-134

Brock, S., and E. D. Mares, 2007, *Realism and anti-realism*, Stocksfield, Acumen

Callender, C., 2002, *Time, Reality & Experience*, Cambridge, Cambridge University Press

Cappelen, H., and E. Lepore, 2005, *Insensitive Semantics*, Malden, Mass, Blackwell

Cappelen, H., and J. Hawthorne, 2009, *Relativism and Monadic Truth*, Oxford, Clarendon Press

Carlson, G.N., 1978, *Reference to Kinds in English*, Bloomington, Ind., Indiana University Linguistics Club

Carnap, R,, 1946, Modalities and quantification. *The Journal of Symbolic Logic*, 11, 33-64

— 1947, *Meaning and necessity*, Chicago, University of Chicago Press

Chalmers, D. J., 1996, *The Conscious Mind*, Oxford, Oxford University Press

Chihara, C.S., 1998, *The Worlds of Possibility: Modal Realism and the Semantics of Modal Logic*, Oxford, Clarendon Press

Chisholm, R.M., 1967, Identity through possible worlds: some questions. *Noûs*, Vol 1, pp. 1-8.

(Reprinted in Loux 1979, pp. 80-87.)
Church, A., 1936, An unsolvable problem of elementary number theory. *American Journal of Mathematics*, Vol 58, pp. 345-363
Cleve, J. van, 2008, The moon and sixpence: a defense of mereological universalism. In Sider, Hawthorne and Zimmerman, 2008, pp. 321-340
Copeland, B.J., 1997, The Church-Turing thesis. *Stanford Encyclopedia of Philosophy*
Craig, W.L., 2000a, *The Tensed Theory of Time*, Dordrecht, Kluwer
— 2000b, *The Tenseless Theory of Time*, Dordrecht, Kluwer
— 2001, *Time and the Metaphysics of Relativity*, Dordrecht, Kluwer
Cresswell, M.J., 1972, The world is everything that is the case. *Australasian Journal of Philosophy*, Vol 50, 1972, pp. 1-13, reprinted in Loux 1979, pp. 129-145
— 1973, *Logics and Languages*, London, Methuen
— 1985a, *Structured Meanings*, Cambridge, Mass, MIT Press, Bradford Books
— 1985b, *Adverbial Modification*, Dordrecht, Reidel
— 1985b, *Adverbial Modification*, Dordrecht, Reidel
— 1990a, Modality and Mellor's McTaggart. *Studia Logica*, Vol 49, pp. 163-170
— 1990b, *Entities and Indices*, Dordrecht, Kluwer
— 1994, *Language in the World*, Cambridge, Cambridge University Press
— 1996, *Semantic Indexicality*, Dordrecht, Kluwer
— 2006, Now is the time. *Australasian Journal of Philosophy*, Vol 84, pp. 311-332
— 2007, The Prior future. *Logique et Analyse*, No 199, pp. 289-302
— 2010, Predicate Wormism: A Quinean account of *de re* modality. *Logique et Analyse*, No. 212, pp. 449-464
Cresswell, M.J., and A.A.Rini, 2010, Are contingent facts a myth? *Analysis*, Vol 70, pp. 1-7
— 2011, Contingent facts: comments on Mellor's reply. *Analysis*, Vol 71, pp. 69-72
Crisp, T.M., 2003, Presentism, *The Oxford Handbook of Metaphysics*, ed M. J. Loux and D. W. Zimmerman. Oxford, Oxford University Press, pp. 211-245
Davidson, D., 1967, Truth and meaning. *Synthese*, Vol 17, pp. 304-323
Davies, M.K., 1978, Weak necessity and truth theories. *Journal of Philosophical Logic*, Vol 7, pp. 415-439
— 1981, *Meaning, Quantification, Necessity*. London, Routledge
— 1983, Actuality and context-dependence I. *Analysis*, Vol 43, pp. 128-133
Divers, J., 2002, *Possible Worlds*, London, Routledge
Dowty, D.R., 1977, Toward a semantic analysis of verb aspect and the English 'imperfective' progressive. *Linguistics and Philosophy*, Vol 1, pp. 45-77
— 1979, *Word Meaning and Montague Grammar: The Semantics of Verbs and Times in Generative Semantics and in Montague's PTQ*, Dordrecht, Reidel
Ducasse, C.J., 1941, Truth, verifiability, and propositions about the future. *Philosophy of Science*, Vol 8, pp. 329-337
Dyke, H., 1998, Real Times and Possible Worlds, in Le Poidevin 1998, pp. 93-117
— 2002a, McTaggart and the truth about time. In Callender 2002, pp. 137-151
— 2002b, Tokens, dates and tenseless truth conditions. *Synthese*, Vol 131, pp. 329-351
— 2003, Temporal Language and Temporal Reality, *The Philosophical Quarterly*, Vol 53, pp. 380-391
Egan, A., J. Hawthorne and B. Weatherson, 2005, Epistemic modals in context In Preyer and Peter 2005, pp. 131-168
Evans, G., 1985, Does tense logic rest upon a mistake? *Collected Papers*, Oxford, Clarendon

Press. pp. 343-363
Field, H., 1972, Tarski's theory of truth. *The Journal of Philosophy*, Vol 69, pp. 347-375
— 1978, Mental representation. Erkenntnis, Vol 13, pp. 9-61
— 1980, *Science Without Numbers: A Defence of Nominalism*, Oxford, Blackwell
Fine, K., 2005, *Modality and Tense: Philosophical Papers*, Oxford, Clarendon Press
Forbes, G., 1983, Actuality and context-dependence I. *Analysis*, Vol 43, pp. 123-128
— 1985, *The Metaphysics of Modality*, Oxford, Oxford University Press
— 1989, *Languages of Possibility*, Oxford, Basil Blackwell
Fraassen, B.C., 1977, The only necessity is verbal necessity. *The Journal of Philosophy*, Vol 74, pp. 71-85
Frege G., 1956, The thought, a logical enquiry. (A.M. and M. Quinton, trans) *Mind*, Vol 65, pp. 289-311
Gallois, A., *Occasions of Identity: A Study in the Metaphysics of Persistence, Change, and Sameness*, Oxford, Clarendon Press, New York, Oxford University Press, 1998
Garson, J.W., 1973, The completeness of an intensional logic: Definite topological logic. *Notre Dame Journal of Formal Logic*, Vol 14, pp. 175-184
Geach, P. T., 1949, Critical notice of Julius Weinberg, *Nicholas of Autricourt*, *Mind*, Vol 58, pp. 238-245
Goranko, V., 1996, Hierarchies of modal and temporal logics with reference pointers. *Language and Information*, Vol 5, pp. 1-24
Gupta, A., 1978, Modal logic and truth. *Journal of Philosophical Logic*, Vol 7, pp. 441-472
Hazen, A.P., 1976, Expressive completeness in modal language, *Journal of Philosophical Logic*, Vol 5, pp. 25-46
— 1979, Counterpart-theoretic semantics for modal logic. *The Journal of Philosophy*, Vol 76, pp. 319-338
— 1996, Actualism again. *Philosophical Studies*, Vol 84, pp. 155-181
Heller, M., 1990, *The Ontology of Physical Objects: Four-dimensional Hunks of Matter*, Cambridge, Cambridge University Press
Hinchliff, M., 1996, The puzzle of change. *Philosophical Perspectives*, Vol 10, (ed. J. Tomberlin), Blackwell, pp. 119-136
Hintikka, K.J.J., 1962, *Knowledge and Belief*, Ithica, Cornell University Press
Hornstein, N., 1990, *As Time Goes By: Tense and Universal Grammar*, Cambridge, Mass., MIT Press
Hughes, G.E. and M.J. Cresswell, 1996, *A New Introduction to Modal Logic*, London, Routledge
Inwagen, P., van, 1980, Indexicality and actuality. *The Philosophical Review*, Vol 89, pp. 403-426
— 1986, Two concepts of possible worlds, *Midwest Studies in Philosophy*, Vol 11, (ed) P. A., French, T.E. Uehling, Jr. and H.K. Wettstein, Minneapolis, University of Minnesota Press, pp. 185-213
Jackson, F., 1998, *From Metaphysics to Ethics: a Defence of Conceptual Analysis*. Oxford, Clarendon Press
Jubien, M., 1975, Ontological commitment to kinds. *Synthese*, Vol 31, pp. 85-106
Judson, L., 1983, Eternity and Necessity in *De Caelo* I.12, in *Oxford Studies in Ancient Philosophy* I, (ed. Julia Annas), Oxford, Clarendon Press, pp. 217-255
Kamp, J.A.W., 1971, Formal Properties of 'Now'. *Theoria*, Vol 40, pp. 76-109
Kaplan, D., 1978, Transworld heir lines. *The Possible and the Actual*, (ed. M. J. Loux), Ithica, Cornell University press, 1979, pp. 88-109

— 1979, On the logic of demonstratives. *Journal of Philosophical Logic*, Vol 8, pp. 81-98
King, J.C., 2003, Tense, modality, and semantic values. *Philosophical Perspectives* 17, pp. 195-245
Kleene, S.C., 1967, *Mathematical Logic*, New York, John Wiley & Sons
Kratzer, A.H.E.S., 1977, What 'must' and 'can' must and can mean. *Linguistics and Philosophy*, Vol 1, pp. 113-126
— 1995, Stage-level and individual-level predicates, in *The Generic Book* ed, G. N. Carlson and F. J. Pelletier, Chicago, University of Chicago Press, pp. 125-175
Kripke, S.A., 1959, A completeness theorem in modal logic. *The Journal of Symbolic Logic*, Vol 24, pp. 1–14
— 1963, Semantical considerations on modal logics. *Acta Philosophica Fennica – Modal and Many-valued Logics*, 83–94
— 1972a, Naming and necessity. *Semantics of Natural Language* (ed. D. Davidson and G. Harman), Dordrecht, Reidel, pp. 253-355
— 1972b, Identity and necessity. *Identity and individuation*, (ed. M.Munitz) New York, New York University Press
Kuhn, S.T., 1980, Quantifiers as Modal Operators, *Studia Logica*, Vol 39, pp. 145-158
Kutschera, F., von, 1997, T × W completeness. *Journal of Philosophical Logic*, Vol 26, pp. 241-250
Le Poidevin, R., 1991, *Change, Cause and Contradiction: A Defence of the Tenseless Theory of Time*, Basingstoke, Macmillan (in association with the Scots Philosophical Club)
— 2003, Why tenses need real times. In Jokić and Smith, 2003, pp. 305-324
Lewis, D. K., 1968, Counterpart theory and quantified modal logic, *The Journal of Philosophy*, Vol 65, pp. 113-126
— 1970, Anselm and actuality. *Noûs*, Vol 4, pp. 175-188
— 1972, General semantics. *Semantics of Natural Language* (ed. D. Davidson and G. Harman), Dordrecht, Reidel, pp. 169-218
— 1973a, *Counterfactuals*, Oxford, Basil Blackwell
— 1973b, Causation. *The Journal of Philosophy*, Vol 70, pp. 556-567 (Reprinted in Lewis 1986b with Appendices.)
— 1974, Tensions, In *Semantics and philosophy* ed. M.K. Munitz and P. K. Unger, New York, New York University Press, 1974., pp. 49-61
— 1976, The paradoxes of time travel. *American Philosophical Quarterly*, Vol 13, pp. 145-152
— 1978, Truth in fiction. *American Philosophical Quarterly*, Vol 15, pp. 37-46
— 1979a, Attitudes *de dicto* and *de se*. *The Philosophical Review*, Vol 88, pp. 513-543
— 1979b, Counterfactual dependence and time's arrow. *Noûs*, Vol 13, pp. 455-476
— 1986a, *On the Plurality of Worlds*, Oxford, Basil Blackwell
— 1986b, Events, *Philosophical Papers*, Vol II, New York, Oxford University Press, pp. 241-269
Linsky B., and E.N. Zalta, 1994, In defense of the simplest quantified modal logic. *Philosophical Perspectives 8: Logic and Language*, ed. J. Tomberlin, Atascaredo, Ridgeview Press, pp. 431-458
Loux, M. J. (ed) 1979, *The Possible and the Actual* Ithica, Cornell University Press
— 2002, *Metaphysics: A Contemporary Introduction*. London, Routledge, Second Edition
Lucas, J.R., 1989, *The Future*. Oxford, Basil Blackwell

Ludlow, P., 1999, *Semantics, Tense, and Time: An Essay in the Metaphysics of Natural Language*, Cambridge, Mass., MIT Press

Łukasiewicz, J., 1922, O determiniźmie. Translated by Z. Jordan in McCall 1967, pp. 19-39, at 38f, as 'On determinism'

Lycan, W.G., 1979, The trouble with possible worlds. *The Possible and the Actual* (ed. M. J. Loux), Ithica, NY, Cornell University Press, pp. 274-316

— 1991, Pot bites kettle: a reply to Miller, *Australasian Journal of Philosophy*, Vol 69, pp. 213f

— 1994, *Modality and Meaning*, Dordrecht, Kluwer

— 2000, *Philosophy of Language: A Contemporary Introduction*, London, Routledge

McCall, S., (ed), 1967, *Polish Logic*, Oxford, Clarendon Press

— 1994, *A Model of the Universe: Space-time, Probability, and Decision*, Oxford, Clarendon press

MacFarlane, J., 2003, Future contingents and relative truth. *Philosophical Quarterly*, Vol 53, pp. 321-336

— 2005, Making sense of relative truth. *Proceedings of the Aristotelian Society*, Vol 105, pp. 305-323

— 2007. Semantic minimalism and nonindexical contextualism. In Pryer and Peter 2007, pp. 240-250

— 2009, Nonindexical contextualism. *Synthese*, Vol 166, pp. 231-250

McGinn, C., 1981, Modal reality. *Reduction, Time and Reality* (ed. R. Healy), Cambridge, Cambridge University Press, pp. 143-187.

McGrath, M., 1998, The Concrete Modal Realist Challenge to Platonism, *Australasian Journal of Philosophy*, Vol 76, pp. 587-610

Mackie, P., 2006, *How Things Might Have Been*, Oxford, Clarendon Press

McTaggart, J.M.E., 1927, *The Nature of Existence*, Cambridge, Cambridge University Press

Maddy, P., 1996, The legacy of 'mathematical truth'. *Benacerraf and his Critics*, A. Morton and S.P. Stich, (eds), Cambridge, Blackwell, pp. 60-72

Markosian, N., 2008, Restricted composition. In Sider, Hawthorne and Zimmerman, 2008, pp. 341-363

Melia, J., 2003, *Modality*, Acumen Publishing

— 2008, Ersatz possible worlds. In Sider, Hawthorne and Zimmerman, 2008, pp. 135-151

Mellor, D. H., 1981, *Real Time*, Cambridge, Cambridge University Press

— 1986, Tense's Tenseless Truth Conditions, *Analysis*, Vol 46, pp. 167-172

— 1987, The singularly affecting facts of causation. *Metaphysics and Morality* (ed. P. Pettit *et al*) Oxford, Blackwell, (Reprinted in *Matters of Metaphysics*, Cambridge, Cambridge University Press, 1991, pp. 201-224.)

— 1995, *The Facts of Causation*, London, Routledge

— 1998, *Real Time II*, London, Routledge

— 2011, Contingent facts: a reply to Cresswell and Rini. *Analysis*, Vol 71, pp.62-68

Mendelson., E., 1964, *Introduction to Mathematical Logic*, Princeton, van Nostrand

Menzel, C., 1989, On an unsound proof of the existence of possible worlds. *Notre Dame Journal of Formal Logic*, Vol 30, pp. 598-603

— 1991, Temporal actualism and singular foreknowledge. *Philosophical Perspectives 5, Philosophy of Religion*, ed. J. Tomberlin, pp. 475-507

Meyer, U., 2006, Worlds and times. *Notre Dame Journal of Formal Logic*, Vol 47, pp. 25-37

— 2009, 'Now' and 'then' in tense logic. *The Journal of Philosophical Logic*, Vol 38, pp.

229-247
Mondadori, F., and A. Morton, 1976, Modal realism: the poisoned pawn, *The Philosophical Review*, Vol 85, pp. 3-20. (Reprinted in Loux 1979, pp. 235-252.)
Montague, R.M., 1974, *Formal Philosophy*, New Haven, Yale University Press
Müller, T, 2007, Prior's tense-logical universalism, *Logique et Analyse* No 199, pp. 223-252
Nolan, D.P., 2002, *Topics in the Philosophy of Possible Worlds*, New York, Routledge
Parsons, J., 2005, I am not now, nor have I ever been, a turnip. *Australasian Journal of Philosophy*, Vol 83, pp. 1-14
Parsons, T., 1980, *Non-existent Objects*, New Haven, Yale University Press
Peacocke, C., 1978, Necessity and truth theories. *Journal of Philosophical Logic*, Vol 7, pp. 473-500
Percival, P., 1989, Indices of truth and temporal propositions. *The Philosophical Quarterly*, Vol 39, pp. 190-199
— 1990, Indices of truth and intensional operators. *Theoria*, Vol 56, pp. 148-172
— 2002, A presentist's refutation of Mellor's McTaggart. In Callender 2002, pp. 91-118
Perry, J., 1977, Frege on demonstratives. *The Philosophical Review*, Vol 86, pp. 474-497. (Reprinted in *Demonstratives*, ed. P. Yourgrau, Oxford, Oxford University Press, pp. 50-70.)
— 1979, The problem of the essential indexical, Noûs, Vol 13, pp. 3-21
Perszyk, K.J., 1993, Against extended modal realism. *Journal of Philosophical Logic*, Vol 22, pp. 205-214
Plantinga, A., 1976, Actualism and Possible Worlds, *Theoria*, Vol 42, pp. 139-160 (Reprinted in Loux 1979, pp. 253-273)
— 1985, Self-profile, in Tomberlin and Van Inwagen 1985, pp. 3-97, and Replies to my colleagues on pp. 313-396
Pollock, J.L., 1985 Plantinga on possible worlds. Tomberlin and Van Inwagen 1985, pp. 121-144)
Preyer, G., and G. Peter, 2005, *Contextualism in Philosophy*, Oxford, Clarendon Press
— 2007, *Context-Sensitivity and Semantic Minimalism*, Oxford, Clarendon Press
Priest, G.G., 1986, Tense and Truth Conditions, *Analysis*, Vol 46, pp. 162-166
— 1987, Tense, *tense* and TENSE, *Analysis*, Vol 47, pp. 184-187
Prior, A.N., 1956, Modality and Quantification in S5, *The Journal of Symbolic Logic* 21, pp. 60-2
— 1957, *Time and Modality*, Oxford, Oxford University Press
— 1959, Thank Goodness That's Over, *Philosophy*, Vol 34, pp. 12-17
— 1960, Identifiable individuals. *Review of Metaphysics*, Vol 13, pp. 684-696. [Reprinted in Prior 2003, pp. 81-92.]
— 1967, *Past, Present and Future*, Oxford, Oxford University Press
— 1968a, 'Now'. *Noûs* 12, pp. 191-207 [Reprinted as chapter 14 of Prior 2003, pp. 171-193.]
— 1968b, Fugitive truth. *Analysis*, Vol 29, pp 5-8. [Reprinted as chapter 2 of Prior 2003, pp. 22-25.]
— 1968c, Modal logic and the logic of applicability, *Theoria*, Vol 34, pp. 183-202. [Reprinted as chapter 20 of Prior 2003, pp. 275-292.]
— 2003, *Papers on Time and Tense*, (New edition, ed, P. Hasle, P, Øhrstrøm, T. Braüner and J. Copeland), Oxford, Oxford University Press
Prior, A.N., and K. Fine, 1977, *Worlds, Times and Selves*, London, Duckworth
Quine, W.V.O., 1940, *Mathematical Logic*, Cambridge, Mass, Harvard University Press (second edition, 1951)

— 1948, On what there is. *Review of Metaphysics*, Vol 1 (Reprinted in Quine 1953a, pp. 1-19)
— 1953a, *From a Logical Point of View* Cambridge, Mass, Harvard University Press (revised edition 1961)
— 1953b, Mr. Strawson on logical theory. *Mind*, Vol 62, pp. 440-443
— 1960, *Word and Object*, Cambridge, Mass, MIT Press
— 1961, Reply to Professor Marcus, *Synthese*, Vol 13, pp. 323-330
— 1969, Ontological relativity. *Ontological Relativity and Other Essays*, New York, Columbia University Press, pp. 26-68
— 1976a, Worlds away. *The Journal of Philosophy*, Vol 73, pp. 859-863 (Reprinted in *Theories and Things*, Cambridge, Mass., Harvard University Press, 1981, pp. 124-128)
— 1976b, *The Ways of Paradox and other Essays*, Cambridge, Mass. Harvard University Press
— 1976c, Whither Physical Objects?, R.Cohen et al. (Eds), *Essays in Memory of Imre Lakatos*, Dordrecht, Reidel, 497-504
Rea, M. C. 1997, (ed) *Material Constitution: A Reader*, Lanham, Rowman & Littlefield
— 2003, Four-Dimensionalism, *The Oxford Handbook of Metaphysics*, ed M. J. Loux and D. W. Zimmerman. Oxford, Oxford University Press, pp. 246-280
— 2006. Presentism and Fatalism, *Australasian Journal of Philosophy*, Vol 84, pp. 511-24;
Reichenbach, H., 1947, *Elements of Symbolic Logic*, New Yor, McMillan
Rescher, N., 1966, On the logic of chronological propositions, *Mind*, Vol 75, pp. 75-96
Rescher, N., and J.W.Garson, 1968, Topological logic, *The Journal of Symbolic Logic*, Vol 33, pp. 357-548
Richard, M., 1982, Tense, propositions and meanings. *Philosophical Studies*, Vol 41, pp. 337-351
— 1990, *Propositional Attitudes: An Essay on Thoughts and how we Ascribe them*, Cambridge, CUP
Richards, T.J., 1975, The worlds of David Lewis, *Australasian Journal of Philosophy*, Vol 53, pp. 105-118
Rini, A.A., 1998, Is There a Modal Syllogistic?, *Notre Dame Journal of Formal Logic*, Vol 39, pp. 554-572
— 2003, When Time is of the Essence: *Prior Analytics* I.15 and *De Caelo* I.12, *Logique et Analyse*, No. 183-184, pp. 419-440
— 2007, Outside Worlds, Outside Times, *Logique et Analyse* No. 199, pp. 1-6
Rini, A.A., and M.J.Cresswell, 2009, On a misapplication of the world-time parallel. *Logique et Analyse*, No. 206, pp. 125-130
Rosen, G., 1990, Modal Fictionalism, *Mind*, Vol 99, pp. 327-354
— 1993, A Problem for Fictionalism About Possible Worlds. *Analysis*, Vol 53, pp. 71-81
— 1995, Modal Fictionalism Fixed. *Analysis*, Vol 55, pp. 67-73
Russell, B.A.W., 1905, On denoting. *Mind*, Vol 14, pp. 479-493
— 1915, On the experience of time, *The Monist* 25: 212-233. (Reprinted as Chapter 6 of Russell 1984, pp. 64-78.)
— 1918, The Philosophy of Logical Atomism. In *Russell's Logical Atomism*, (ed. D. Pears), London, Collins, 1972
— 1919, *Introduction to Mathematical Philosophy*, London, George Allen and Unwin
— 1984, *The Collected Papers of Bertrand Russell*, Vol 7, London, George Allen and

Unwin
Salmon, N., 1986, *Frege's Puzzle*, Cambridge, Mass, MIT Press
— 1987, Existence. *Philosophical Perspectives, 1 Metaphysics* (ed. J.E. Tomberlin), Atascadero, Ridgeview Publishing Co., pp. 49-108
— 2003, Tense and intension. in Jokić and Smith, 2003, pp. 107-154
Schiffer, S., 1987, *Remnants of Meaning*, Cambridge, Mass. MIT Press
Scott, D.S., 1970, Advice on modal logic. *Philosophical Problems in Logic*, ed. K. Lambert, Dordrecht, Reidel, pp. 143-173
Sider, T., 1996, All the world's a stage. *Australasian Journal of Philosophy*, Vol 74, pp. 433-453
— 1999a, Presentism and ontological commitment. *The Journal of Philosophy*, Vol 96, pp. 325-347
— 1999b, Global Supervenience and Identity Across Times and Worlds, *Philosophy and Phenomenological Research* 49(4), pp. 913-937
— 2000, The stage view and temporary intrinsics. *Analysis*, Vol 60, pp. 84-88
— 2001, *Four Dimensionalism: An Ontology of Persistence and Time*, Oxford, Clarendon Press
— 2003, Reductive theories of modality. in Michael J. Loux and Dean W. Zimmerman (eds.) *The Oxford Handbook of Metaphysics*. Oxford, Oxford University Press, 2003: 180-208
Sider, T., J. Hawthorne and D.W. Zimmerman, (eds) 2008, *Contemporary Debates in Metaphysics*, Oxford, Blackwell
Skyrms, B., 1976, Possible worlds, physics and metaphysics. *Philosophical Studies*, Vol 30, 323-332
Smart, J.J.C., 1955, Spatialising time. *Mind*, Vol 64, pp. 239-241
— 1987, Replies. *Metaphysics and Morality: Essays in Honour of J.J.C. Smart*, Oxford, Blackwell, pp. 173-195
— 2008, The tenseless theory of time. In Sider, Hawthorne and Zimmerman, 2008, pp. 226-238
Smith, Q., 1993, *Language and Time*, New York, Oxford University Press
Smullyan, A.F., 1948, Modality and description. *The Journal of Symbolic Logic*, 13, 31–7
Stalnaker, R.C., 1968, A theory of conditionals. *Studies in Logical Theory* (ed. N. Rescher), Oxford, Basil Blackwell, pp. 98-112
— 1984, *Inquiry*, Cambridge, Mass, MIT Press, Bradford Books
— 1989, On what's in the head. *Philosophical Perspectives* 3 *Philosophy of Mind and Action Theory* (ed. J. E. Tomberlin) Atascadero CA, Ridgeview Publishing Co, pp. 287-316
— 1996, On what possible worlds could not be. *Benacerraf and his Critics*, A. Morton and S.P. Stich, (eds), Cambridge, Blackwell, pp. 103-118. (Reprinted in Stalnaker 2003, pp. 40-54
— 2003, *Ways a World Might Be*, Oxford, Clarendon Press
Tarski, A., 1936, The concept of truth in formalized languages. *Logic, semantics and Metamathematics*, Oxford, Clarendon Press, 1956, pp. 152-278
Thomason, R. H., 1970, Indeterminist time and truth value gaps. *Theoria*, Vol 36, pp. 264-281
— 1976, Necessity, quotation, and truth: an indexical theory. *Language in Focus*, (ed. A Kasher) Dordrecht, Reidel, pp. 119-138
— 1984, Combinations of tense and modality. *Handbook of Philosophical Logic* (ed. F. Guenthner and D.M. Gabbay), Vol II, Dordrecht, Reidel, 1984, pp. 135-165

Tomberlin, J., and P. Van Inwagen (eds) 1985, *Alvin Plantinga*, Dordrecht, Reidel

Tooley, M., 1997, *Time, Tense, and Causation*, Oxford, Clarendon Press

Wagner, S., 1996, Prospects for Platonism, *Benacerraf and his Critics*, A. Morton and S.P. Stich, (eds), Cambridge, Blackwell, pp. 73-99

Wahlberg, T.H., 2010, The tenseless copula in temporal predication. *Erkenntnis*, Vol 72, pp. 267-280

Williamson, T., 1999, Logic and existence. *Proceedings of the Aristotelian Society*, Supplementary, Vol 73, pp. 181-203

Wilson, N.L., 1959 Substances without substrata. *Review of Metaphysics*, Vol 12, pp. 521-539

Wölfl, S., 1999, Combinations of tense and modality for predicate logic. *Journal of Philosophical Logic*, Vol 28, pp. 371-398

Wittgenstein, L., 1921, *Tractatus Logic-Philosophicus*. (Translated by D.F.Pears and B.F.McGinness), 2nd printing 1963. London, Routledge and Kegan Paul

— 1953, *Philosophical Investigations*, Oxford, Blackwell

Wright, C., 1980, *Frege's Conception of Natural Numbers as Objects*, Aberdeen, Aberdeen University Press

Yagisawa, T., 1988, Extended modal realism. *Philosophical Studies*, Vol 53, pp. 175-204

— 2008, Modal realism with modal tense. *Australasian Journal of Philosophy*, Vol, 86, pp. 309-327

Yourgrau, P., 1986, On Time and Actuality; The Dilemma of the Privileged Position, *British Journal for the Philosophy of Science*, Vol 37 pp. 405-417

Zimmerman, D., 2004, (ed), *Oxford Studies in Metaphysics, Vol 1*, Oxford, Oxford University Press

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